

# Correctness and Verification using Software Contracts

Thomas Gilray

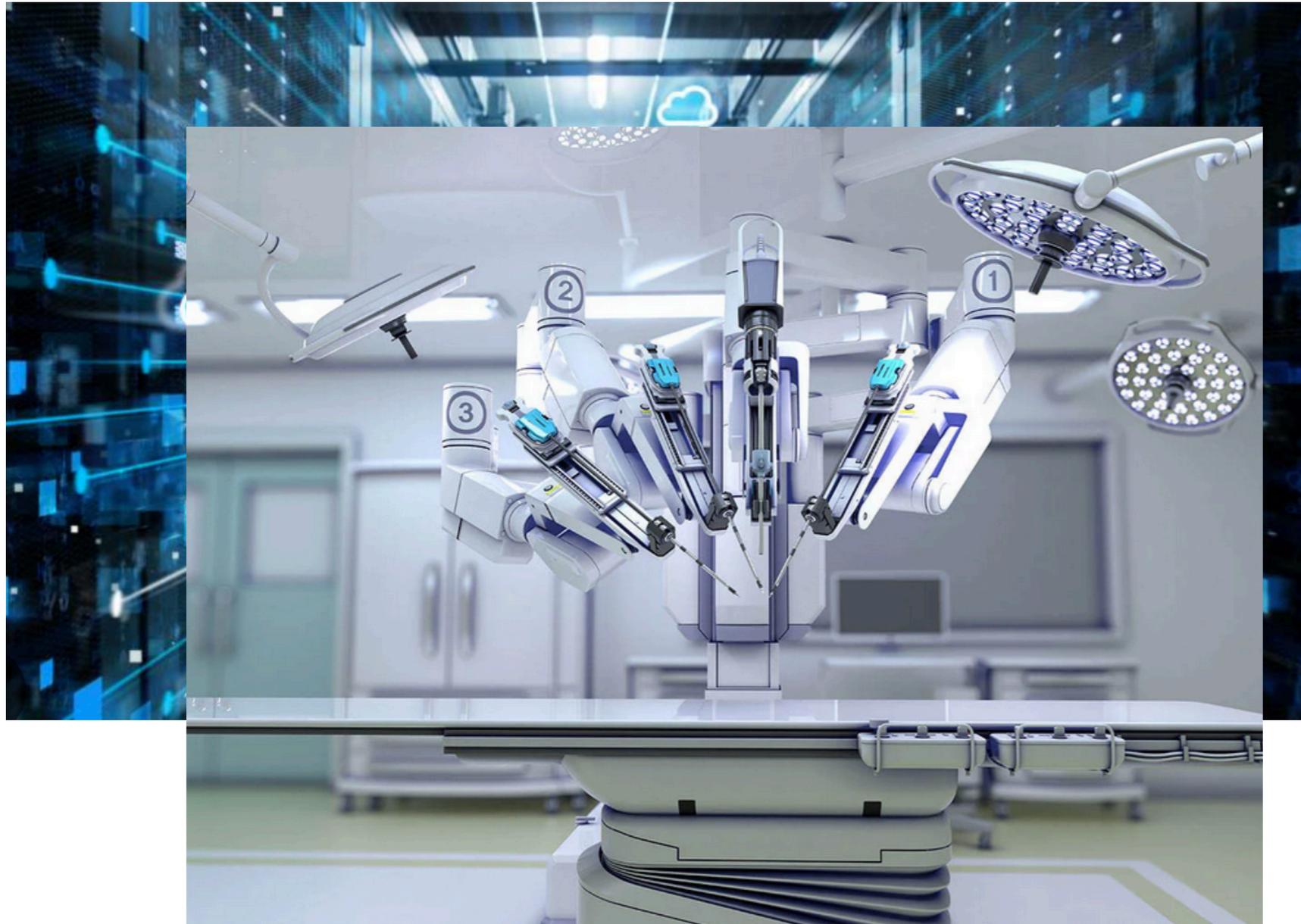
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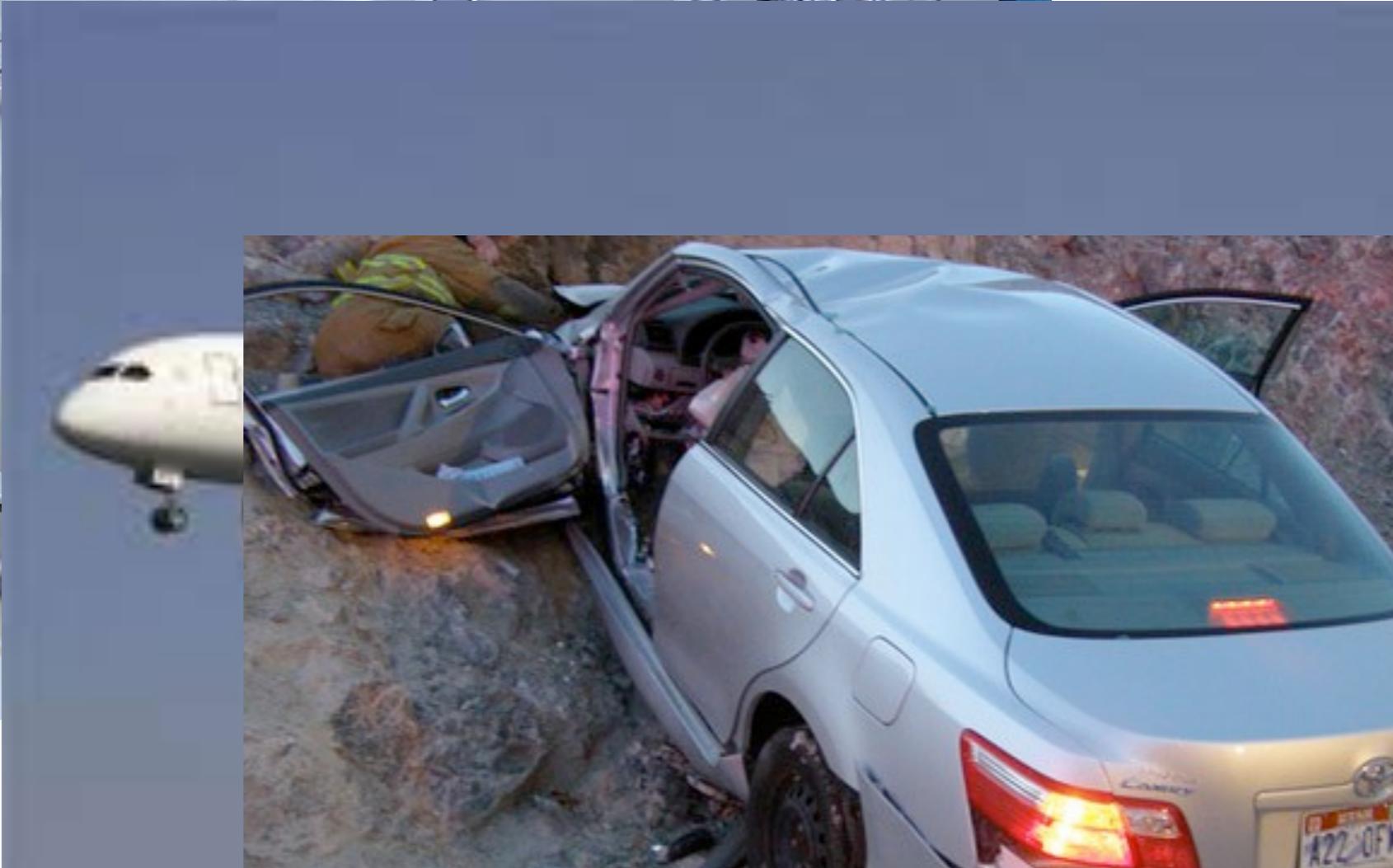
**WASHINGTON STATE**  
UNIVERSITY











“riddled with bugs”

“spaghetti-like”

<http://www.safetyresearch.net/blog/articles/toyota-unintended-acceleration-and-big-bowl-%E2%80%9Cspaghetti%E2%80%9D-code>

What to do about “worse is better”?

Richard P. Gabriel. “The rise of worse is better.”

## **Firm/discrete concerns**

- Correctness properties
- Security/Robustness
- Can the program “go wrong” — exhibit a class of bugs, vulnerabilities

## **Soft/continuous concerns**

- Simplicity/Maintainability
- Time to market
- Performance/Optimization
- Feature set/Completeness

# “Worse is better”

## **Soft/continuous concerns**

- Correctness properties
- Security/Robustness
- Can the program “go wrong” — exhibit a class of bugs, vulnerabilities

## **Firm/discrete concerns**

- Simplicity/Maintainability
- Time to market
- Performance/Optimization
- Feature set/Completeness

# Test-driven Development

# Test-driven Development



Test 0



Test 1

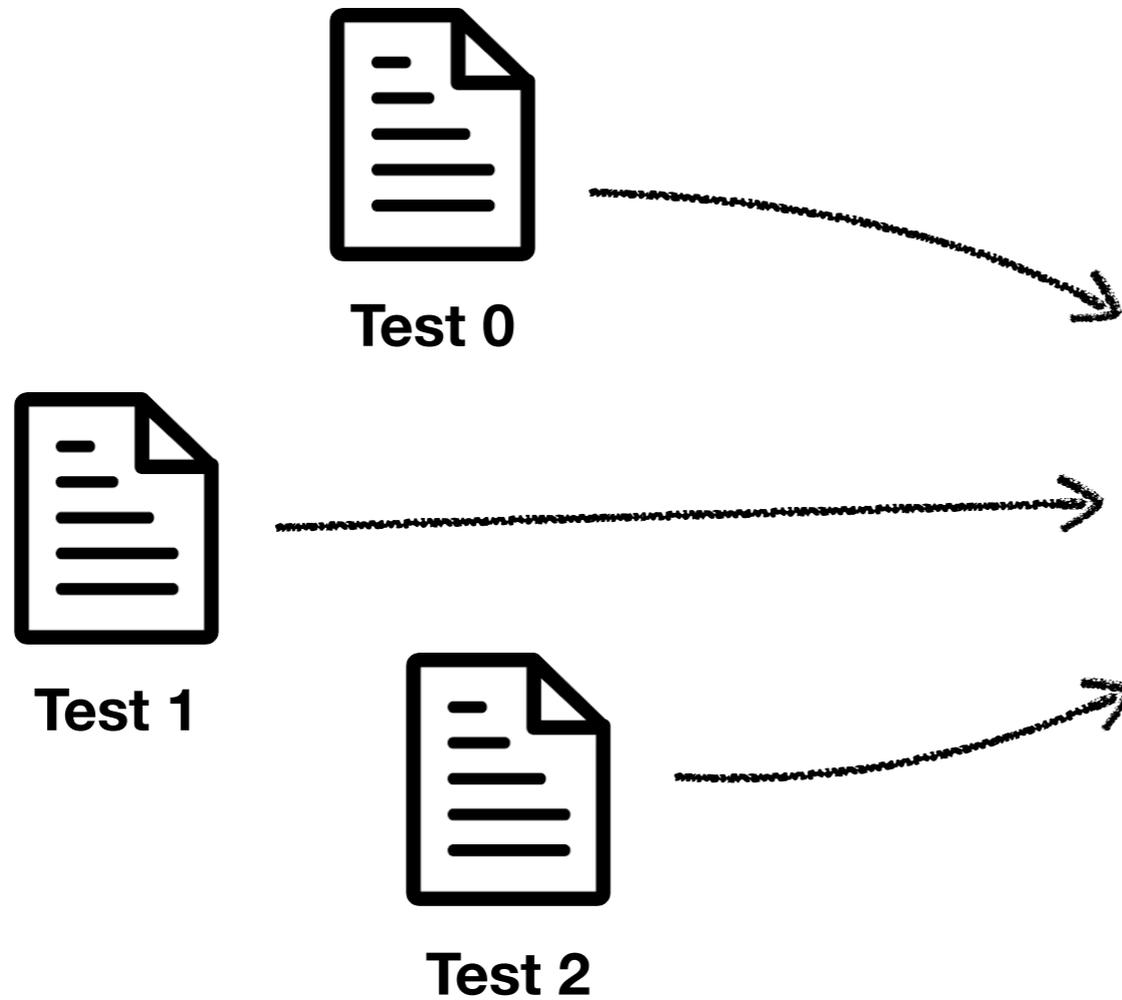


Test 2



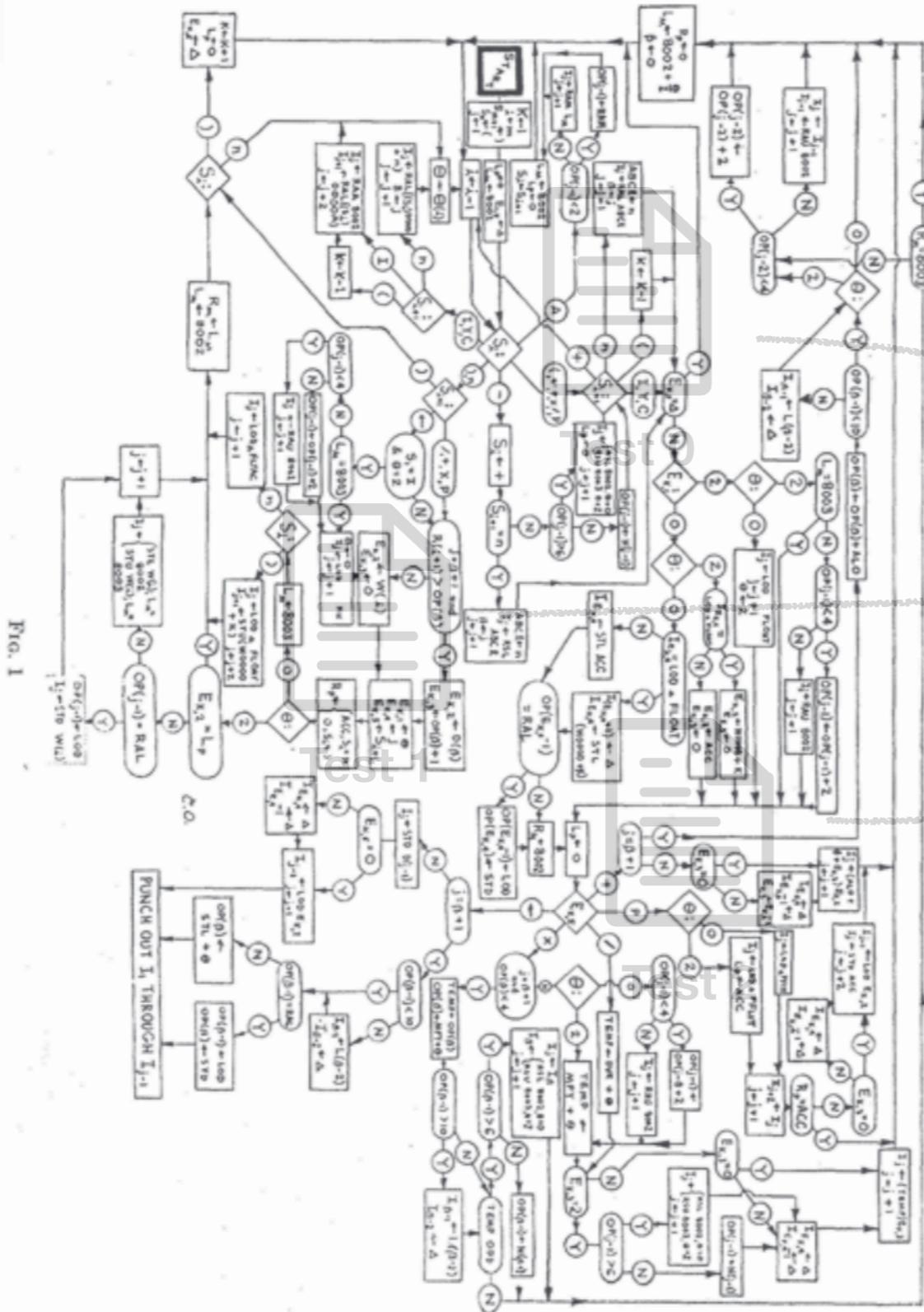


# Specification-driven Development



```
2 }  
3     else if ([keyPath isEqualToString:kAttributedTextKey]) {  
4         self._placeholderTextView.font = [change valueForKey:NSKeyValueChangeNewKey]  
5     }  
6     else if ([keyPath isEqualToString:kAttributedString] & newAttributedString = [change valueForKey:NSKeyValueChangeNewKey]) {  
7         NSAttributedString *newAttributedString = [change valueForKey:NSKeyValueChangeNewKey].string;  
8         [self setPlaceholderVisibleForText:newAttributedString];  
9     }  
10    else if ([keyPath isEqualToString:kTextKey]) {  
11        NSString *newText = [change valueForKey:NSKeyValueChangeNewKey];  
12        [self setPlaceholderVisibleForText:newText];  
13    }  
14    else if ([keyPath isEqualToString:kExclusionPathsKey]) {  
15        self._placeholderTextView.textContainer.exclusionPaths = [change valueForKey:NSKey  
16        [self resizePlaceholderFrame];  
17    }  
18    else if ([keyPath isEqualToString:kLineFragmentPaddingKey]) {  
19        self._placeholderTextView.textContainer.lineFragmentPadding = [change valueForKey:  
20        [self resizePlaceholderFrame];  
21    }  
22    else if ([keyPath isEqualToString:kTextContainerInsetKey]) {  
23        self._placeholderTextView.textContainer.inset = [change valueForKey:  
24        [self resizePlaceholderFrame];  
25    }  
26    else if ([keyPath isEqualToString:kTextAlignmentKey]) {  
27        NSValue *value = [change valueForKey:NSKeyValueChangeNewKey];  
28        self._placeholderTextView.textContainerInset = value.CGRectInsetValue;  
29    }  
30    else if ([keyPath isEqualToString:kTextAlignmentKey]) {  
31        NSNumber *alignment = [change valueForKey:NSKeyValueChangeNewKey];  
32        self._placeholderTextView.textAlignment = alignment.intValue;  
33    }  
34    else if ([keyPath isEqualToString:kTextAlignmentKey]) {  
35        self._placeholderTextView.textAlignment = alignment.intValue;  
36    }  
37    }  
38 }
```

# Specification-driven Development



```
2 } else if ([keyPath isEqualToString:kAttributedTextKey]) {  
3     self._placeholderTextView.font = [change valueForKey:NSKeyValueChangeNewKey];  
4 } else if ([keyPath isEqualToString:kAttributedString] {  
5     NSAttributedString *newAttributedString = [change valueForKey:NSKeyValueChangeNewKey];  
6     [self setPlaceholderVisibleForText:newAttributedString];  
7 } else if ([keyPath isEqualToString:kTextKey]) {  
8     NSString *newText = [change valueForKey:NSKeyValueChangeNewKey];  
9     [self setPlaceholderVisibleForText:newText];  
10 } else if ([keyPath isEqualToString:kExclusionPathsKey]) {  
11     self._placeholderTextView.textContainer.exclusionPaths = [change objectForKey:NSKeyValueChangeNewKey];  
12     [self resizePlaceholderFrame];  
13 } else if ([keyPath isEqualToString:kLineFragmentPaddingKey]) {  
14     self._placeholderTextView.textContainer.lineFragmentPadding = [change objectForKey:NSKeyValueChangeNewKey];  
15     [self resizePlaceholderFrame];  
16 } else if ([keyPath isEqualToString:kTextContainerInsetKey]) {  
17     self._placeholderTextView.textContainer.inset = [change objectForKey:NSKeyValueChangeNewKey];  
18     [self resizePlaceholderFrame];  
19 } else if ([keyPath isEqualToString:kTextContainerInsetInsetKey]) {  
20     NSInteger *value = [change objectForKey:NSKeyValueChangeNewKey];  
21     self._placeholderTextView.textContainerInsetInset = value; [self resizePlaceholderFrame];  
22 } else if ([keyPath isEqualToString:kTextAlignmentKey]) {  
23     NSTextAlignment *value = [change objectForKey:NSKeyValueChangeNewKey];  
24     self._placeholderTextView.textAlignment = value; [self resizePlaceholderFrame];  
25 }
```

# Contract-driven Development

*"design by contract"*

```
2 }
3   else if ([keyPath isEqualToString:kAttributedTextKey]) {
4     self._placeholderTextView.font = [change valueForKey:NSKeyValueChangeNewKey];
5   }
6   else if ([keyPath isEqualToString:kAttributedStringKey]) {
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8     [self setPlaceholderVisibleForText:newAttributedString];
9   }
10  }
11  else if ([keyPath isEqualToString:kTextKey]) {
12    NSString *newText = [change valueForKey:NSKeyValueChangeNewKey];
13    [self setPlaceholderVisibleForText:newText];
14  }
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19  }
20  }
21  else if ([keyPath isEqualToString:kLineFragmentPaddingKey]) {
22    self._placeholderTextView.textContainer.lineFragmentPadding = [change objectForKey:NSKeyValueChangeNewKey];
23    [self resizePlaceholderFrame];
24  }
25  }
26  else if ([keyPath isEqualToString:kTextContainerInsetKey]) {
27    NSValue *value = [change objectForKey:NSKeyValueChangeNewKey];
28    self._placeholderTextView.textContainerInset = value.unsignedIntegerValue;
29  }
30  }
31  else if ([keyPath isEqualToString:kTextAlignmentKey]) {
32    NSValue *value = [change objectForKey:NSKeyValueChangeNewKey];
33    self._placeholderTextView.textAlignment = alignment.intValue;
34  }
35  }
36  }
37  }
38  }
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89  }
90  }
91  }
92  }
93  }
94  }
95  }
96  }
97  }
98  }
99  }
100 }
```

# Contract-driven Development

*"design by contract"*



So, what is a *contract* anyway?

