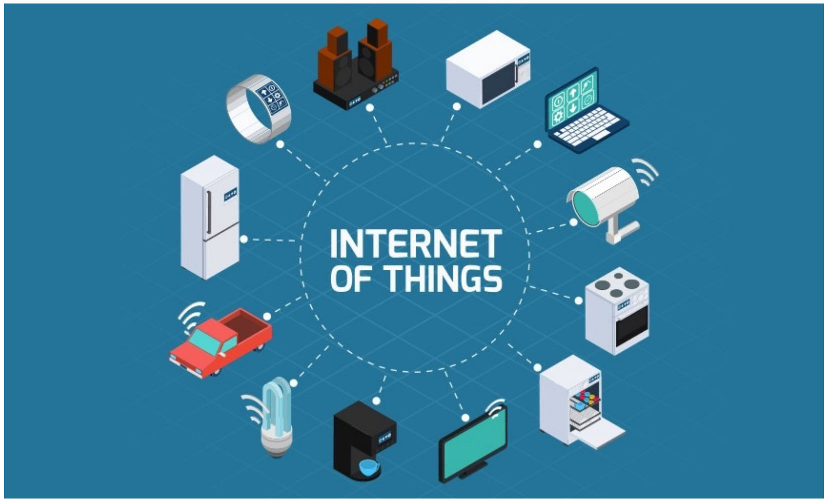
Recent Progress in Modern Cryptography

Feng-Hao Liu
feng-hao.liu@wsu.edu
Associate Professor
Washington State University

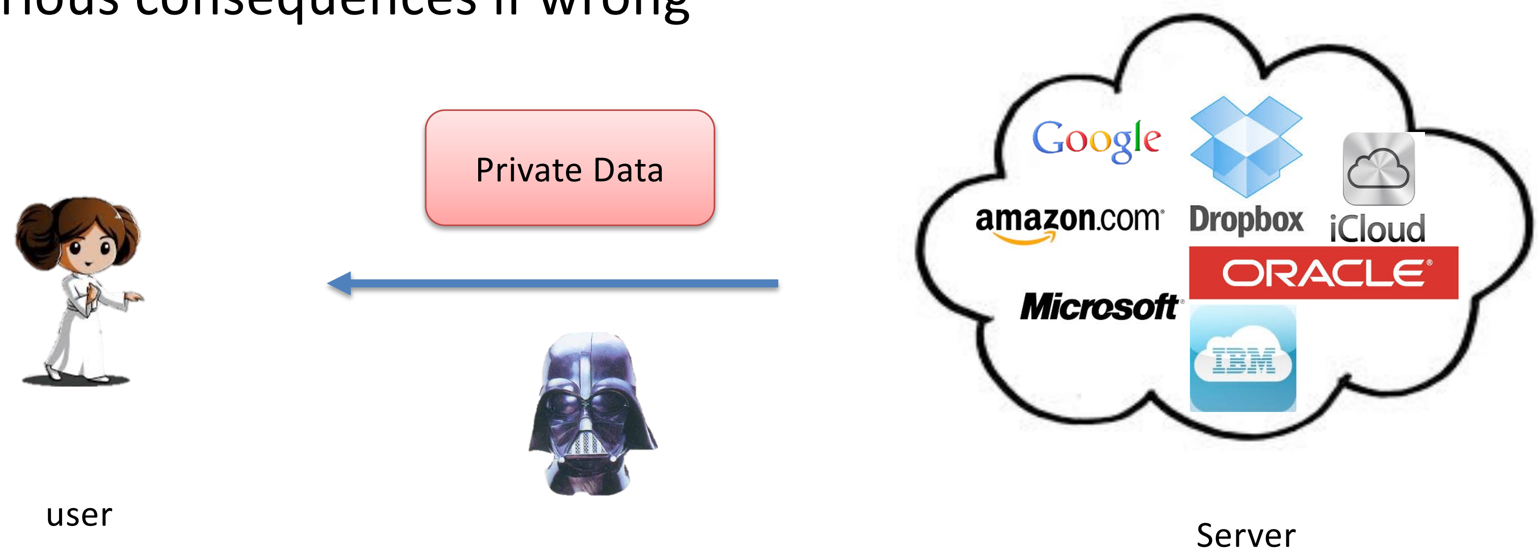
Internet Technology

- Build a connected world
  - Online/mobile banking
  - Email
  - Social media
  - Online conference