# contract **noun**

con·tract | \ˈkän-ˌtrakt  \

## Definition of *contract* (Entry 1 of 3)

- 1 a** : a binding agreement between two or more persons or parties  
*especially* : one legally enforceable  
*//* If he breaks the *contract*, he'll be sued.

# contract **noun**

con·tract | \ˈkän-ˌtrakt  \

## Definition of *contract* (Entry 1 of 3)

- 1 a** : a binding agreement between two or more persons or parties  
*especially* : one legally enforceable  
*//* If he breaks the *contract*, he'll be sued.

**An agreement between multiple parties for mutual benefit.**

**contract** noun

con·tract | \ˈkän-ˌtrakt  \

**Definition of *contract* (Entry 1 of 3)**

**1 a** : a binding agreement between two or more persons or parties

*especially* : one legally enforceable

// If he breaks the *contract*, he'll be sued.

**The agreement is enforced and violations are blamed on an offending party.**



A reallocating array<T> class in C++

```
    return e;  
}
```

```
void insert(const T& ele, u64 index = 0)  
{  
    assert(length >= index);  
  
    if (length+1 > buff_length)  
    {  
        // reallocate buffer  
        T* oldbuff = buff;  
  
        buff_length *= 2;  
        buff = allocator.alloc(buff_length);  
  
        // copy old data  
        for (u64 i = 0; i < length; ++i)  
        {
```

A reallocating array<T> class in C++

```
    return e;
}

void insert(const T& ele, u64 index = 0)
{
    assert(length >= index);

    if (length+1 > buff_length)
    {
        // reallocate buffer
        T* oldbuff = buff;

        buff_length *= 2;
        buff = allocator.alloc(buff_length);

        // copy old data
        for (u64 i = 0; i < length; ++i)
    }
```

## A reallocating array<T> class in C++

```
    return e;  
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```
void insert(const T& ele, u64 index = 0)  
{  
    assert(length >= index);  
  
    if (length+1 > buff_length)  
    {  
        // reallocate buffer  
        T* oldbuff = buff;  
  
        buff_length *= 2;  
        buff = allocator.alloc(buff_length);  
  
        // copy old data  
        for (u64 i = 0; i < length; ++i)  
        {
```

A reallocating array<T> class in C++

```
    }  
    return e;  
}  
  
void insert(const T& ele, u64 index = 0)  
{  
    // Precondition:  
    assert(length >= index);  
    assert(length <= buff_length);  
  
    // ... insert, possible reallocation ...  
  
    // Postcondition:  
    assert(length <= buff_length);  
}
```

A reallocating array<T> class in C++

```
    return e;  
}
```

```
void insert(const T& ele, u64 index = 0)  
{  
    // Precondition:  
    assert(length >= index);  
    assert(length <= buff_length);  
  
    // ... insert, possible reallocation ...  
  
    // Postcondition:  
    assert(length <= buff_length);  
}
```



# Meyer's "Design by Contract"

Implemented in Meyer's **Eiffel** programming language, a typed, object-oriented language with contracts at the center.

"A contract carries mutual **obligations** and **benefits**."

"Design by contract". **Bertrand Meyer. 1986.**

# Applying “Design by Contract”

```
put_child (new: NODE) is
  require
    new /= Void
  do
    -- Insertion algorithm

  ensure
    new.parent = Current
    child_count = old child_count + 1
end
```

“Applying design by contract”. **Bertrand Meyer. 1992.**

# Applying “Design by Contract”

Precondition

```
put_child (new: NODE) is  
  require  
    new /= Void  
  do
```

-- Insertion algorithm

```
  ensure  
    new.parent = Current  
    child_count = old child_count + 1  
  end
```

Postcondition

“Applying design by contract”. **Bertrand Meyer. 1992.**

^

To call `put_child`, calling code must satisfy its **obligations**

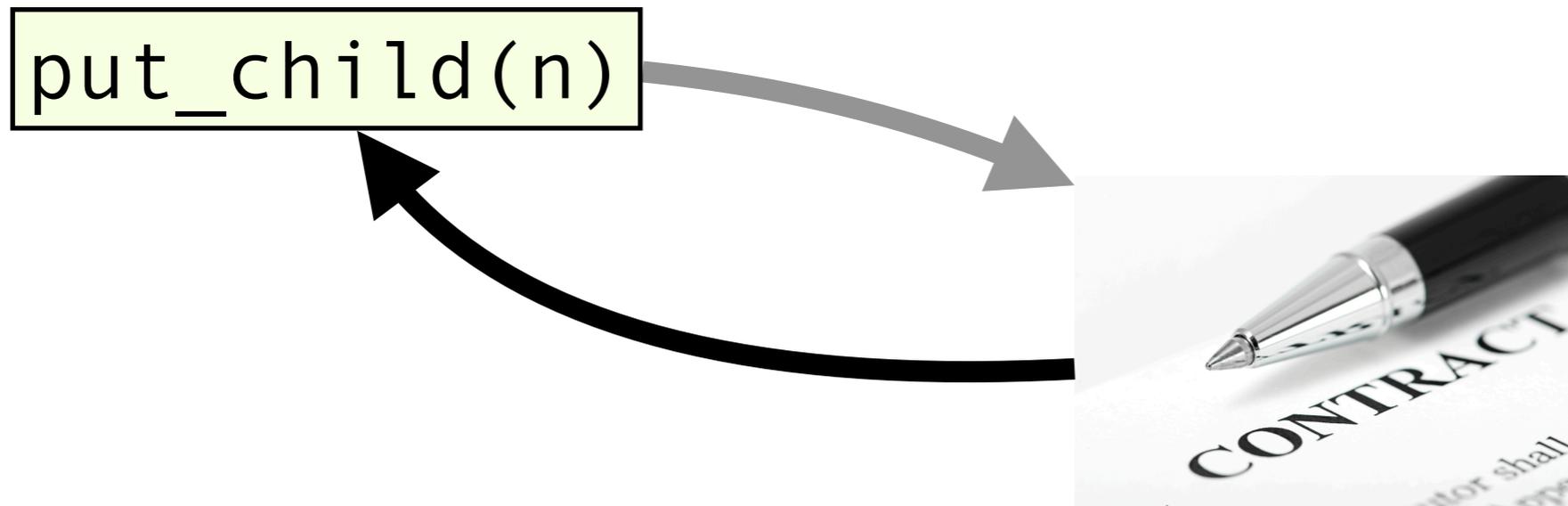
`put_child(n)`



```
put_child (new: NODE) is
  require
    new /= Void
  do
    -- Insertion algorithm

  ensure
    new.parent = Current
    child_count = old child_count + 1
  end
```

To return `put_child`, must ensure it provides **benefits**<sup>^</sup>



```
put_child (new: NODE) is
  require
    new /= Void
  do

    -- Insertion algorithm

  ensure
    new.parent = Current
    child_count = old child_count + 1
  end
```

If client **breaks** contract,  
put\_child is not obligated to provide benefits

put\_child(Void)



```
put_child (new: NODE) is
  require
    new /= Void
  do
    -- Insertion algorithm

  ensure
    new.parent = Current
    child_count = old child_count + 1
  end
```

If client **breaks** contract,  
put\_child is not obligated to provide benefits

put\_child(Void)

not (Void /= Void)



```
put_child (new: NODE) is
  require
    new /= Void
  do
    -- Insertion algorithm

  ensure
    new.parent = Current
    child_count = old child_count + 1
  end
```

If client **breaks** contract,  
put\_child is not obligated to provide benefits

put\_child(Void)

not (Void /= Void)



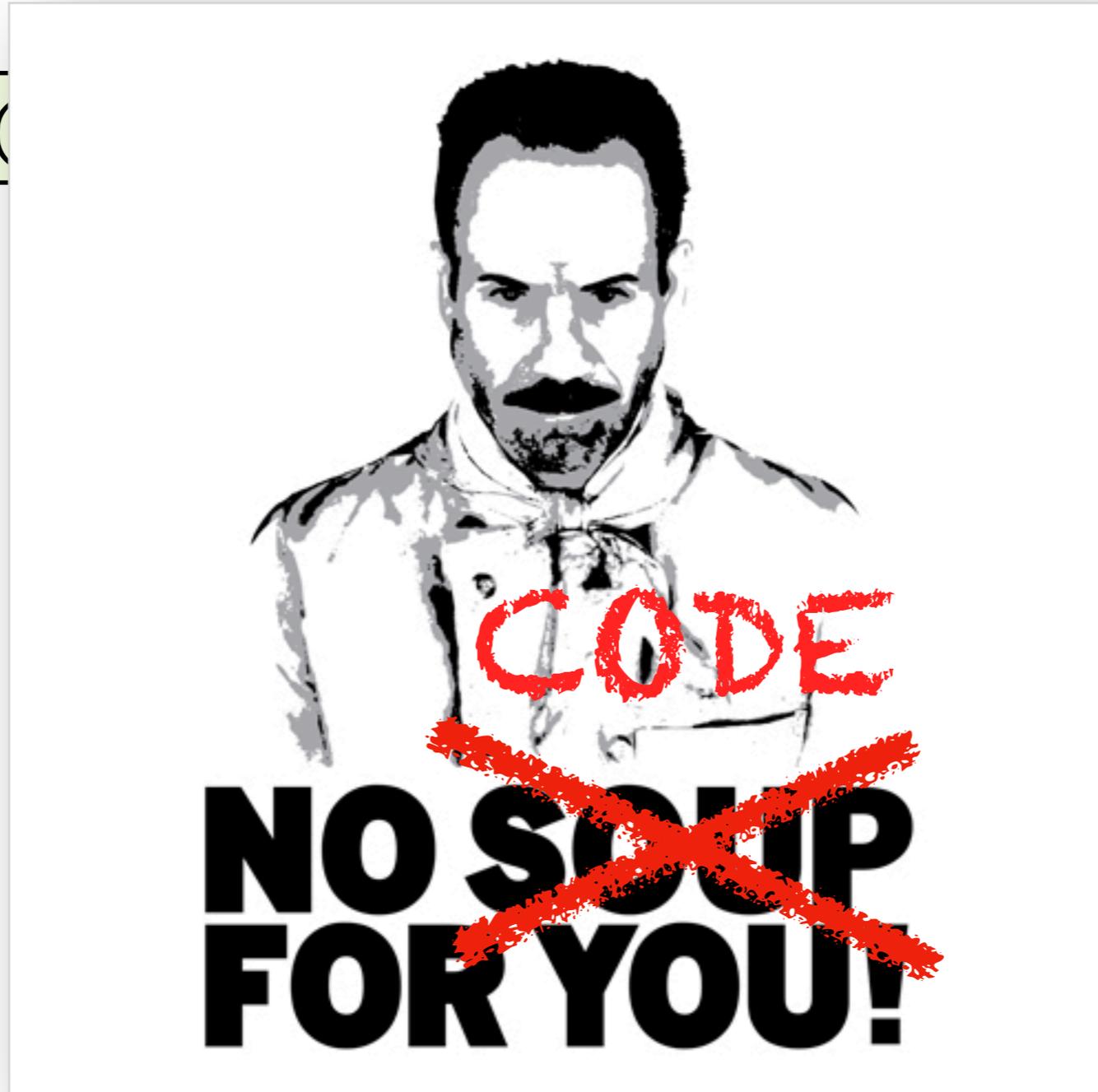
```
put_child (new: NODE) is
  require
    new /= Void
  do
    -- Insertion algorithm

  ensure
    new.parent = Current
    child_count = old child_count + 1
  end
```

If client **breaks** contract,  
put\_child is not obligated to provide benefits

put\_child(

Void) 😞



```
ensure  
  new.parent = Current  
  child_count = old child_count + 1  
end
```

Contracts are a ***linguistic mechanism*** implemented as a *built-in feature* of the language, using *source-to-source translation*, or *using a macro system*.

```
/**
 * @pre n >= 0
 * @post return >= 1
 */
public static int fact(int n) {
    if (n <= 1) return 1;
    else return n * fact(n-1);
}
```

## factorial in Java (e.g., using jContract)

```
/**
 * @pre n >= 0
 * @post return >= 1
 */
public static int fact(int n) {
    if (n <= 1) return 1;
    else return n * fact(n-1);
}
```

## factorial in Java (e.g., using jContract)

```
public static int fact(int n) {  
    assert n >= 0;  
    if (n <= 1) {  
        assert 1 >= 1;  
        return 1;  
    } else {  
        int rv = n * fact(n-1);  
        assert rv >= 1;  
        return rv;  
    }  
}
```

## factorial in Java (e.g., using jContract)

```
public static int fact(int n) {  
    assert n >= 0;  
    if (n <= 1) {  
        assert 1 >= 1;  
        return 1;  
    } else {  
        int rv = n * fact(n-1);  
        assert rv >= 1;  
        return rv;  
    }  
}
```

The contract bakes dynamic checks into the source code, executed at every evaluation of fact(n)!

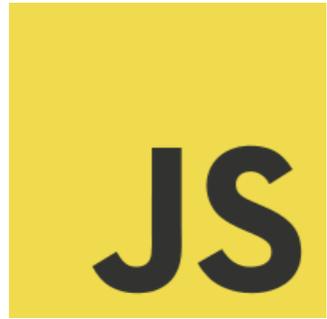


“Contracts for higher-order functions”. **Findler, Felleisen. 2002.**



# Contracts.js

*“Hygienic Macros for JavaScript”*. **Tim Disney. 2015**



**Contracts.js**

**Try it out online!**

<http://www.contractsjs.org/#/examples>

*“Hygienic Macros for JavaScript”*. **Tim Disney. 2015**

```
// [number] -> [string]
function array_numtostr(arr) {
    assert(arr instanceof Array);

    var str_arr = [];
    for (var i = 0; i < arr.length; ++i)
        str_arr.push(arr[i]+"");

    assert(str_arr instanceof Array);
    return str_arr
}
```

```
// [number] -> [string]
function array_numtostr(arr) {
    assert(arr instanceof Array);

    var str_arr = [];
    for (var i = 0; i < arr.length; ++i)
        str_arr.push(arr[i]+"");

    assert(str_arr instanceof Array);
    return str_arr
}
```

```
// [number] -> [string]
function array_numtostr(arr) {
    assert(arr instanceof Array
           && arr.reduce((a,n)=>(typeof n)
== "number" && a, true));

    var str_arr = [];
    for (var i = 0; i < arr.length; ++i)
        str_arr.push(arr[i]+"");

    assert(str_arr instanceof Array
           && str_arr.reduce((a,s)=>(typeof s)
== "string" && a, true));
    return str_arr
}
```

```
@ ([...Num]) -> [...Str]
function array_numtostr(arr) {

    var str_arr = [];
    for (var i = 0; i < arr.length; ++i)
        str_arr.push(arr[i]+"");

    return str_arr
}
```

```
➔ @ ([...Num]) -> [...Str]
function array_numtostr(arr) {

    var str_arr = [];
    for (var i = 0; i < arr.length; ++i)
        str_arr.push(arr[i]+"");

    return str_arr
}
```

**contracts.js's @** macro allows the programmer to associate a function contract with `array_numtostr`

```
@ ([...Num], (Num)->Str) -> [...Str]
function array_numtostr(arr, format) {

    var str_arr = [];
    for (var i = 0; i < arr.length; ++i)
        str_arr.push(format(arr[i]));

    return str_arr
}
```

```
array_numtostr([14,18],
    (n) => "0x"+n.toString(16))
```

```
// => ["0xe", "0x12"]
```

```
@ ([...Num], (Num)->Str) -> [...Str]
function array_numtostr(arr, format) {

    var str_arr = [];
    for (var i = 0; i < arr.length; ++i)
        str_arr.push(format(arr[i]));

    return str_arr
}
```

```
array_numtostr([14,18],
    (n) => "0x"+n.toString(16))
```

```
// => ["0xe", "0x12"]
```

**We can check that `arr` is an array,  
and that its elements are numbers...**

```
@ ([...Num], (Num) -> Str) -> [...Str]
function array_numtostr(arr, format) {
  assert(...??...);
  var str_arr = [];
  for (var i = 0; i < arr.length; ++i)
    str_arr.push(format(arr[i]));
}
```

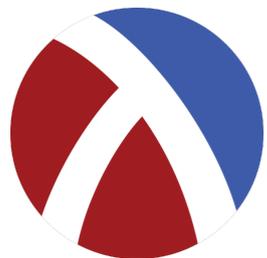
**...but how can we check that `format` satisfies  
`(Num) -> Str` before `array_numtostr` is evaluated?**

```
array_numtostr([14, 18],
  (n) => "0x"+n.toString(16))
```

**`format` is a first-class function—a behavioral value!**

# Contracts on behavioral values are *delayed*.

The contract `array_numtostr` requires on its argument `format` is enforced in the same way as the contract on `array_numtostr`!



“Contracts for higher-order functions”. **Findler, Felleisen. 2002.**

```
@ ([...Num], (Num) -> Str) -> [...Str]
function array_numtostr(arr, format) {

    var str_arr = [];
    for (var i = 0; i < arr.length; ++i)
        str_arr.push(format(arr[i]));

    return str_arr
}
```

```
array_numtostr([14, 18], (n) => n)
```

```
// => ["0xe", "0x12"]
```

```
@ ([...Num], (Num)->Str) -> [...Str]
function array_numtostr(arr, format) {

    var str_arr = [];
    for (var i = 0; i < arr.length; ++i)
        str_arr.push(format(arr[i]));

    return str_arr
}
```

```
array_numtostr([14,18], (n) => n) ←
```

```
// => ["0xe", "0x12"]
```

```
@ ([...Num], (Num)->Str) -> [...Str]
function array_numtostr(arr, format) {
    var str_arr = [];
    for (var i = 0; i < arr.length; ++i)
        str_arr.push(format(arr[i]));
    return str_arr
}
```



```
array_numtostr([14,18], (n) => n) ←
// => ["0xe", "0x12"]
```

```
Error: array_numtostr: contract violation
expected: Str
given: 14
in: the return of
      the 2nd argument of
      ([...Num], (Num) -> Str) -> [...Str]
function array_numtostr guarded at line: 4
blaming: (calling context for array_numtostr)
```

Higher-order contract systems track program labels alongside contracts to ***properly assign blame*** when failure occurs.

“Correct blame for contracts”. **Dimoulas. 2011.**

```

@ (a: [...Num], f: (Num)->Str)
  -> r: [...Str] | a.length == r.length
function array_numtostr(arr, format) {

    var str_arr = [];
    for (var i = 0; i < arr.length; ++i)
        str_arr.push(format(arr[i]));

    return str_arr
}

array_numtostr([14,18],
    (n) => "0x"+n.toString(16))

// => ["0xe", "0x12"]

```

```
@ (a: [...Num], f: (Num)->Str)
  -> r: [...Str] | a.length == r.length ←
function array_numtostr(arr, format) {

    var str_arr = [];
    for (var i = 0; i < arr.length; ++i)
        str_arr.push(format(arr[i]));

    return str_arr
}

array_numtostr([14,18],
    (n) => "0x"+n.toString(16))

// => ["0xe", "0x12"]
```

**Software contracts**

**Static typechecking**

**"Refinement types for ML", Freeman, Pfenning. 1991.**

## Software contracts

- **Same expressivity** as the host programming language

## Static typechecking

- **Separate type language** with a static semantics

"Refinement types for ML", **Freeman, Pfenning. 1991.**

## Software contracts

- **Same expressivity** as the host programming language
- **Reports actual/observed errors** & witnesses to errors

## Static typechecking

- **Separate type language** with a static semantics
- Reports **potential errors** in abstract terms

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- Can add **significant run-time overhead**, breaks tail calls

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- **Separate type language** with a static semantics
- Reports **potential errors** in abstract terms
- Produces fast code **without run-time monitoring** of types

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## Software contracts

- **Same expressivity** as the host programming language
- **Reports actual/observed errors** & witnesses to errors
- Can add **significant run-time overhead**, breaks tail calls
- Error discovery is **delayed until runtime**

## Static typechecking

- **Separate type language** with a static semantics
- Reports **potential errors** in abstract terms
- Produces fast code **without run-time monitoring** of types
- Potential failures are **discovered ahead of time**

"Refinement types for ML", **Freeman, Pfenning. 1991.**

The future of dynamically enforced contracts is ***static verification!***

# **Dynamic enforcement of program properties with software contracts**



**Formal verification  
of program properties**

**Dynamic enforcement of program  
properties with software contracts**

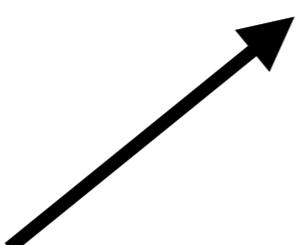
```
@ ((Num) -> Num, Num) -> Num
function f(g, x) {

    // y = ...

    return g(y)
}
```

```
@ ((Num) -> Num, Num) -> Num
function f(g, x) {
    // y = ...
    return g(y)
}
```

**Control-flow at  $g(y)$  here...**



```
@ ((Num) -> Num, Num) -> Num  
function f(g, x) {
```

```
  // y = ...
```

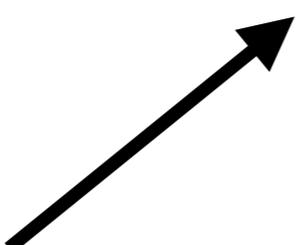
```
  return g(y)
```

```
}
```

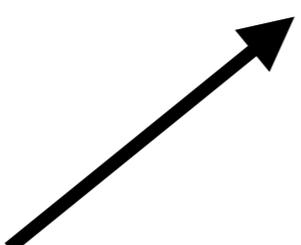
**...and on the  
callers of f.**



**...depends on data-flow to g...**



**Control-flow at g(y) here...**



`f((n) => n+1, 0)`

`f((n) => n*n, 2)`

`@ ((Num) -> Num, Num) -> Num`

`function f(g, x) {`

`// y = ...`

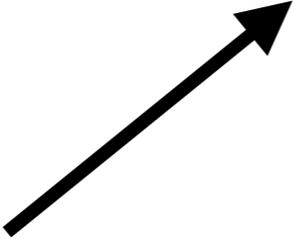
`return g(y)`

`}`

**...and on the  
callers of f.**



**...depends on data-flow to g...**



**Control-flow at g(y) here...**

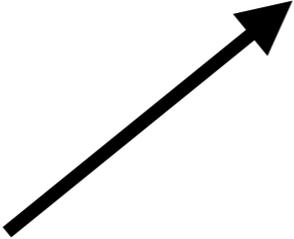
```
f((n) => n+1, 0)
f((n) => n*n, 2)
f(h, 0)
```

```
@ ((Num) -> Num, Num) -> Num
function f(g, x) {
  // y = ...
  return g(y)
}
```

**...and on the  
callers of f.**



**...depends on data-flow to g...**



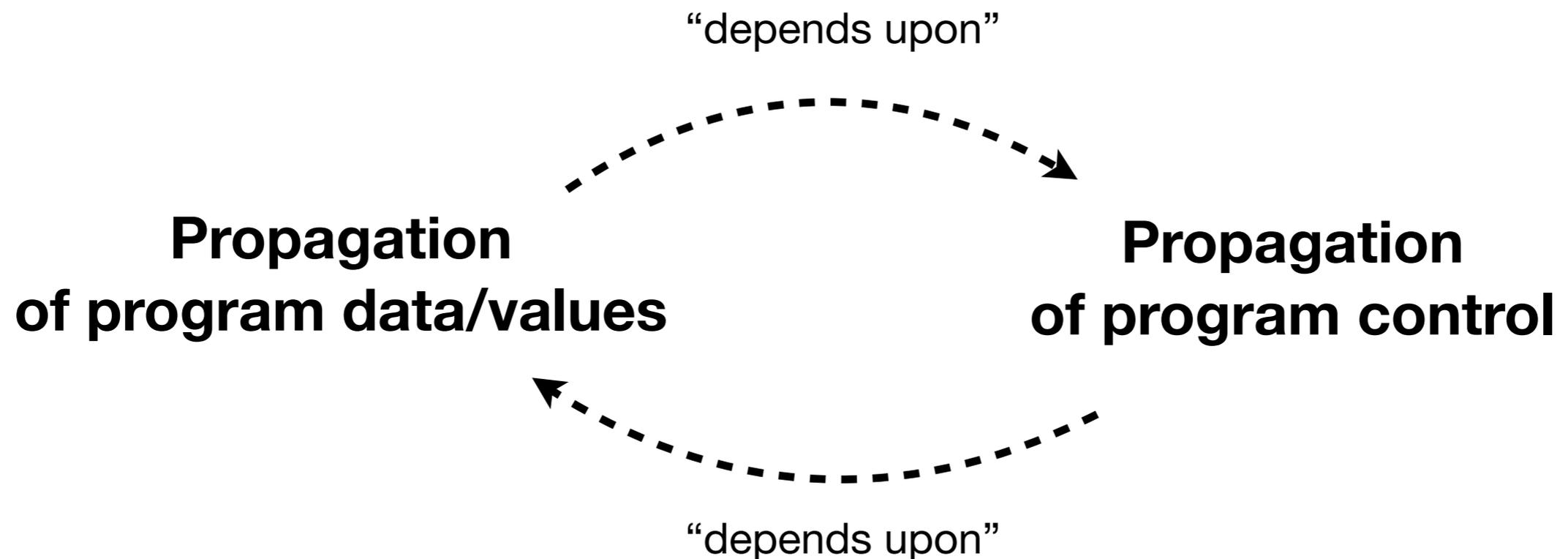
**Control-flow at g(y) here...**

**Propagation  
of program data/values**

**Propagation  
of program control**

This is called the

# higher-order control-flow problem



and to tackle this problem, we use

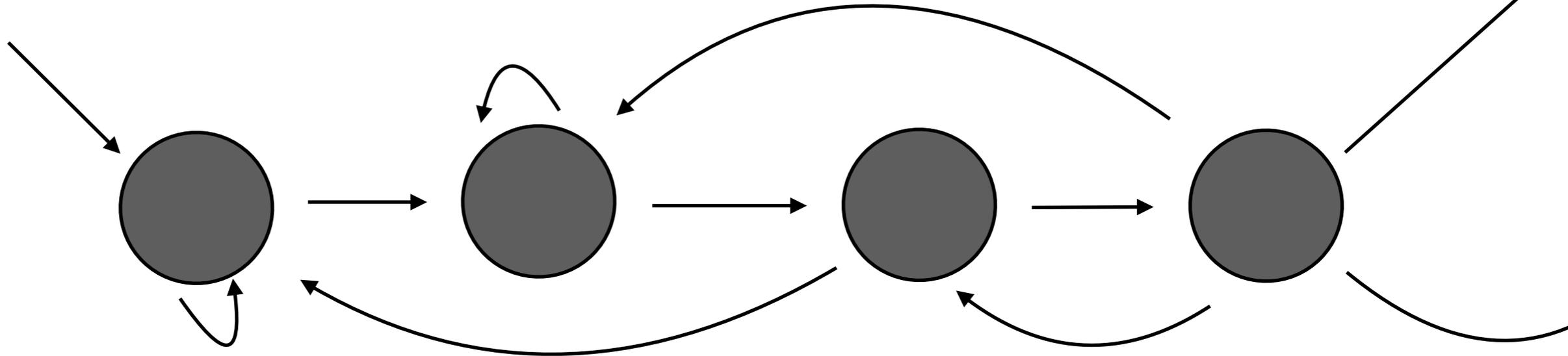
**AI**



and to tackle this problem, we use

# AI





# Abstract Interpretation

*“Do you know the way?”*

*“Do you know the way?”*

“Sure, you take a left just past  
that **tree** there.”

*“The little poplar just there?”*



Ah no, the tree over *there*,  
the ***one with 51 branches***  
***& 8206 leaves.***

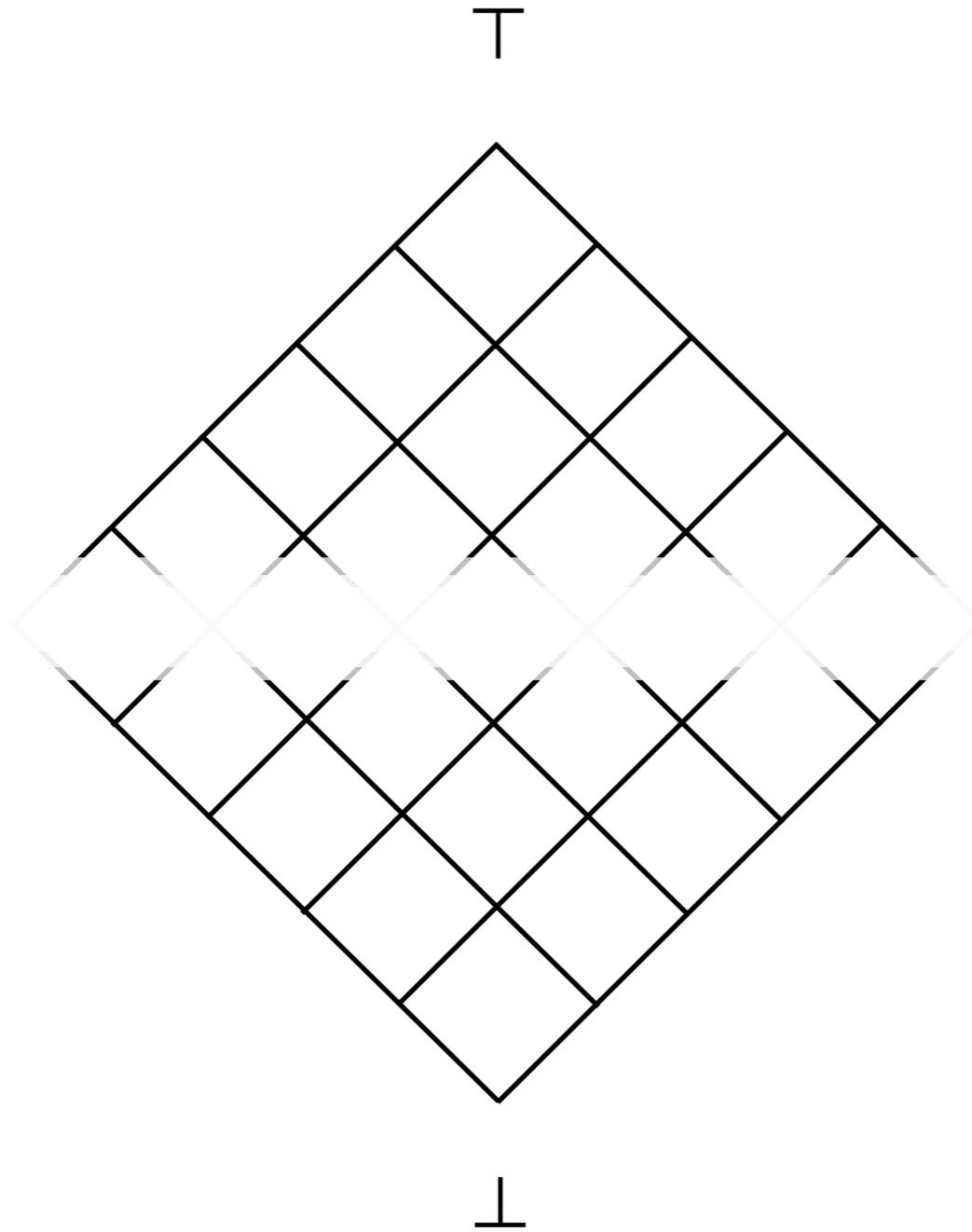
Ah no, the tree over *there*,  
the ***one with 51 branches***  
***& 8206 leaves.***

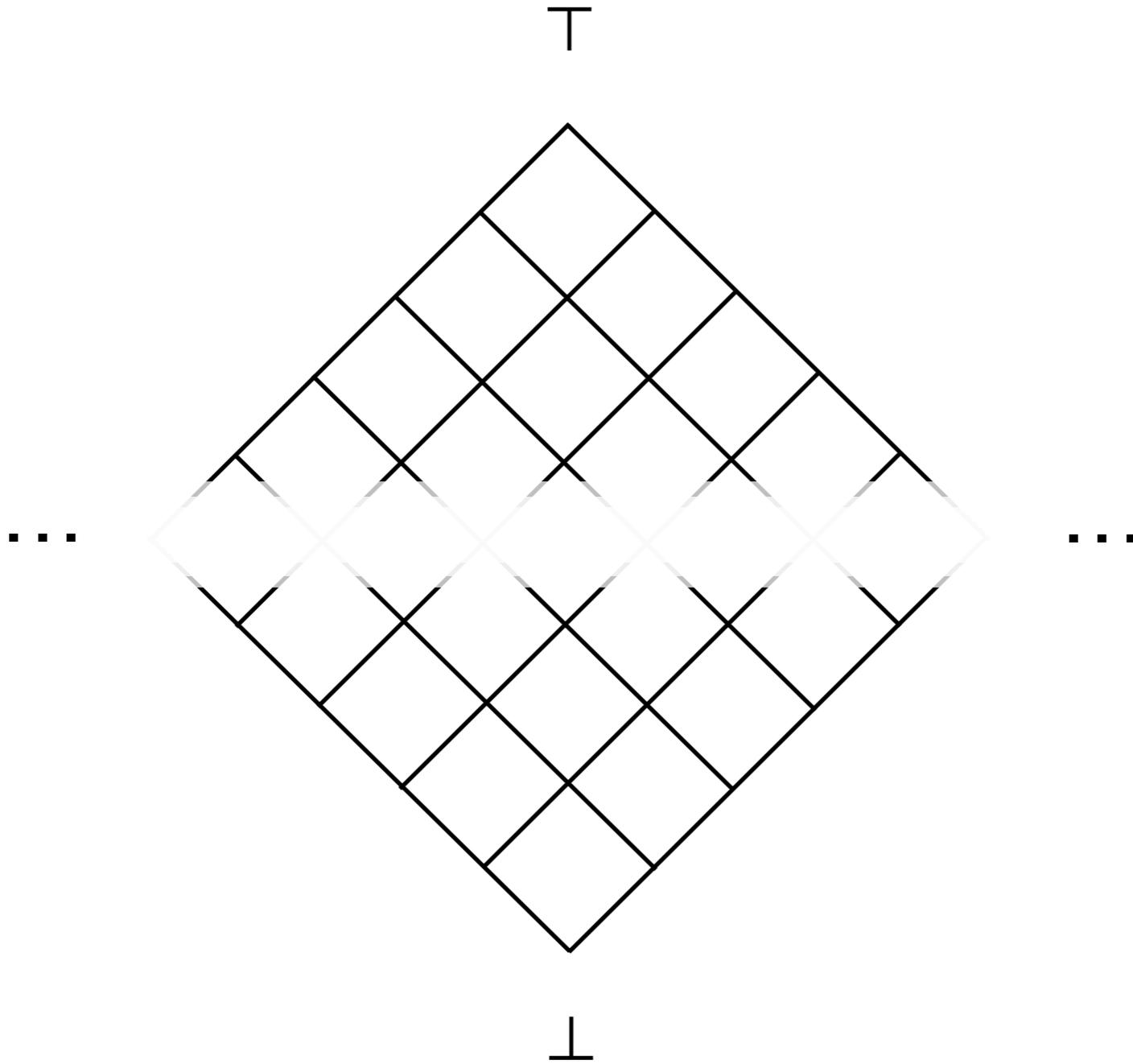


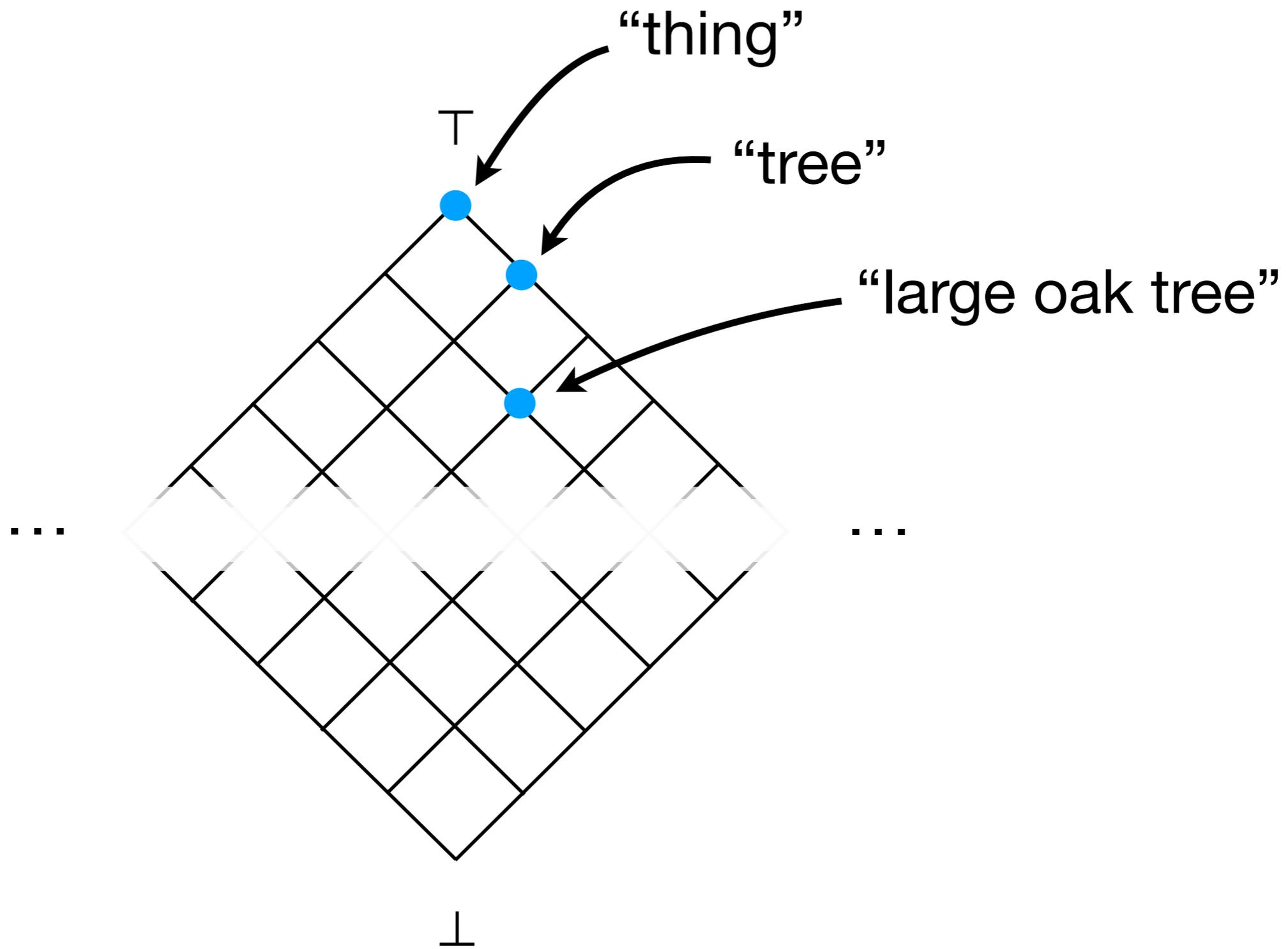
Ah no, the tree over *there*,  
the *one with 51 branches*

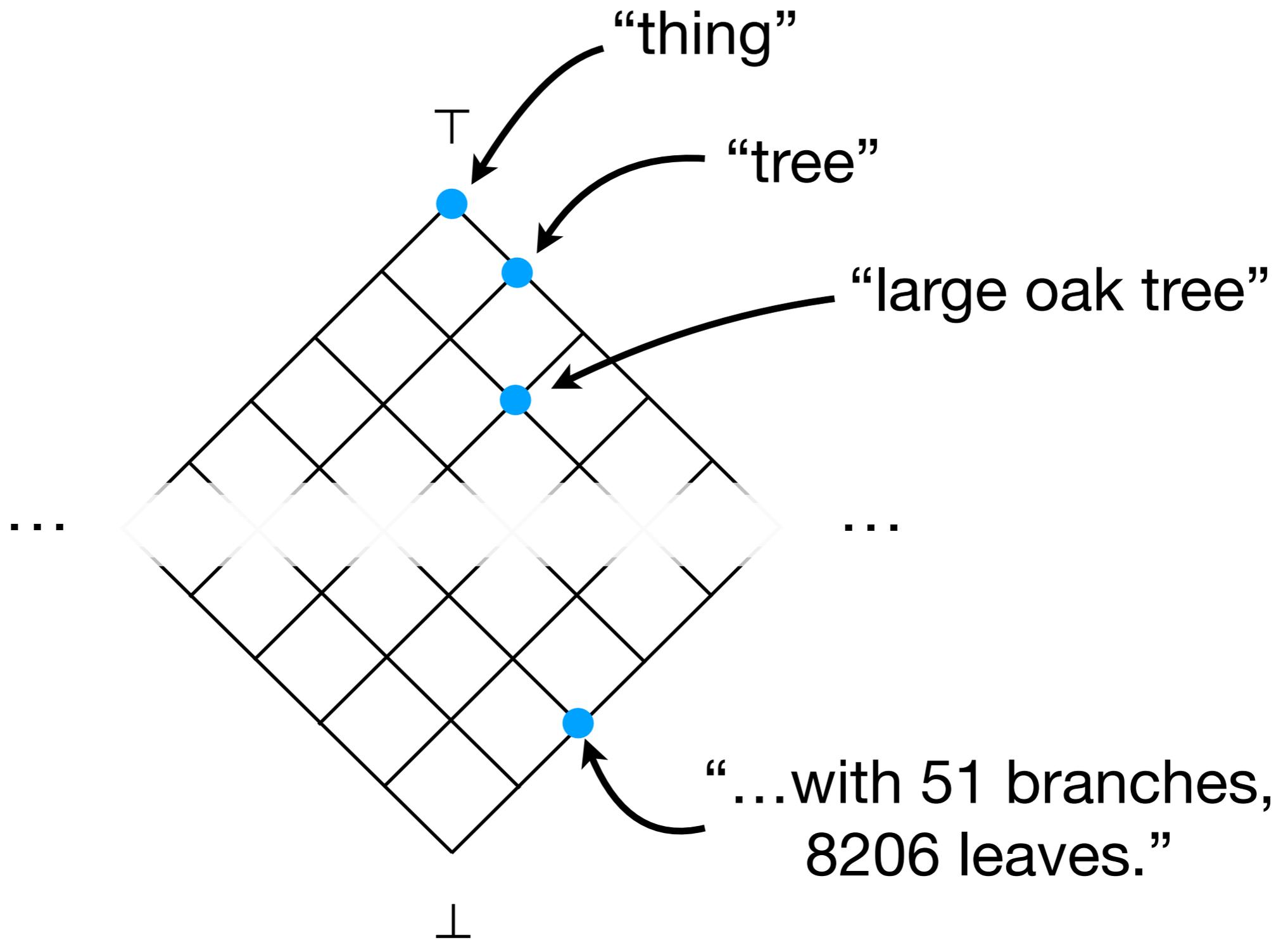
Ah no, the ***large oak tree*** there.