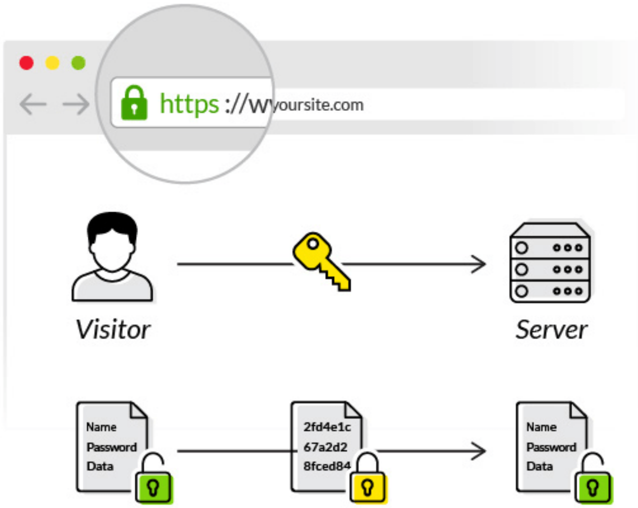
Security of Cyberspace

- Privacy and Authentication are important!
  - Sensitive data in cyberspace
  - Serious consequences if wrong



Important Technology

- Public-key cryptography (PKC)
  - Foundation of https
  - Email, secure payment, social media logins, etc.



Foundation of PKC

- Need math problems not solvable by even super computers
  - Only a few candidates

Factoring:

Secret key: (p, q), Public key: N=pq

RSA crypto systems

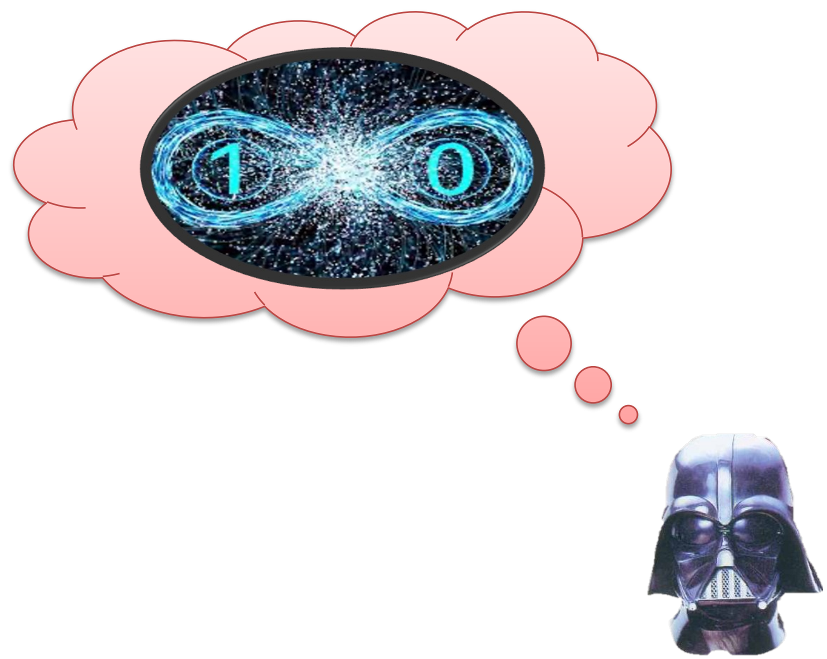
Discrete Log:

Secret key x, Public key: (g, g^x)

Diffie-Hellman Key Exchange

Foundation is Challenged!

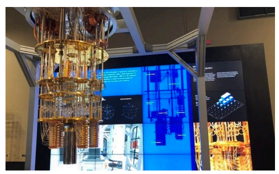
- [Shor94] poly-time quantum algorithm to solve factoring and DLog
  - Quantum is more powerful