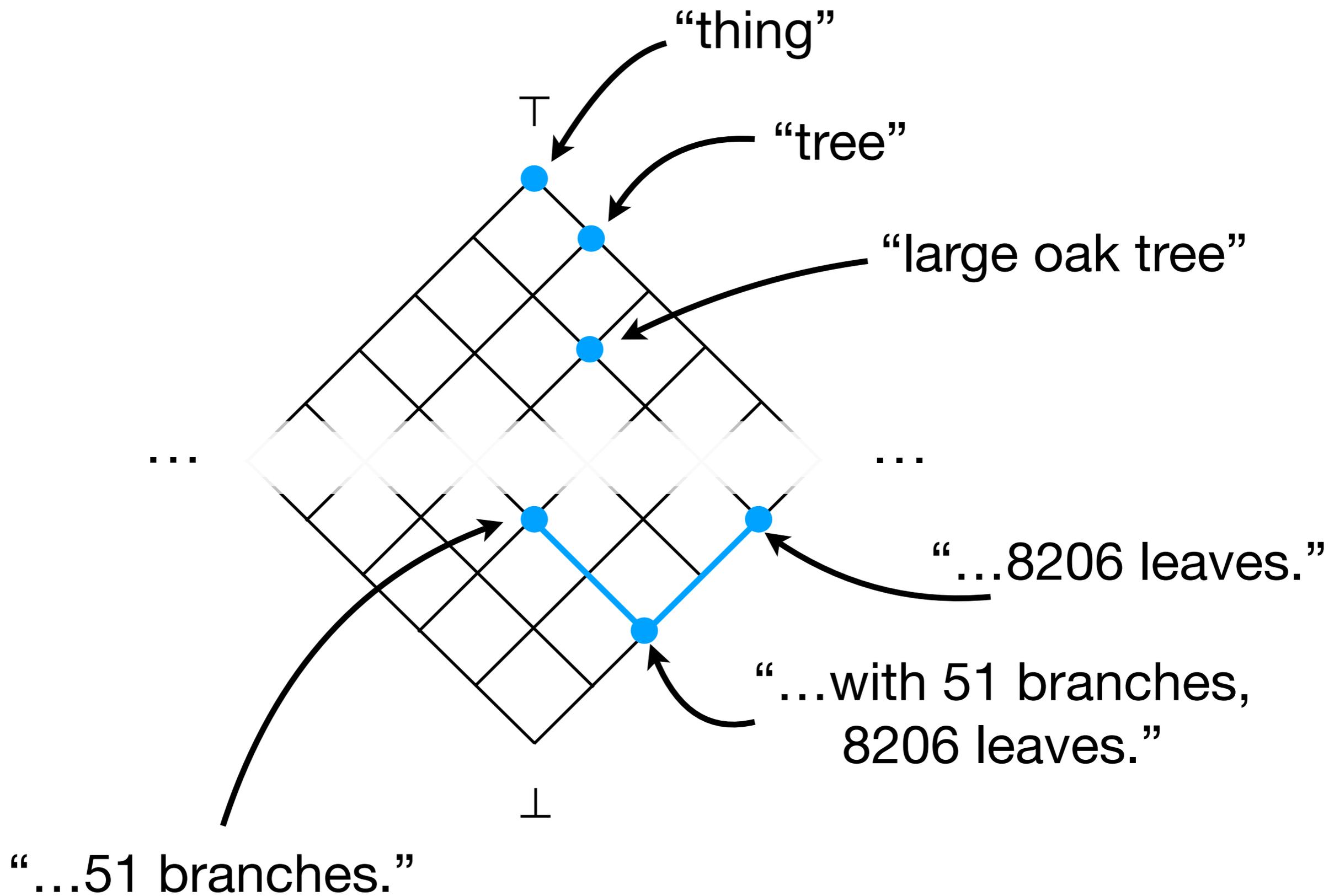








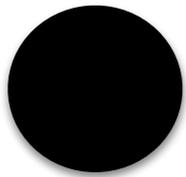




prog

Plotkin (1981), Tarski (1955)

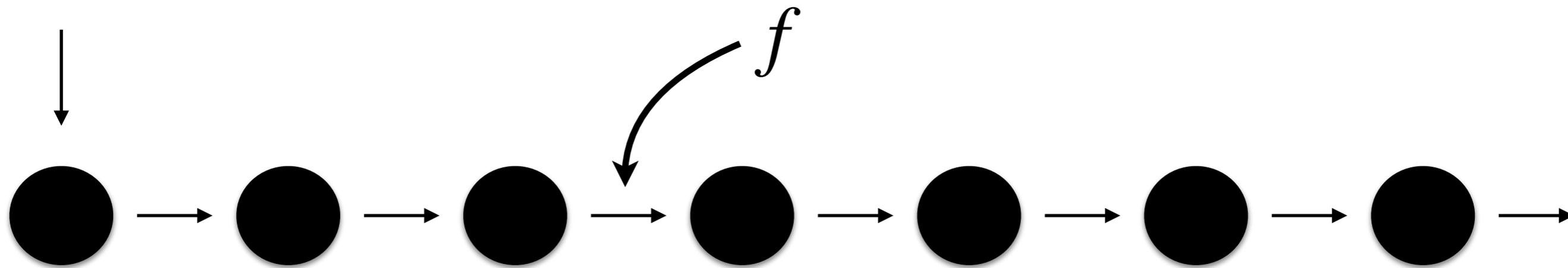
prog



**Initial State**

Plotkin (1981), Tarski (1955)

prog

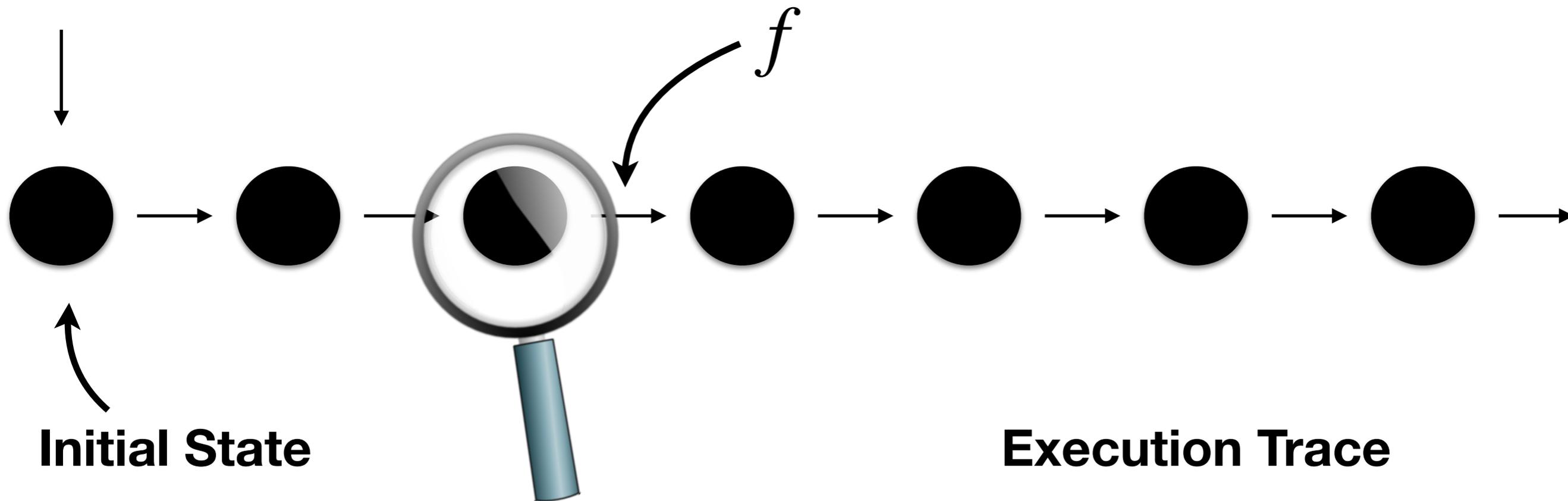


**Initial State**

**Execution Trace**

Plotkin (1981), Tarski (1955)

prog

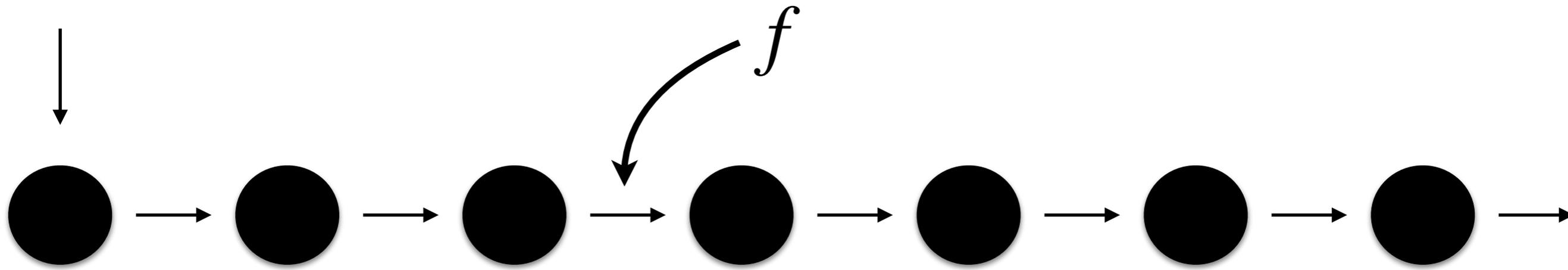


**States may contain:**

- the program counter,
  - a binding environment,
  - a model of the heap,
  - a model of the stack,
- etc...

Plotkin (1981), Tarski (1955)

prog



**Initial State**

**Execution Trace**

Plotkin (1981), Tarski (1955)

prog

```
// @post !isEven(return)
public static int nextOdd(int x) {
    if (isEven(x))
        return x+1;
    else return x+2;
}
```

Plotkin (1981), Tarski (1955)

prog

```
// @post !isEven(return)
public static int nextOdd(int x) {
    if (isEven(x))
        return x+1;
    else return x+2;
}
```

9

11

Plotkin (1981), Tarski (1955)

prog

{..., -3, -2, -1, 0, 1, 2, 3, ...}

```
// @post !isEven(return)
public static int nextOdd(int x) {
    if (isEven(x))
        return x+1;
    else return x+2;
}
```

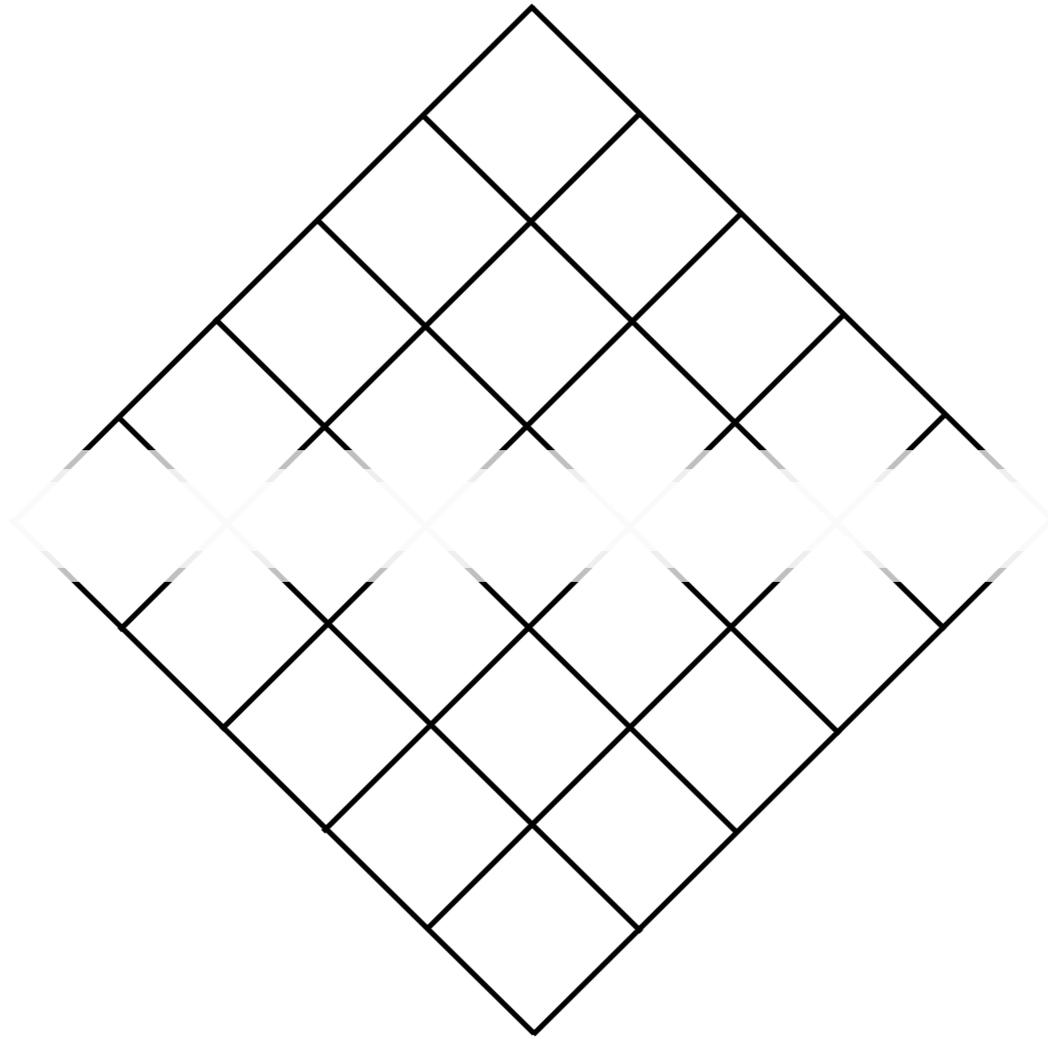
Initial State

Execution Trace

{..., 5, -3, -1, 1, 3, 5, ...}

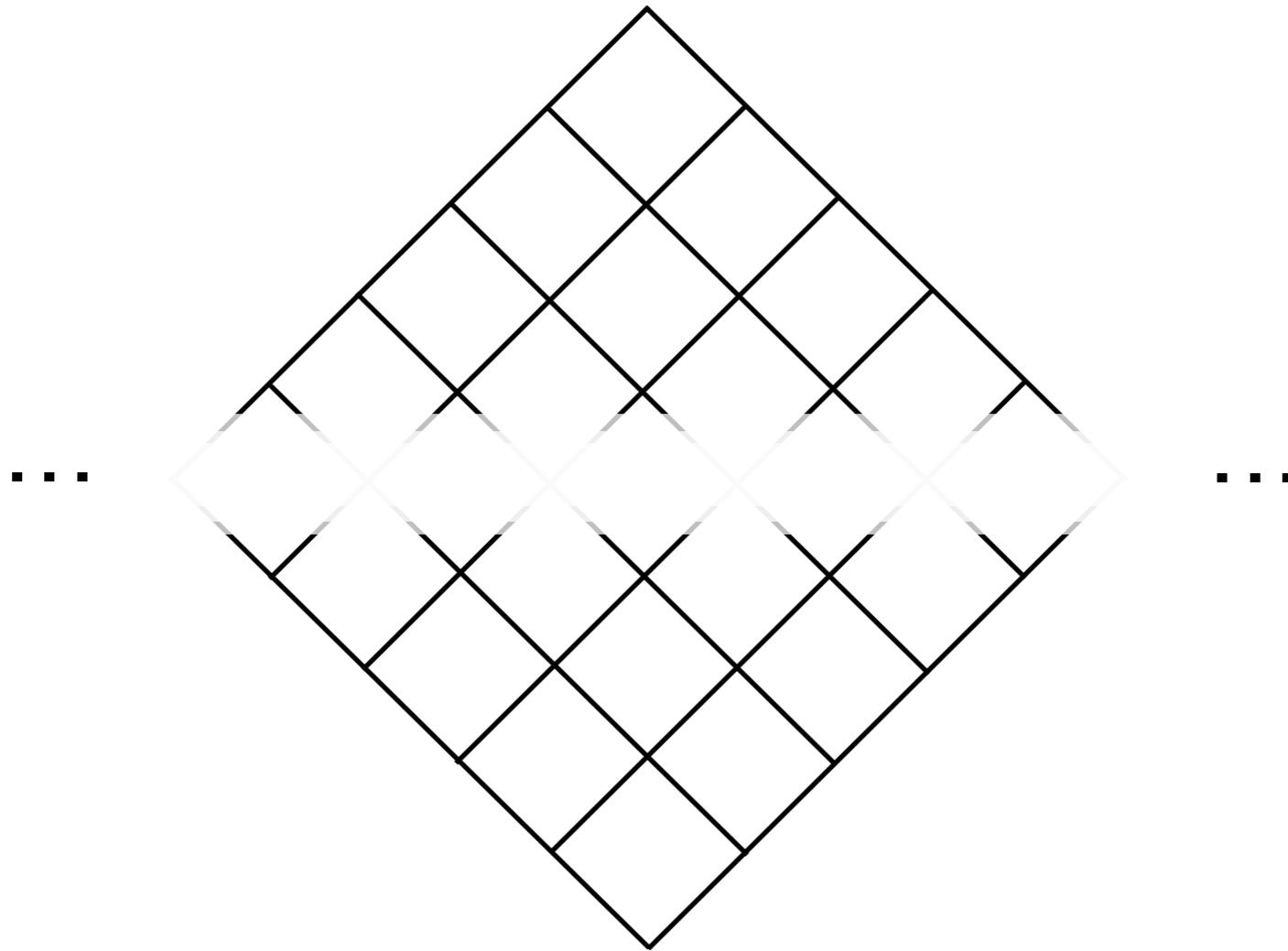
Plotkin (1981), Tarski (1955)

$$\top = \mathbb{Z}$$



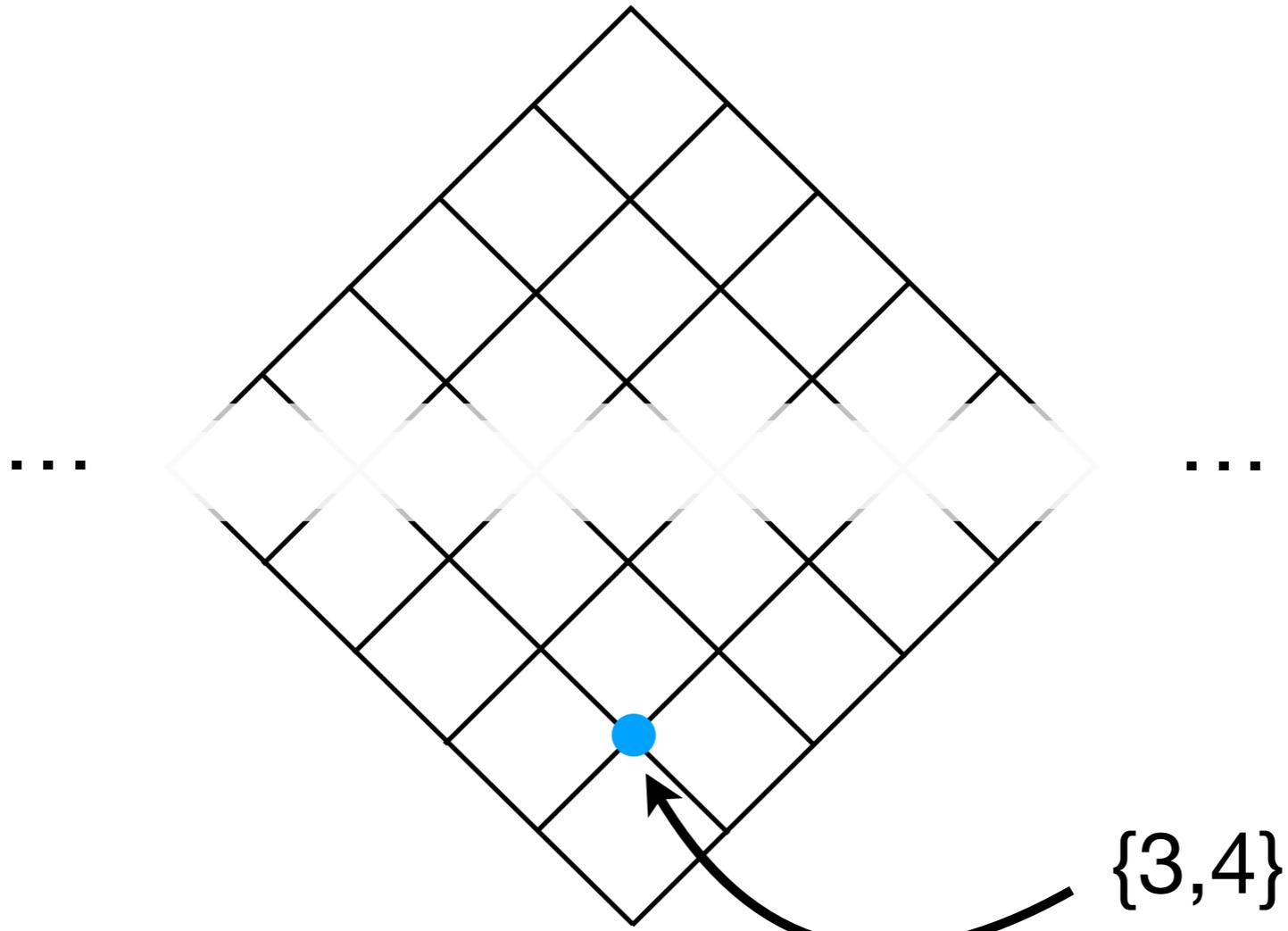
$$\perp = \emptyset$$

$$\top = \mathbb{Z}$$

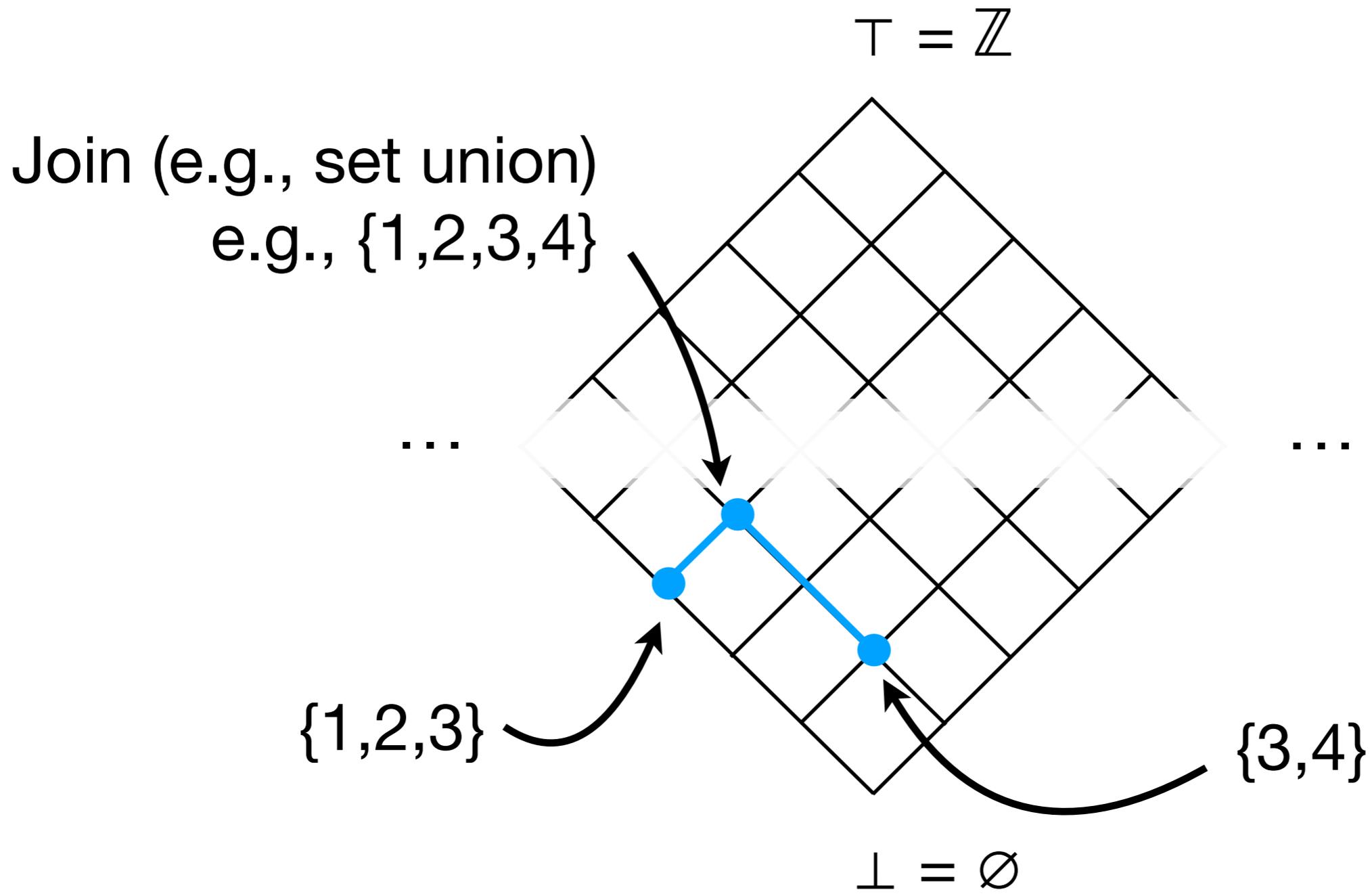


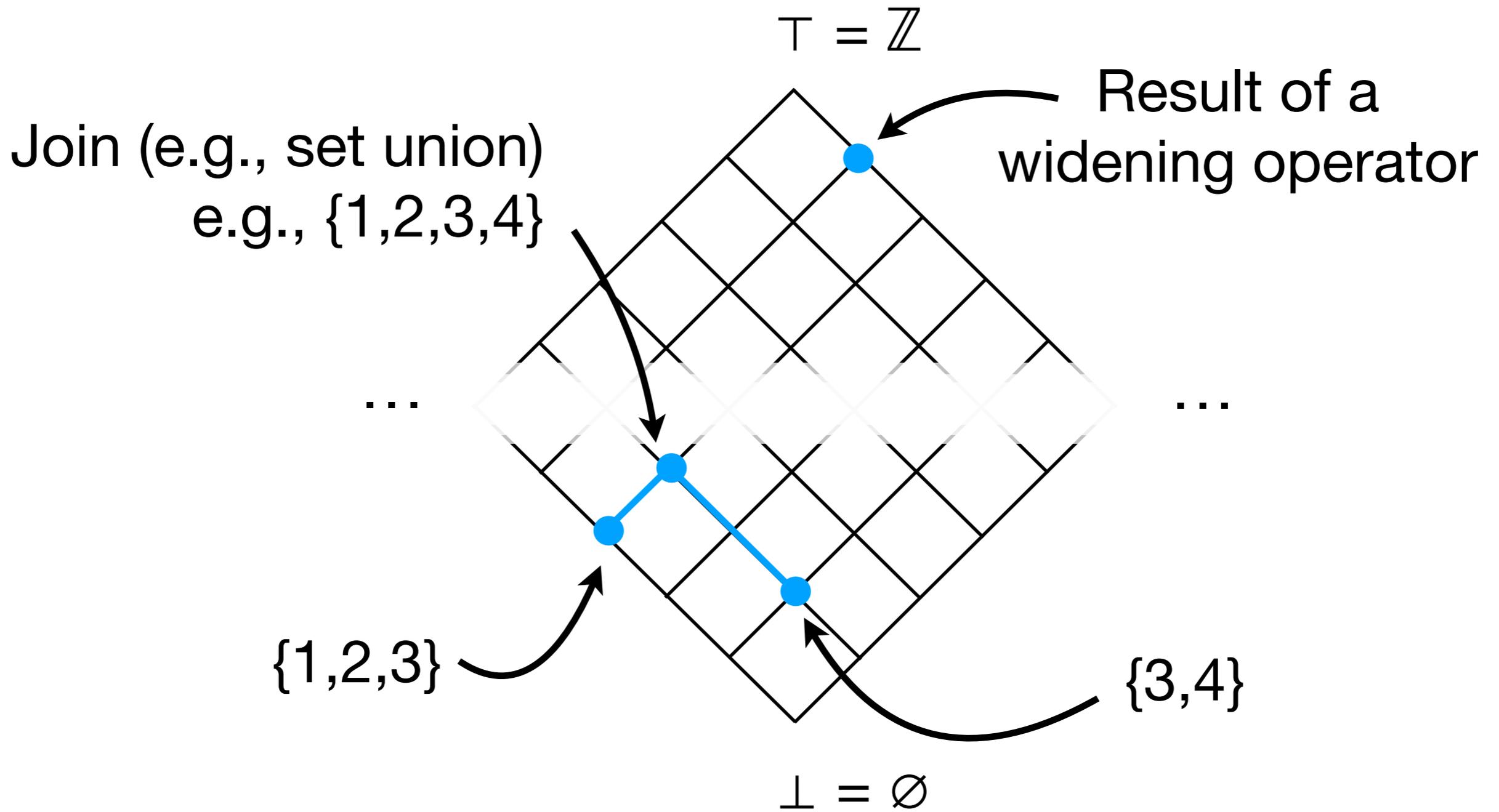
$$\perp = \emptyset$$

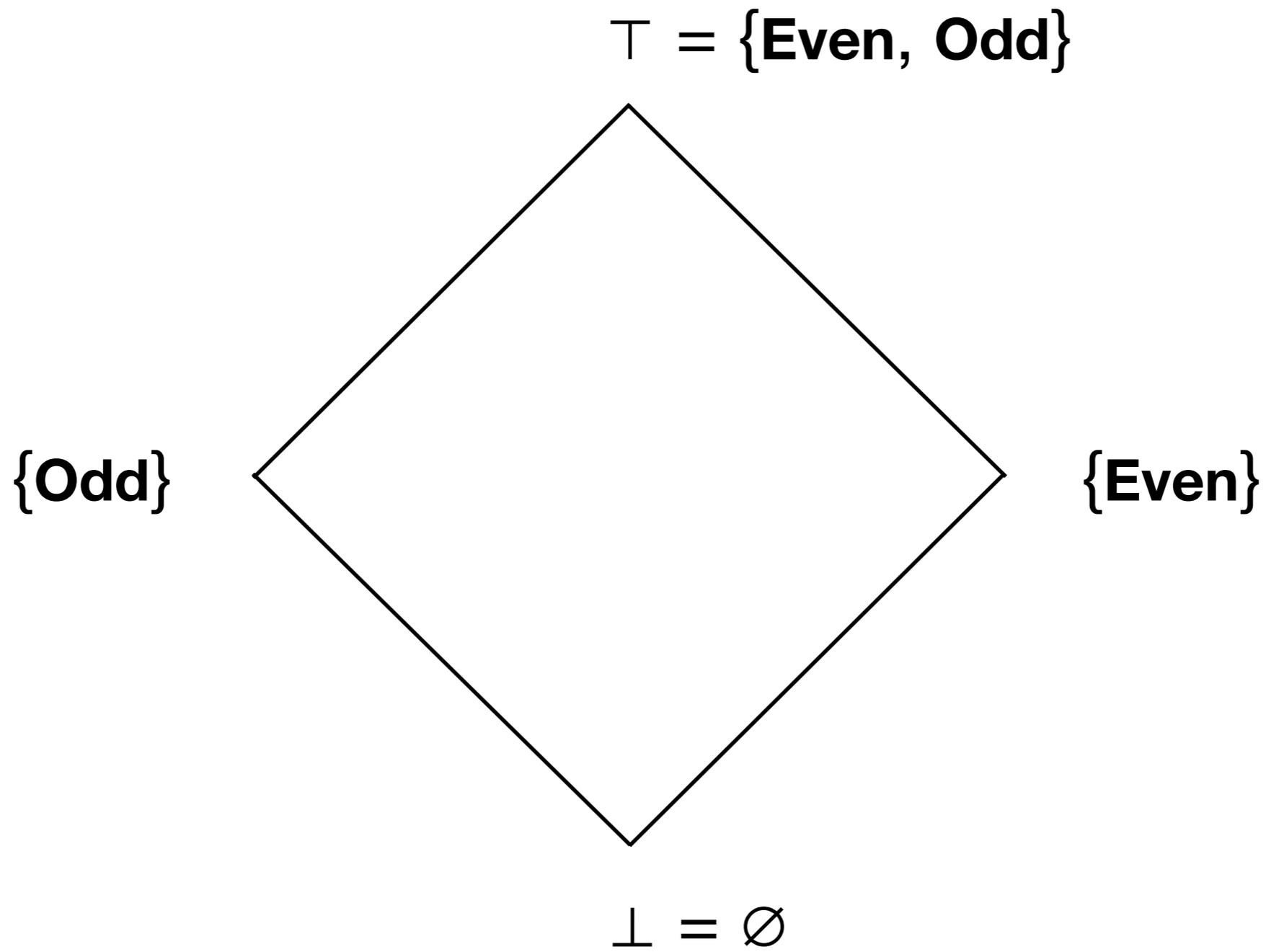
$$\top = \mathbb{Z}$$

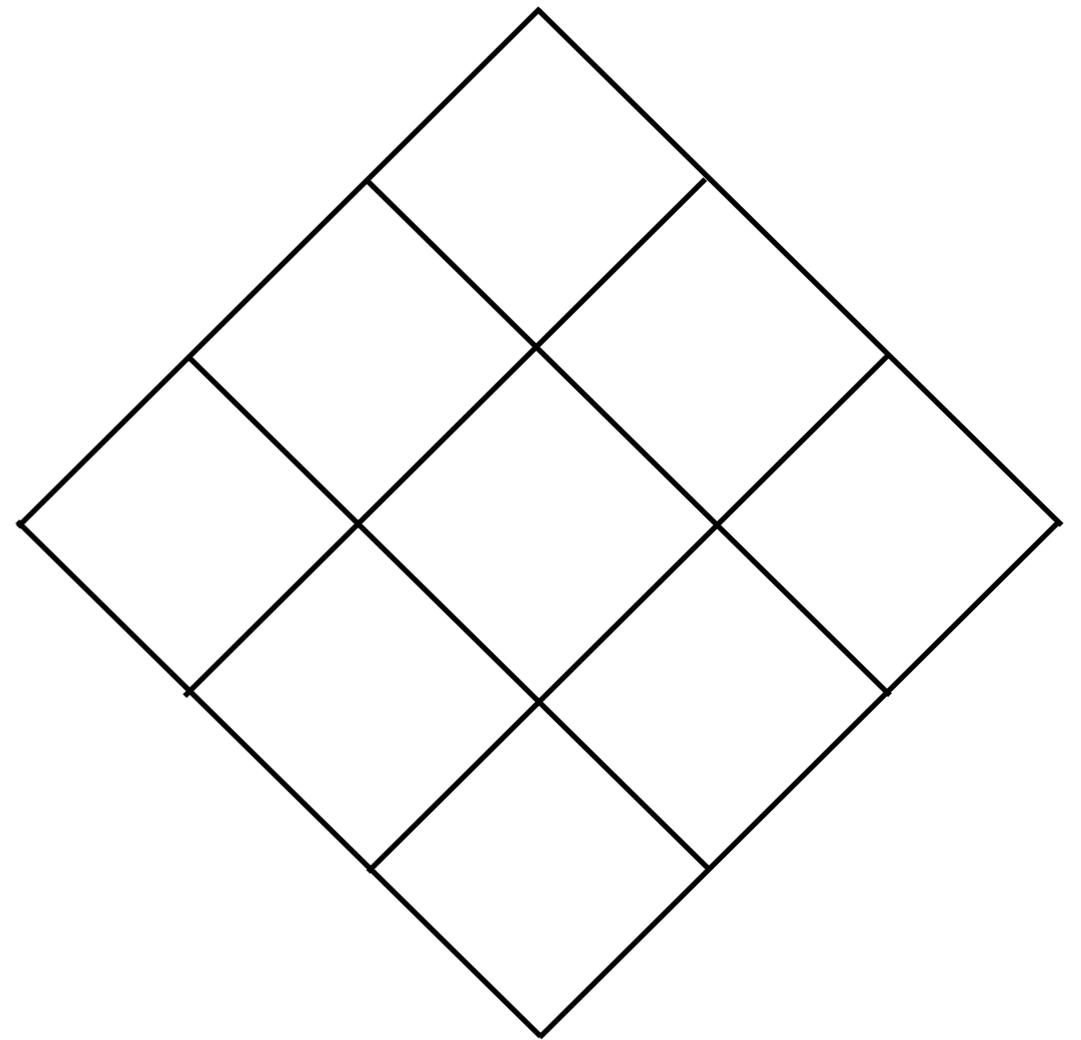
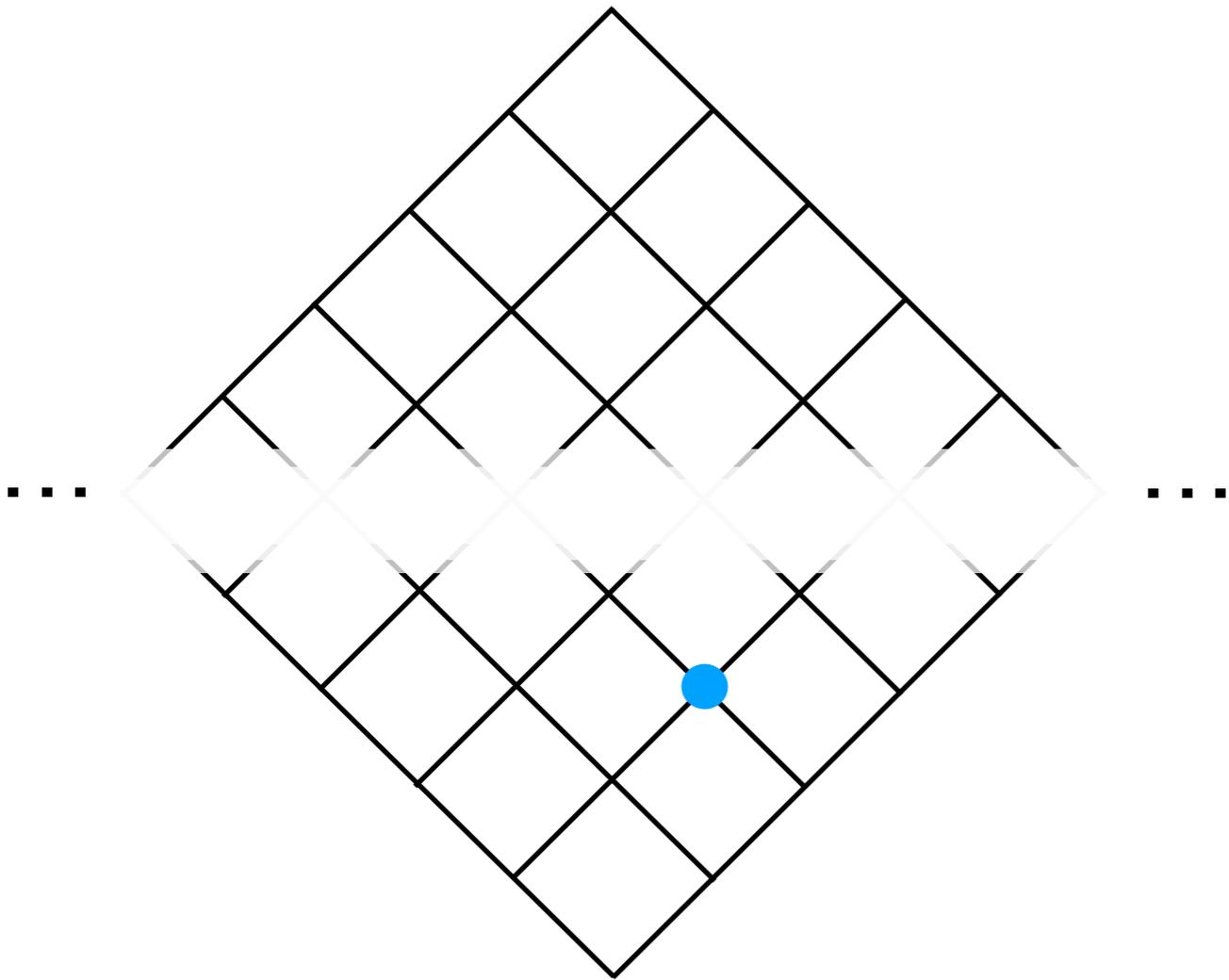