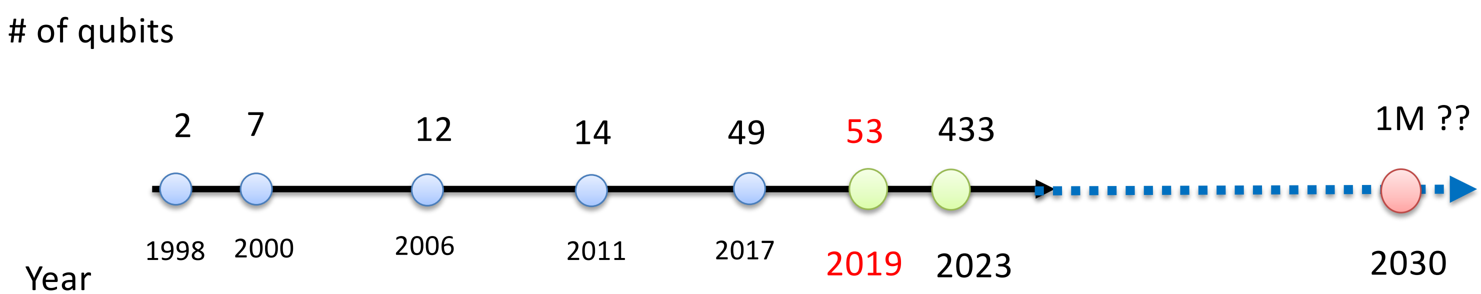
- Technology was not there.
  - E.g., “15 = 3 * 5” (2001)
  - Not practical yet

Technology Advances!

- Towards building larger quantum computers



IBM



First Quantum Supremacy
for specific task (Google)

Facing New Reality

- NIST: Post Quantum (PQ) PKC standardization call [2016 - ongoing]
- Industry: Evaluate performances of PQ candidates

AWS Security Blog

Post-quantum TLS now supported in AWS KMS

by Andrew Hopkins | on 04 NOV 2019 | in Advanced (300), AWS Key Management Service, Security, Identity, & Compliance | Permalink | Comments | Share

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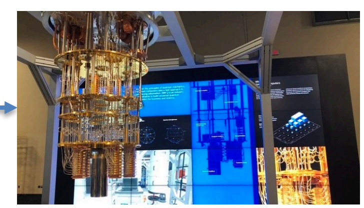
Traditional PKC Crisis

- Obsolete eventually at some point…
  - New security infrastructure

- Critical time to develop new sciences
  - New foundation for PQ Crypto
  - New advanced Crypto capabilities

Cryptography



Quantum Computing

My Research

- Basic Mission: Rebuild basic crypto tools against quantum computing
  - Post-quantum (PQ) cryptography [NIST current efforts]
  - Future security of internet applications

Foundation

- Vision: Enable efficient richer crypto capabilities
  - Computing on encrypted data, Fully homomorphic encryption (FHE)
  - Applications to private ML and data analytics
  - Numerous advanced crypto designs