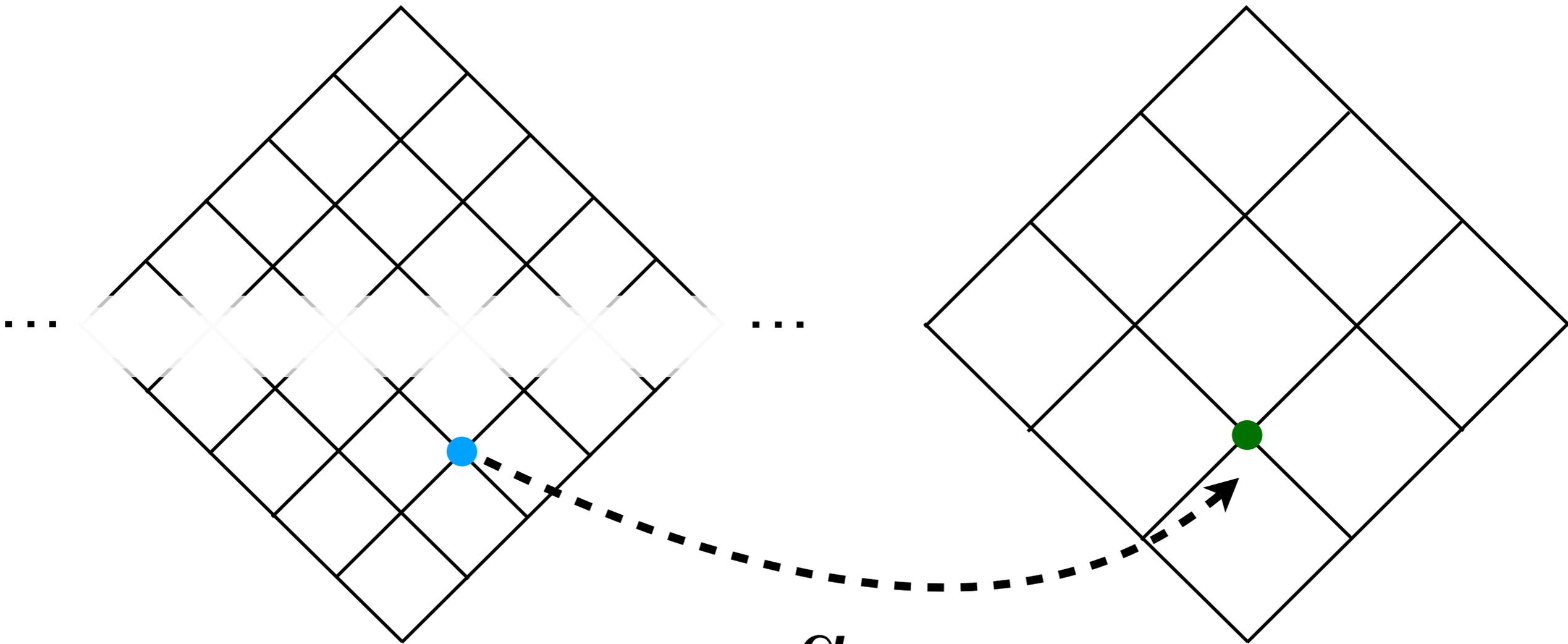
$$\perp = \emptyset$$







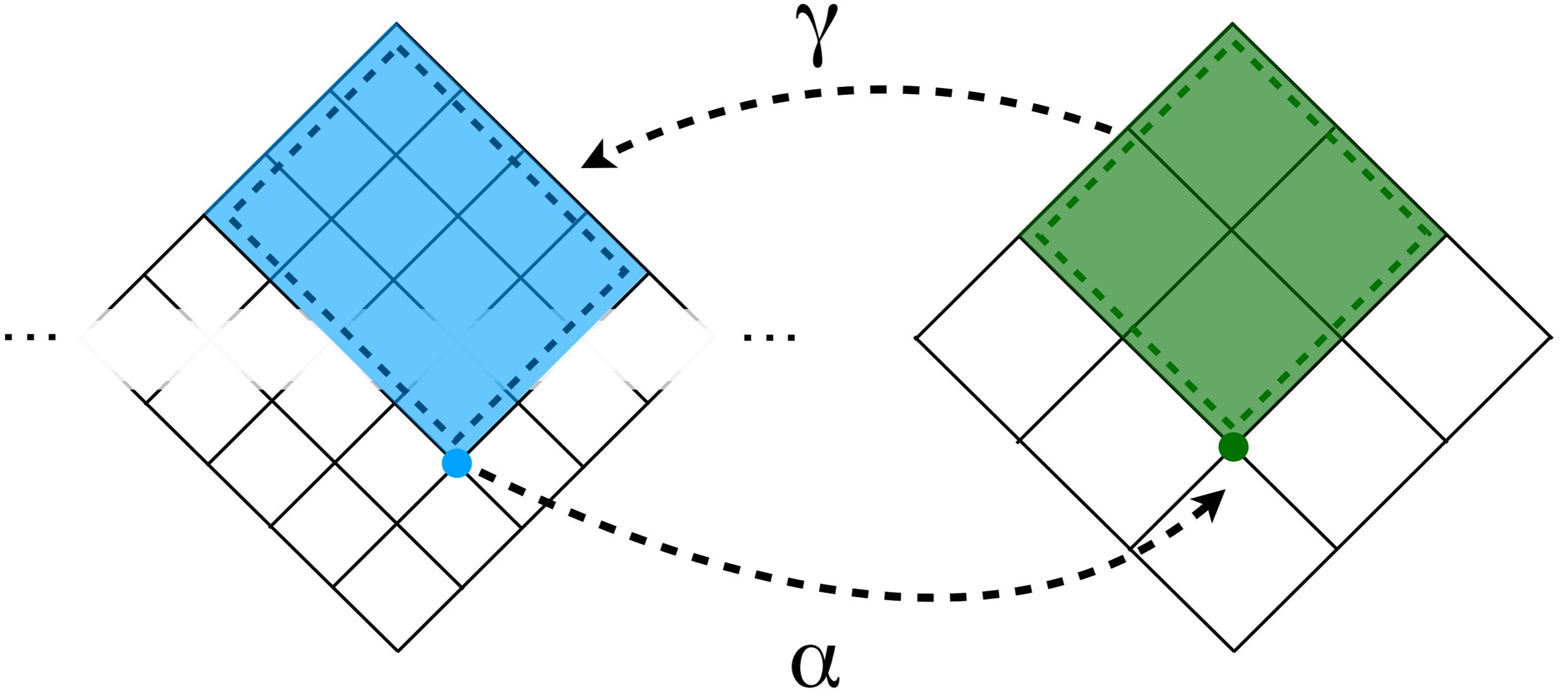




$\alpha$

Abstraction

Concretization

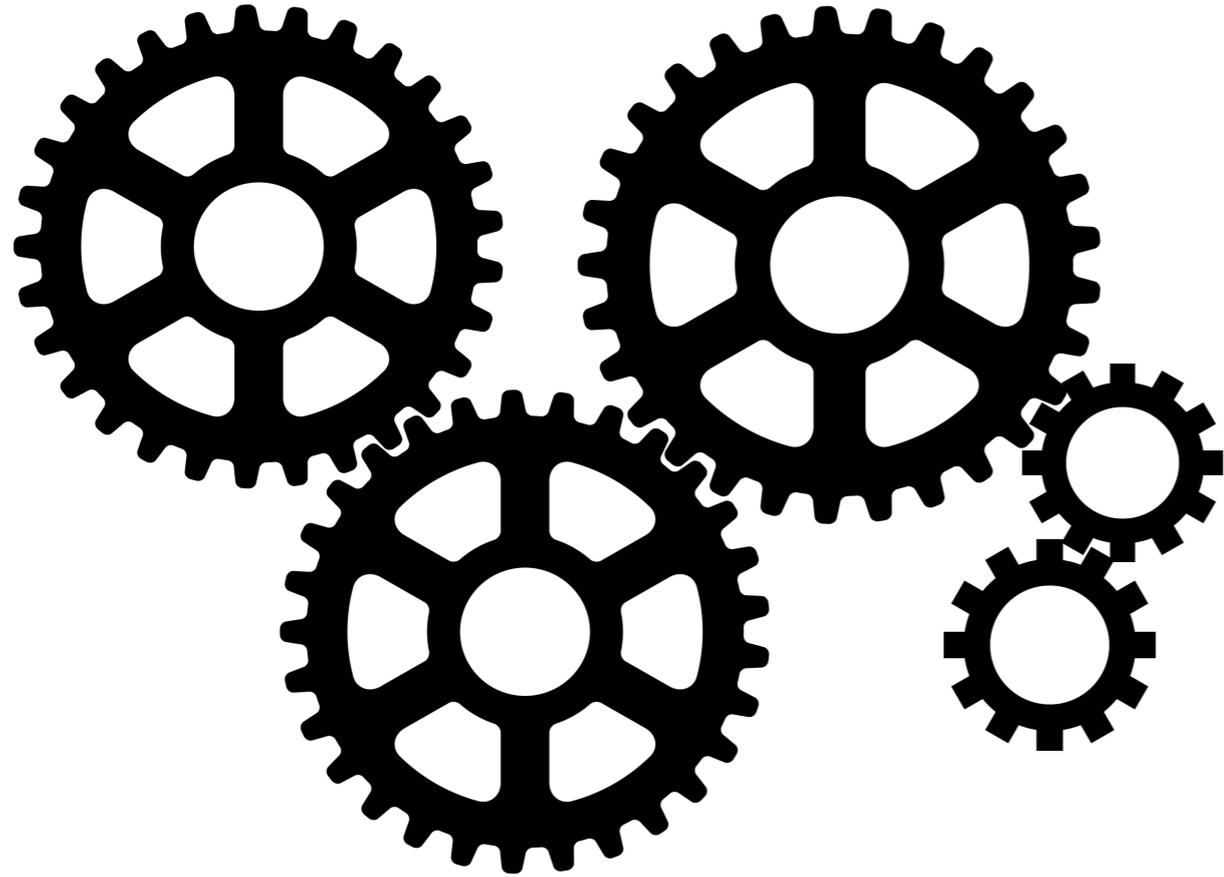


Abstraction

Galois connection

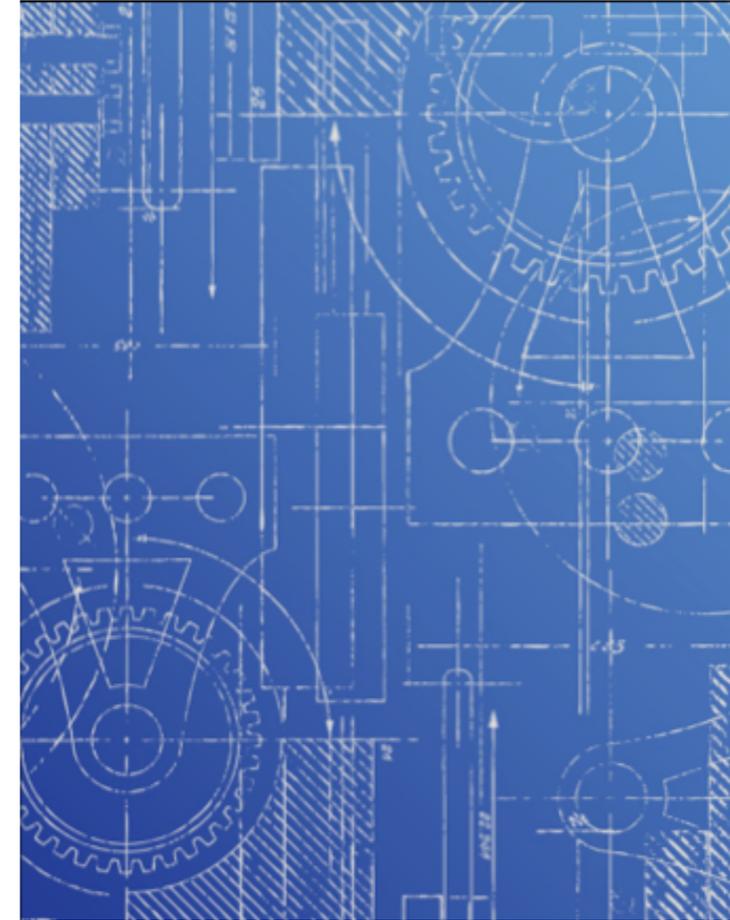
$$\alpha(c) \sqsubseteq a \iff c \sqsubseteq \gamma(a)$$

# Concrete Interpreter



+

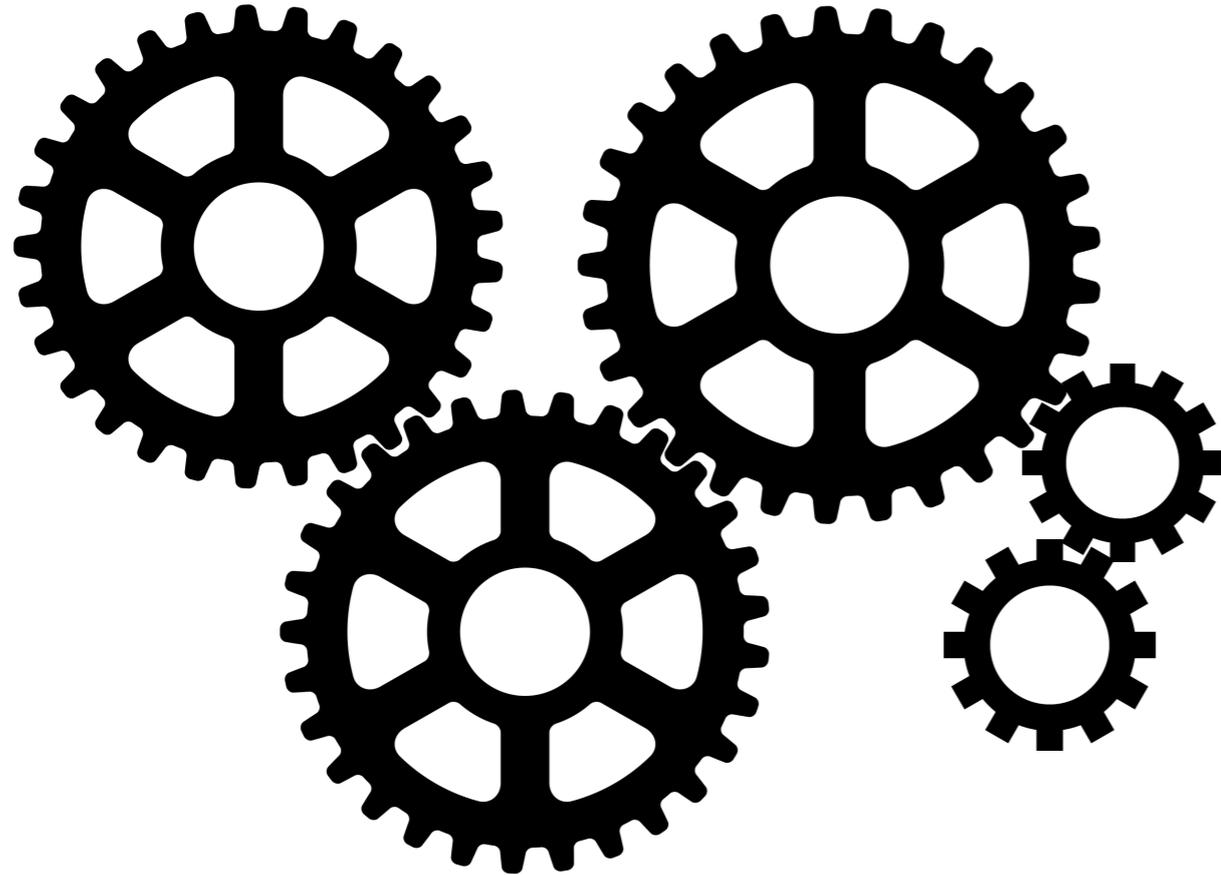
# Abstraction Specification



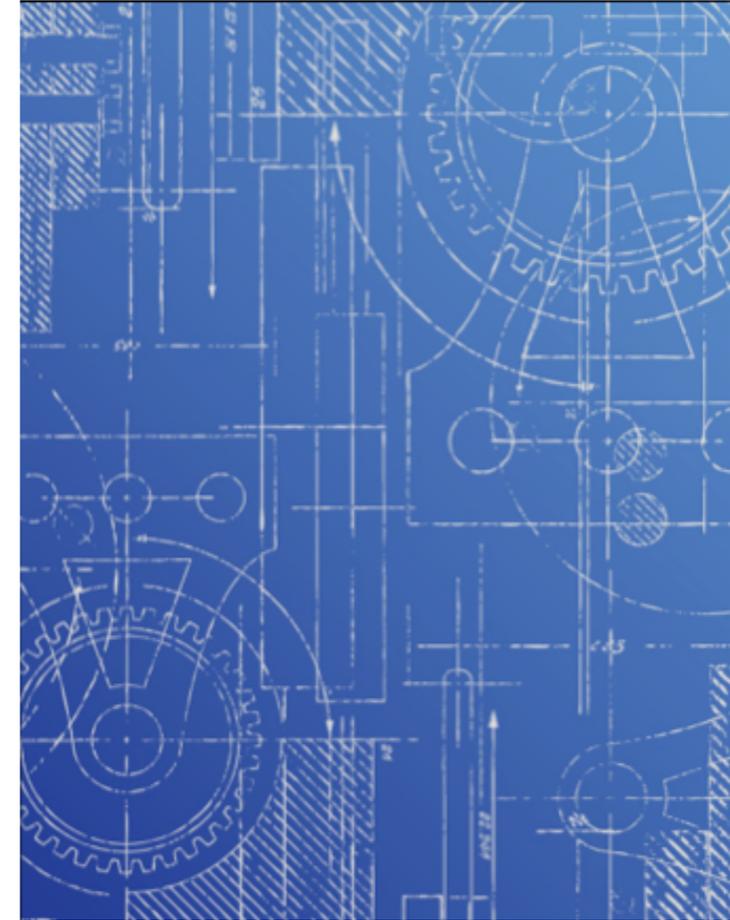
# Abstract Interpreter

# Concrete Interpreter

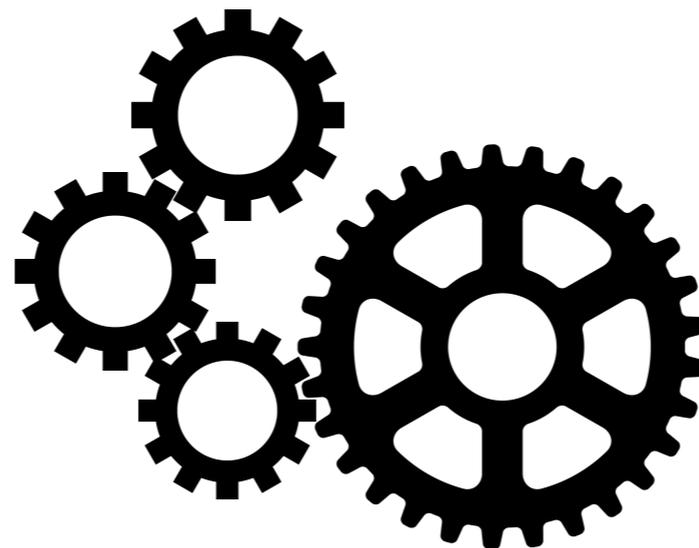
# Abstraction Specification



+



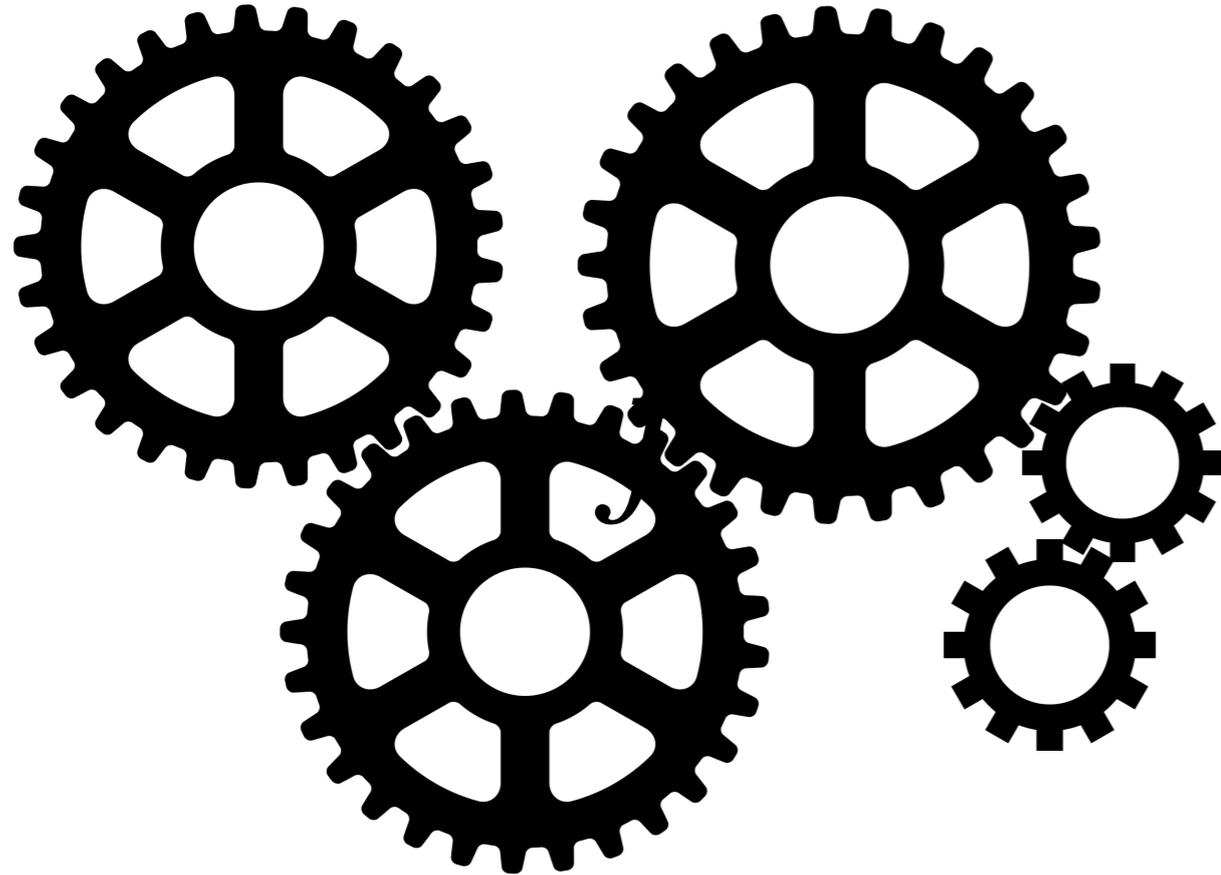
=



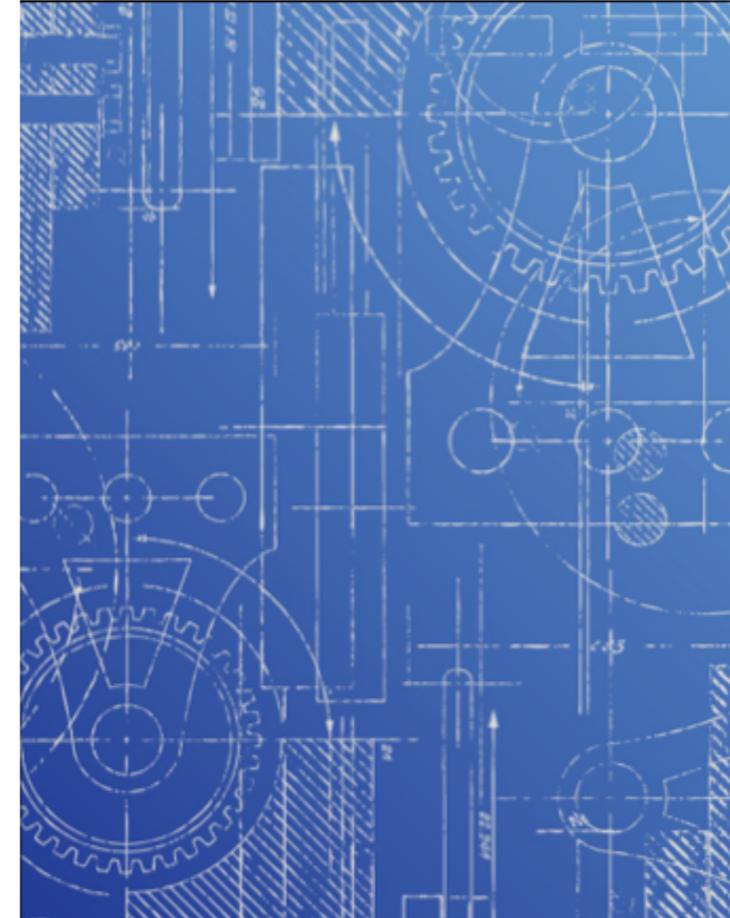
# Abstract Interpreter

# Concrete Interpreter

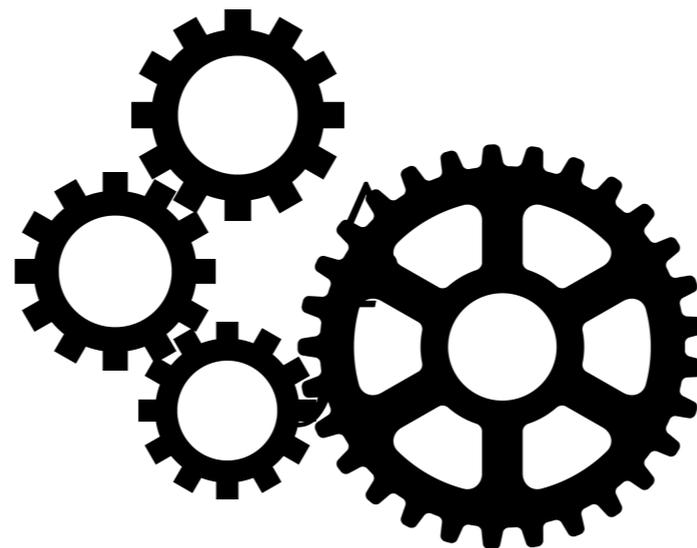
# Abstraction Specification



+



=



# Abstract Interpreter

# Concrete Interpreter

# Abstraction Specification

$$f + (\alpha, \gamma)$$

$$= \hat{f} \quad \text{Abstract Interpreter}$$

# The calculational approach to Abstract Interpretation

$$\hat{f} = \alpha \circ f \circ \gamma$$

Cousot, Cousot (1976, 1977, 1979)

```
public static int nextOdd(int x) {  
    if (isEven(x))  
        return x+1;  
    else return x+2;  
}
```

```
public static int nextOdd(int x) {  
    if (isEven(x))  
        return x+1;  
    else return x+2;  
}
```

**{Odd}  $\hat{+}$  {Odd}**

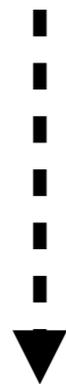
```

public static int nextOdd(int x) {
    if (isEven(x))
        return x+1;
    else return x+2;
}

```

**{Odd} <sup>^</sup> + {Odd}**

**$\gamma$**



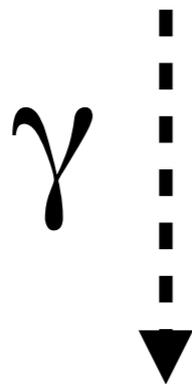
**{..., -3, -1, 1, 3, ...} + {..., -3, -1, 1, 3, ...}**

```

public static int nextOdd(int x) {
    if (isEven(x))
        return x+1;
    else return x+2;
}

```

$\{\text{Odd}\} \overset{\wedge}{+} \{\text{Odd}\}$



$\{\dots, -3, -1, 1, 3, \dots\} + \{\dots, -3, -1, 1, 3, \dots\} = \{\dots, -6, -4, -2, 0, 2, 4, 6, \dots\}$

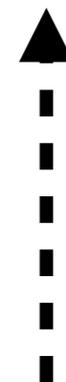
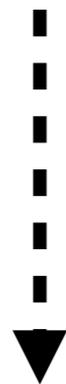
```

public static int nextOdd(int x) {
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}

```

$\{\text{Odd}\} \overset{\wedge}{+} \{\text{Odd}\}$

$\gamma$



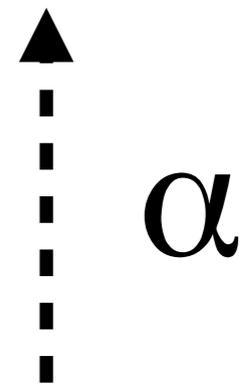
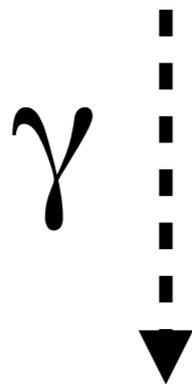
$\alpha$

$\{\dots, -3, -1, 1, 3, \dots\} + \{\dots, -3, -1, 1, 3, \dots\} = \{\dots, -6, -4, -2, 0, 2, 4, 6, \dots\}$

$$(\hat{+}) = \alpha \circ (+) \circ \gamma$$

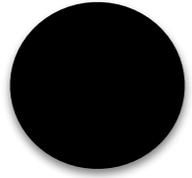
Calculated abstract implementation

$$\{\text{Odd}\} \hat{+} \{\text{Odd}\} = \{\text{Even}\}$$

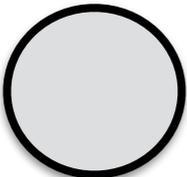
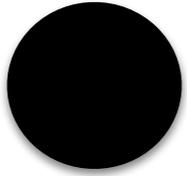


$$\{\dots, -3, -1, 1, 3, \dots\} + \{\dots, -3, -1, 1, 3, \dots\} = \{\dots, -6, -4, -2, 0, 2, 4, 6, \dots\}$$

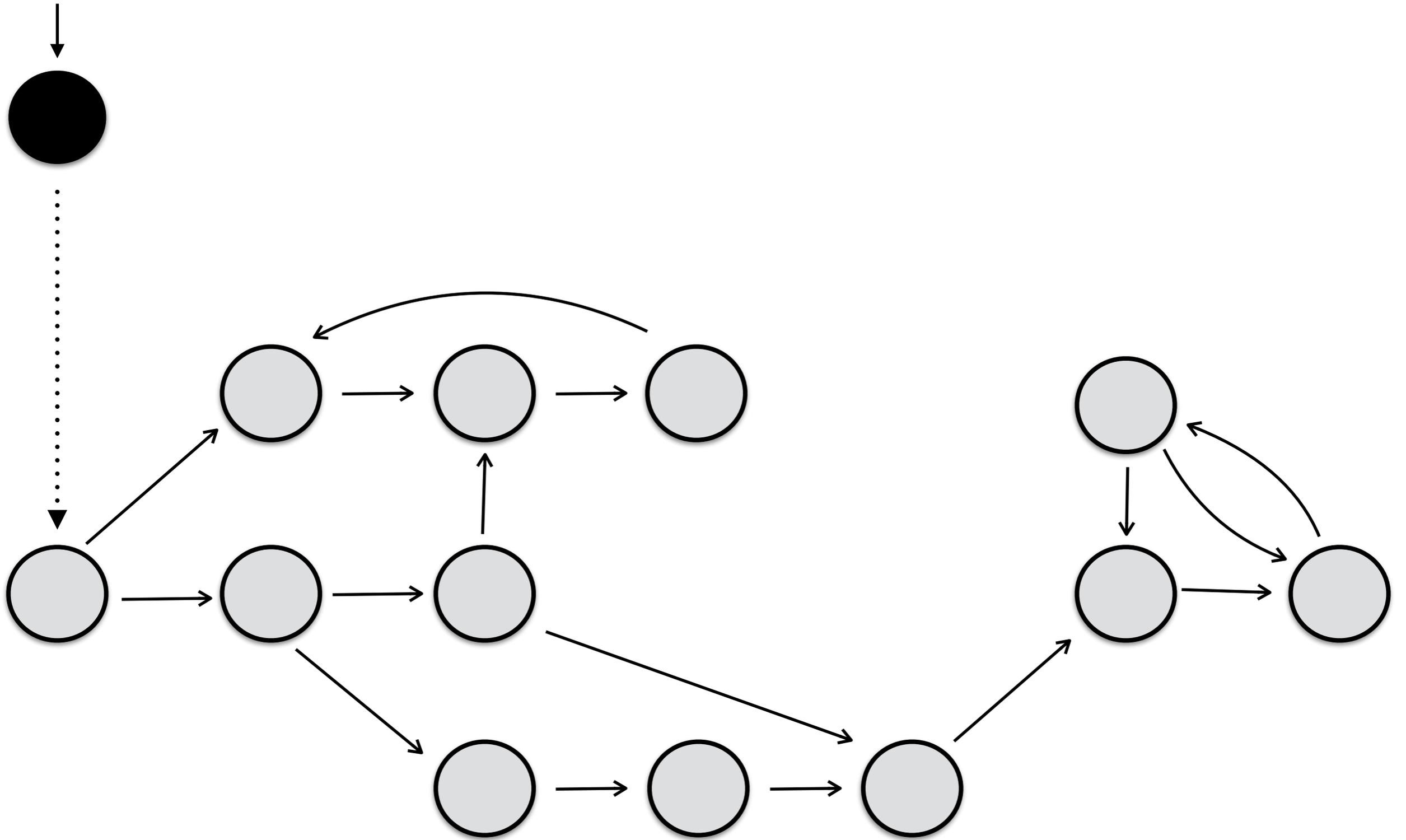
prog



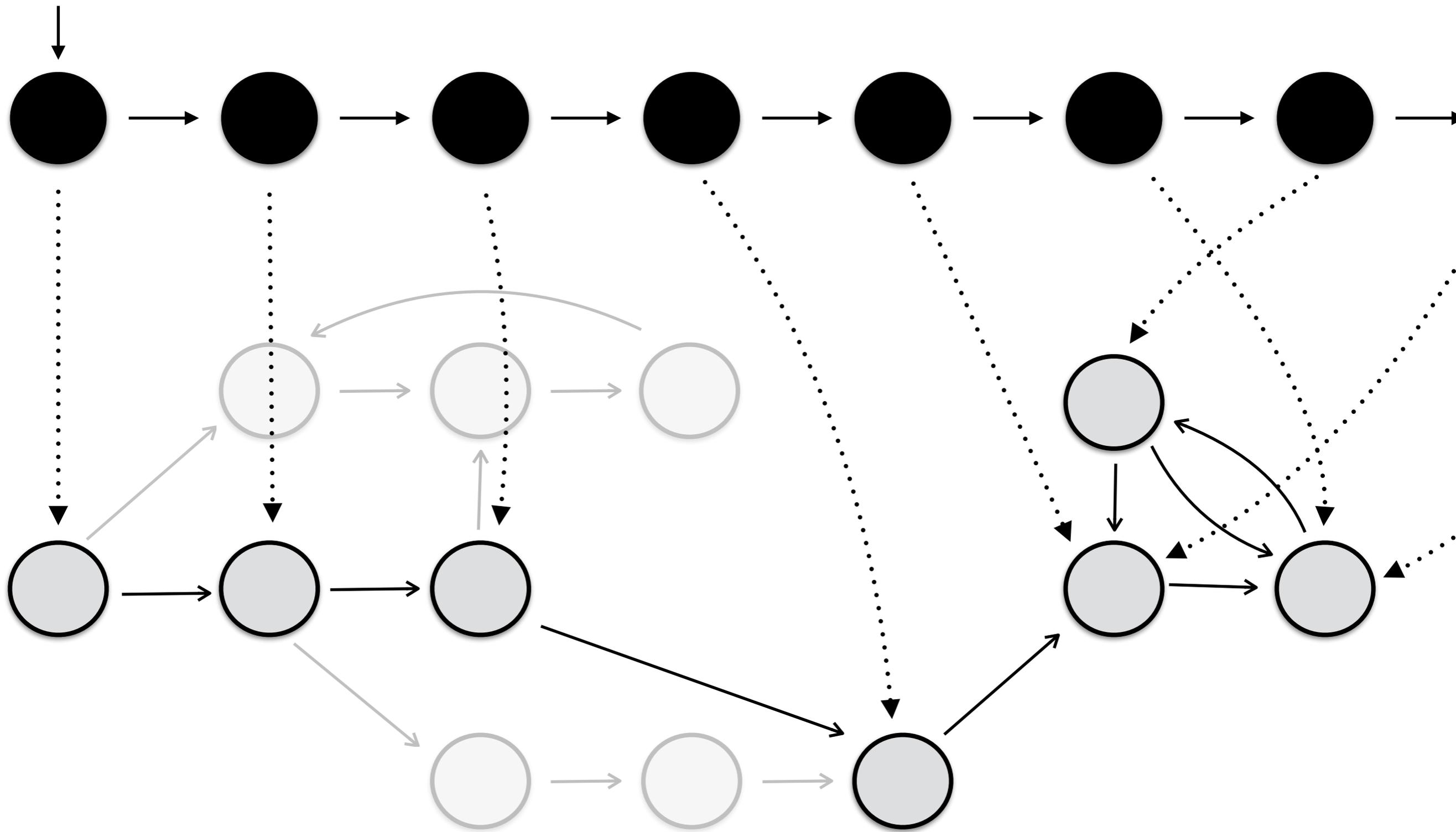
prog



prog



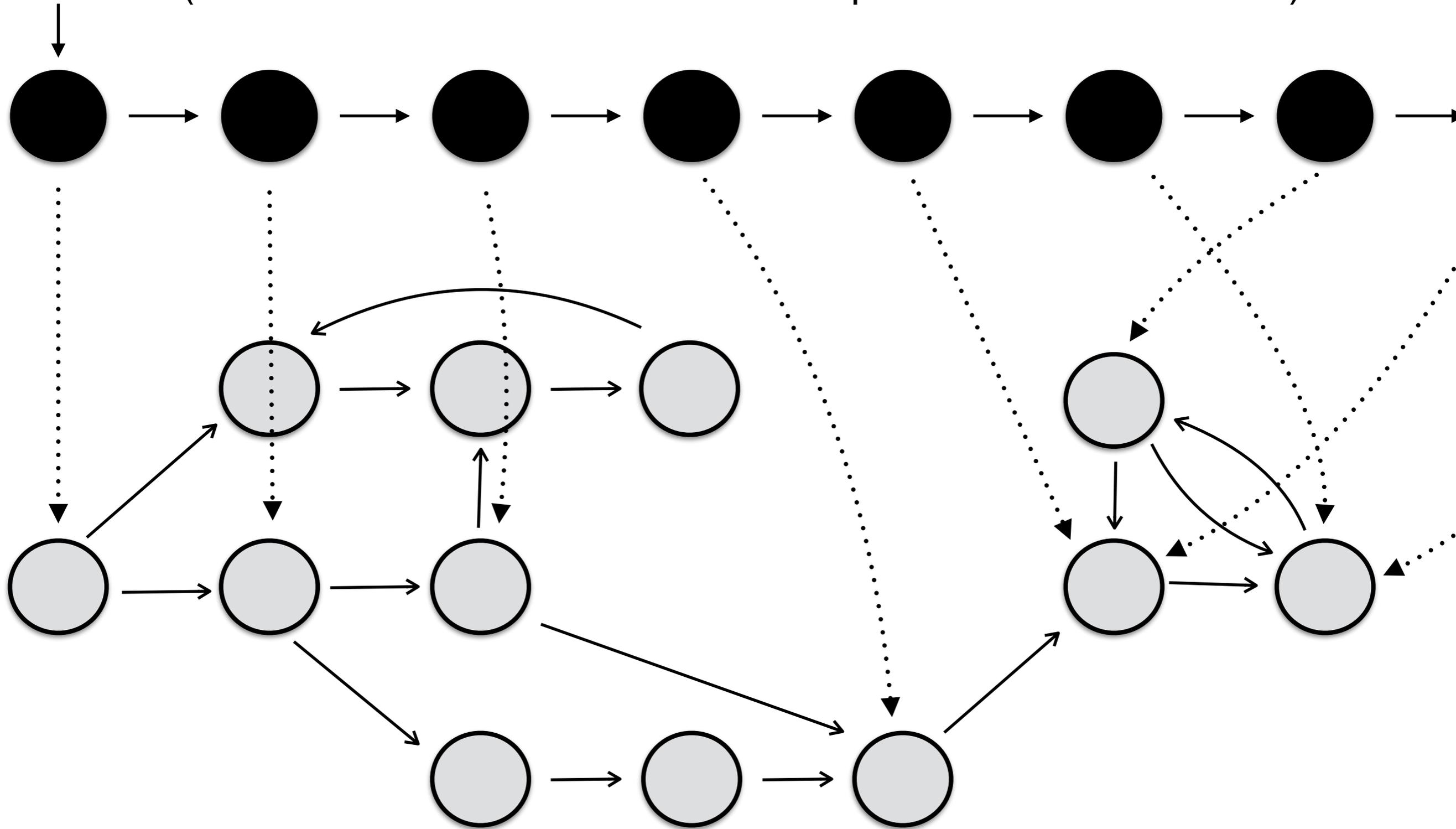
prog



# Soundness Condition

prog

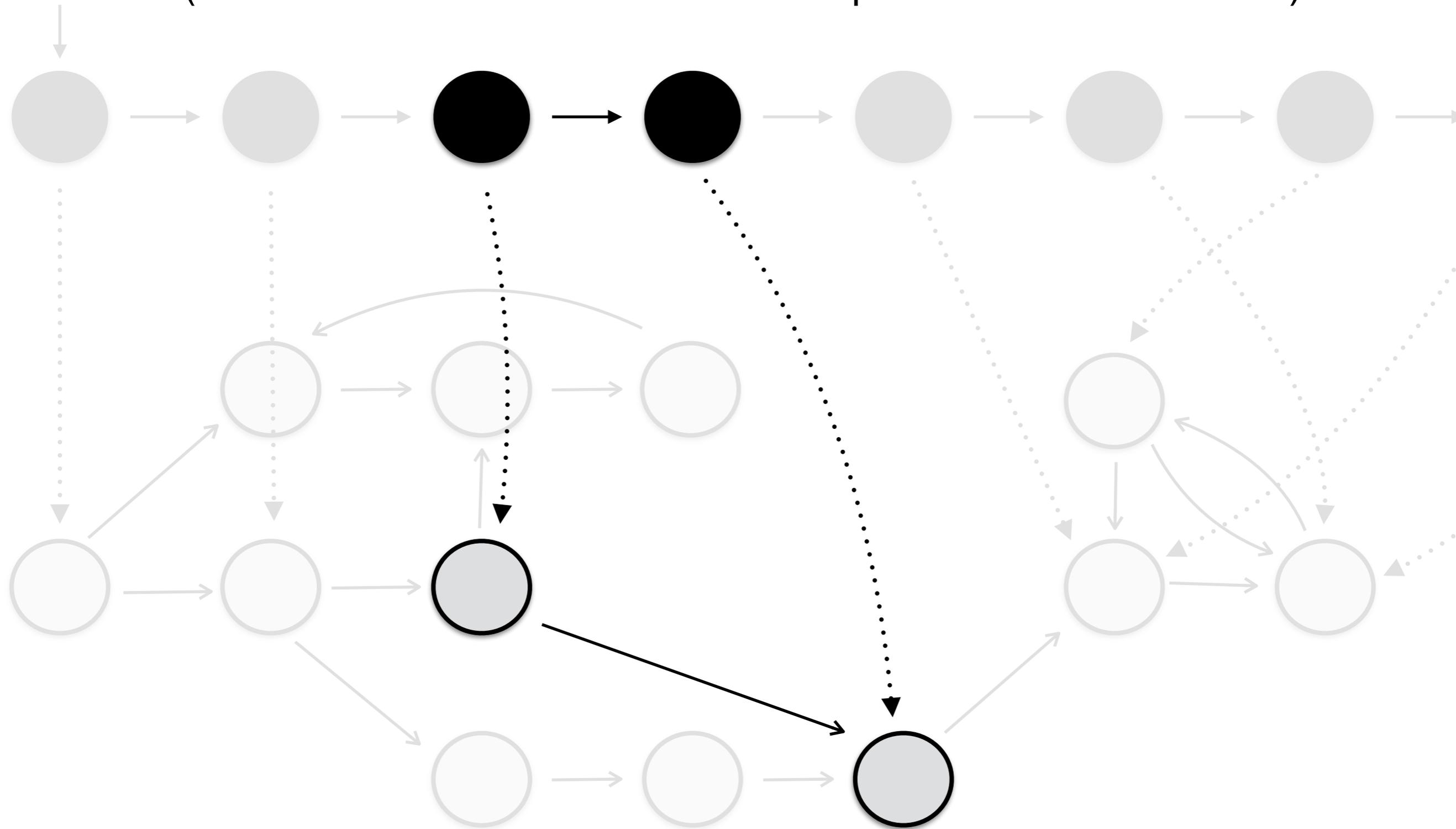
(all observable behaviors must be represented in our model!)



# Soundness Condition

(all observable behaviors must be represented in our model!)

prog



# Abstract Interpretation + Symbolic Execution



```

    }

    return e;
}

void insert(const T& ele, s64 index = 0)
{
    // Precondition:
    assert(length >= index);

    // Possible reallocation, shift-back

    // Placement-new a T at index
    new (&buff[index]) T(ele);

    // Postcondition:
    assert(length <= buff_length);
}

```

```
    }  
    return e;  
}  
  
void insert(const T& ele, s64 index = 0)  
{  
    // Precondition:  
    if (!(length >= index))  
        err("Assert failed.");  
  
    // Possible reallocation, shift-back  
  
    // Placement-new a T at index  
    new (&buff[index]) T(ele);  
  
    // Postcondition:  
    if (!(length <= buff_length))  
        err("Assert failed.");  
}
```

```

    return e;
}
this =  $\alpha$    ele =  $\beta$    index =  $\gamma$ 

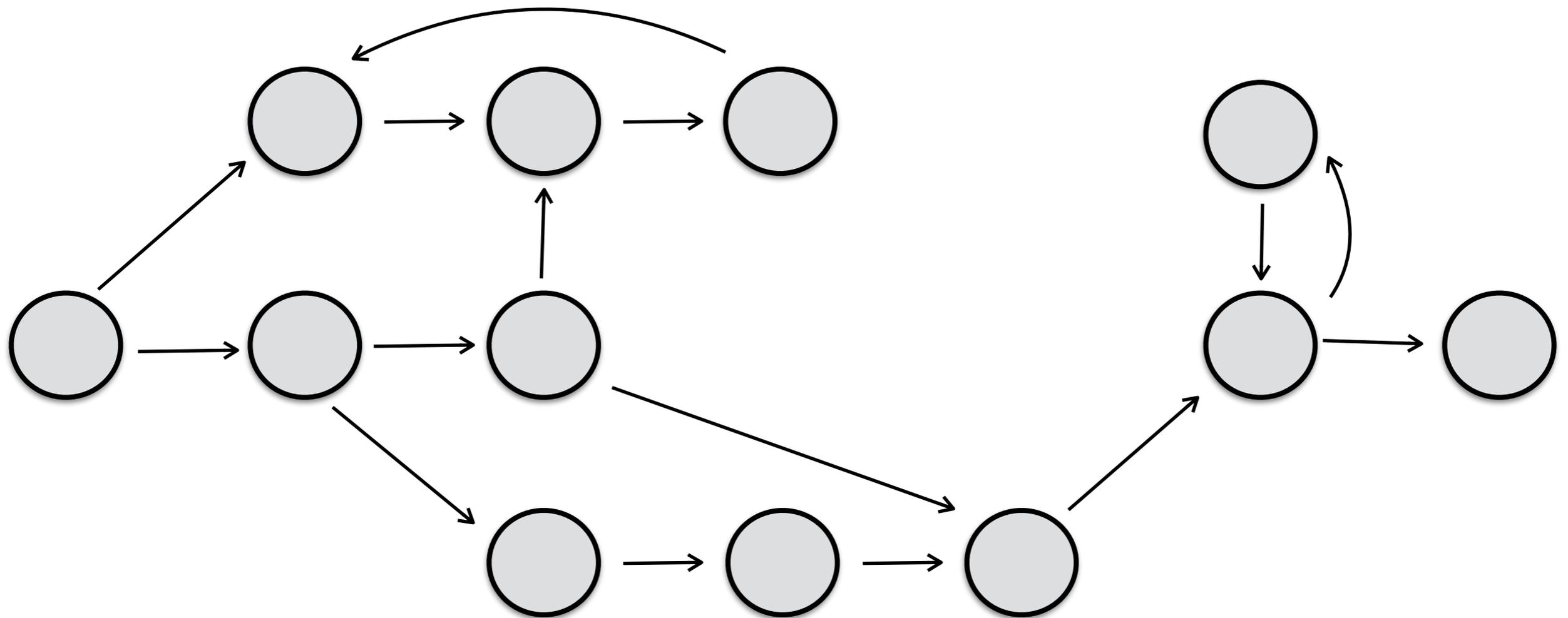
```

```

void insert(const T& ele, s64 index = 0)
{
    // Precondition:  $\alpha.\mathbf{length} < \gamma$ 
    if (!(length >= index))
        err("Assert failed.");
    // Possible reallocation, shift-back
    // Placement-new a T at index
    new (&buff[index]) T(ele);
    // Postcondition:  $\alpha.\mathbf{length} \geq \gamma$ 
    if (!(length <= buff_length))
        err("Assert failed.");
}

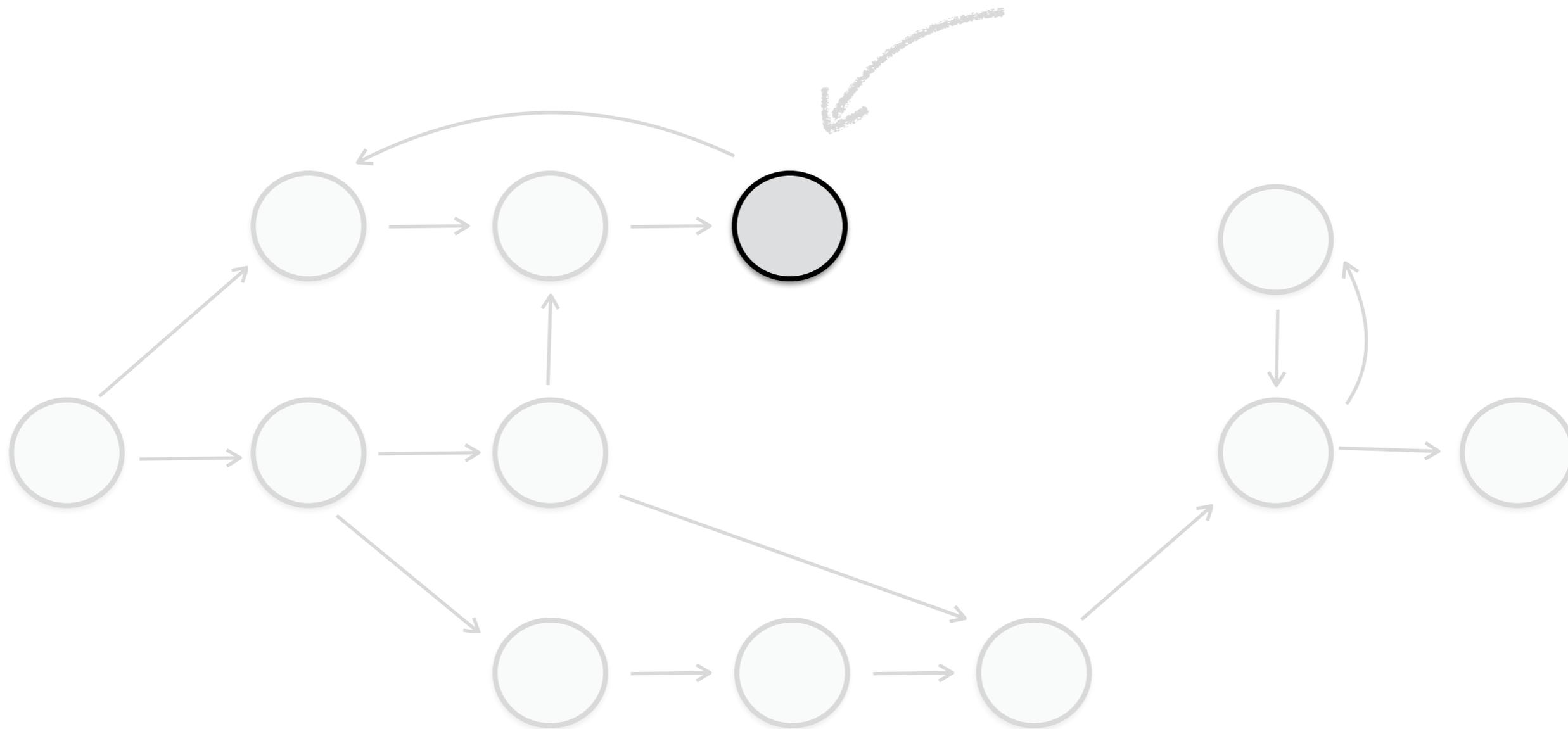
```

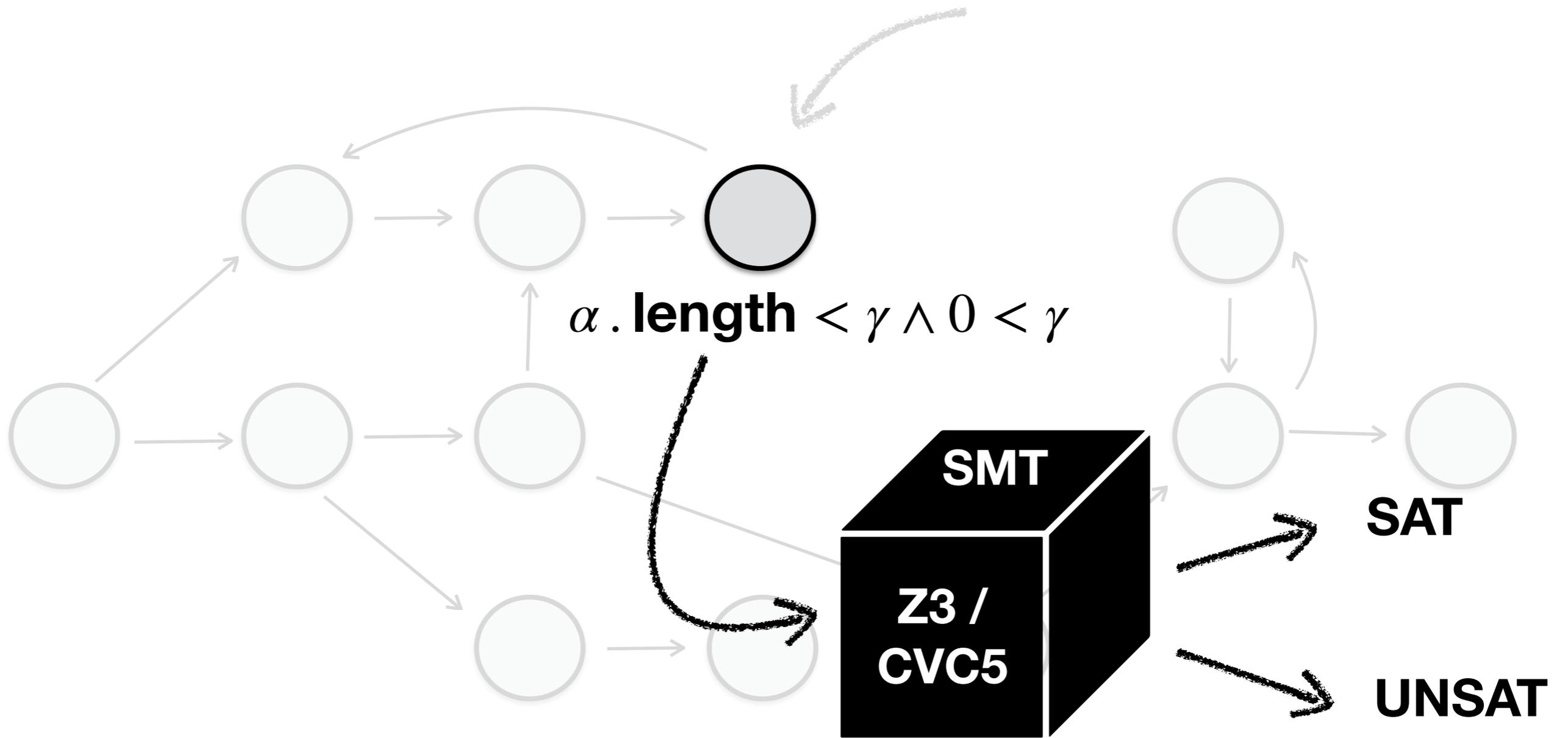
# Abstract Symbolic Execution



“Soft Contract Verification for Higher-order Stateful Programs”.  
**Nguyễn, Gilray, Tobin-Hochstadt, Van Horn. 2018.**

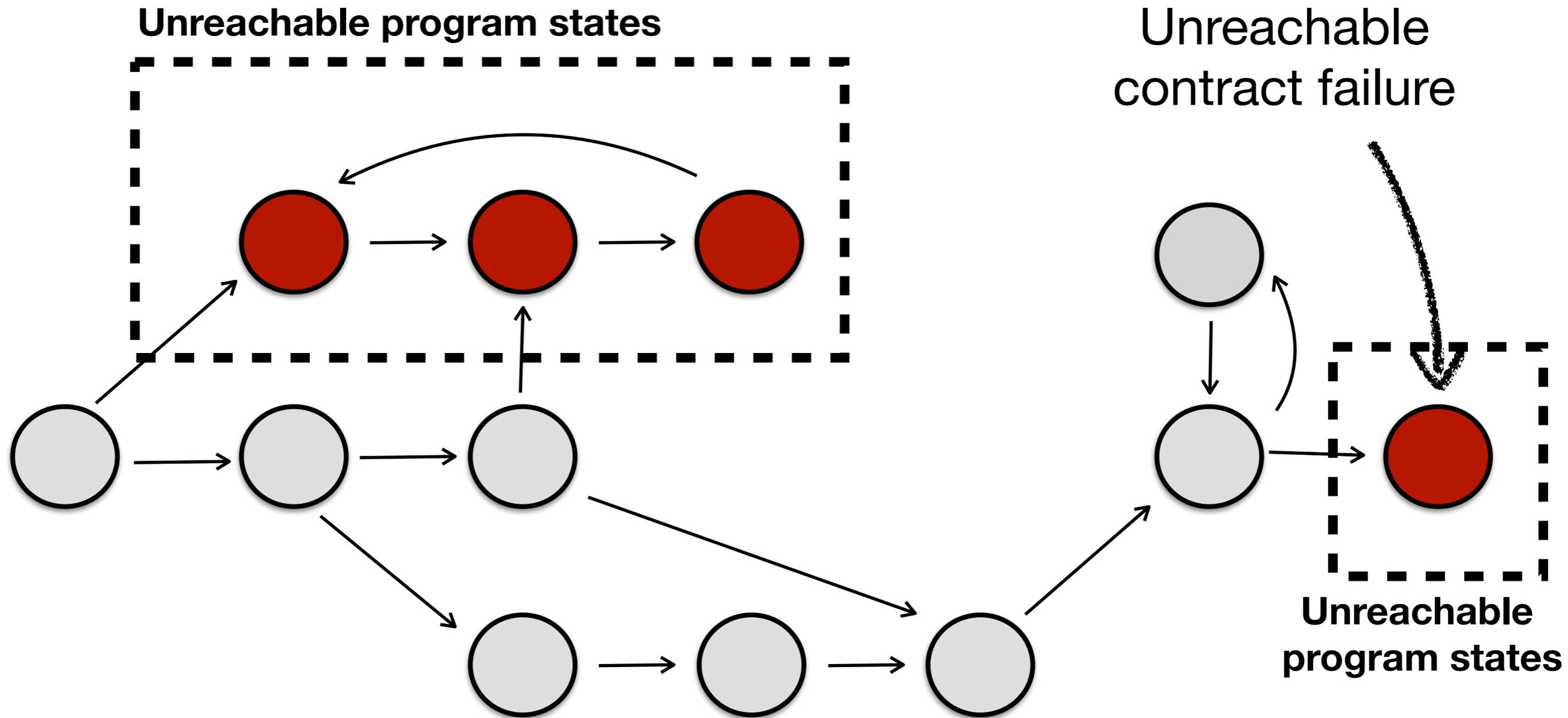








# Abstract Symbolic Execution



“Soft Contract Verification for Higher-order Stateful Programs”.  
**Nguyễn, Gilray, Tobin-Hochstadt, Van Horn. 2018.**

```

// [number] -> [string]
function array_numtostr(arr) {
    assert(arr instanceof Array
           && arr.reduce((a,n)=>(typeof n)
                        == "number" && a, true));

    var str_arr = [];
    for (var i = 0; i < arr.length; ++i)
        str_arr.push(arr[i]+"");

    assert(str_arr instanceof Array
           && str_arr.reduce((a,s)=>(typeof s)
                        == "string" && a, true));
    return str_arr
}

```

```

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  return str_arr
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```





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- *Abstract symbolic execution* (AI+SE) gives us a way to verify contracts *on a best-effort basis* where a failure to verify a contract *degrades gracefully* to run-time monitoring.

# Thanks

- Contracts are a *linguistic mechanism* for embedding *dynamic monitors to enforce program correctness*.
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