Roadmap

- Background
- Recent Progress
- Challenges and Vision

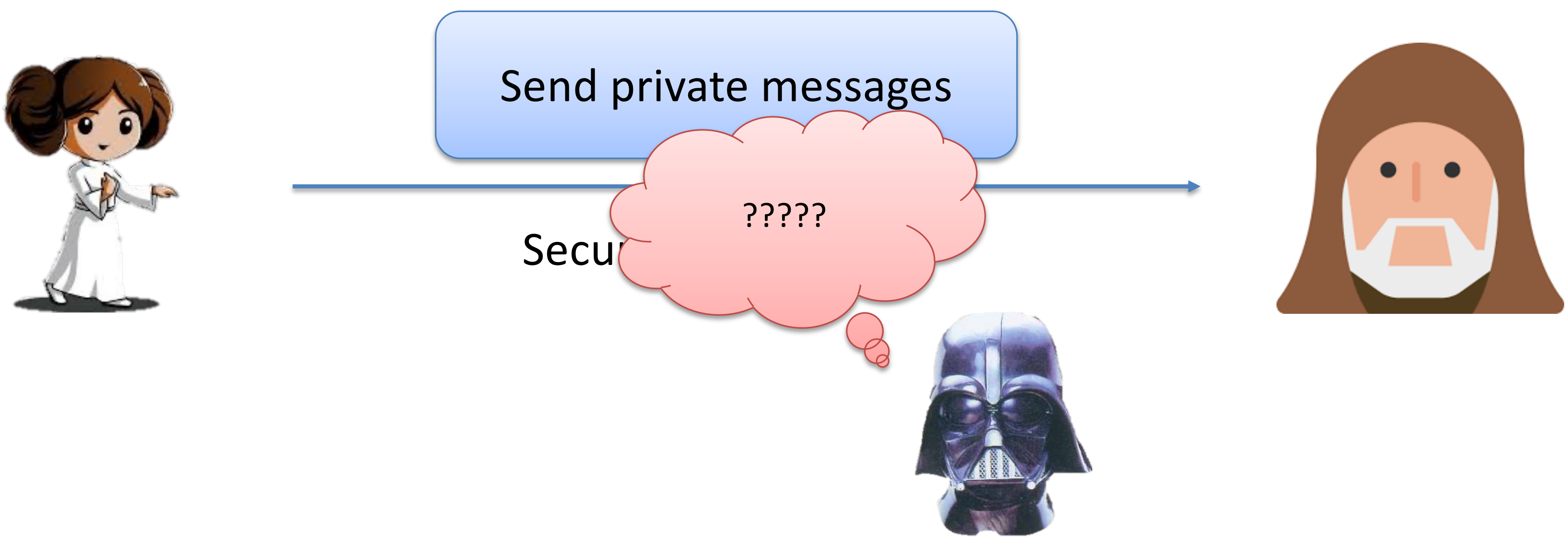
Cryptography in General

- What is "security"?
  - No attacker can "break" the system
  - What does that mean?

- How to achieve "security"?
  - How to defend against infinitely many possible attacks?

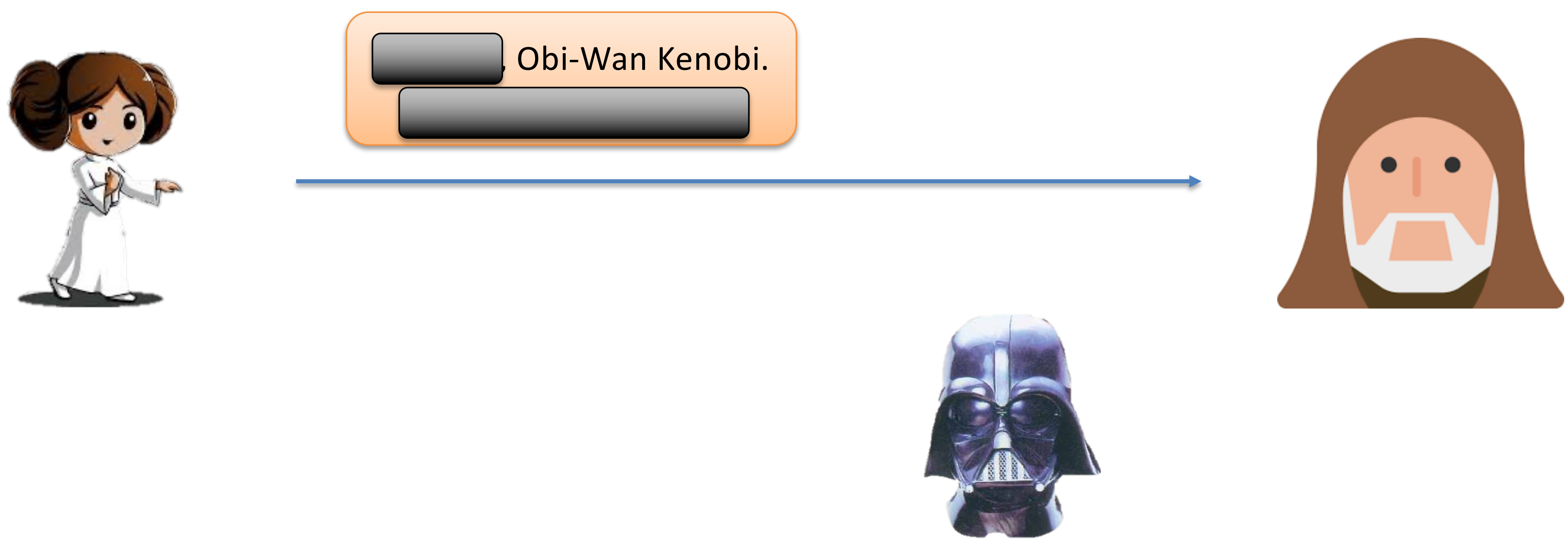
Modern Cryptography

- Define a Clear Security Goal
  - E.g., Secure Channel



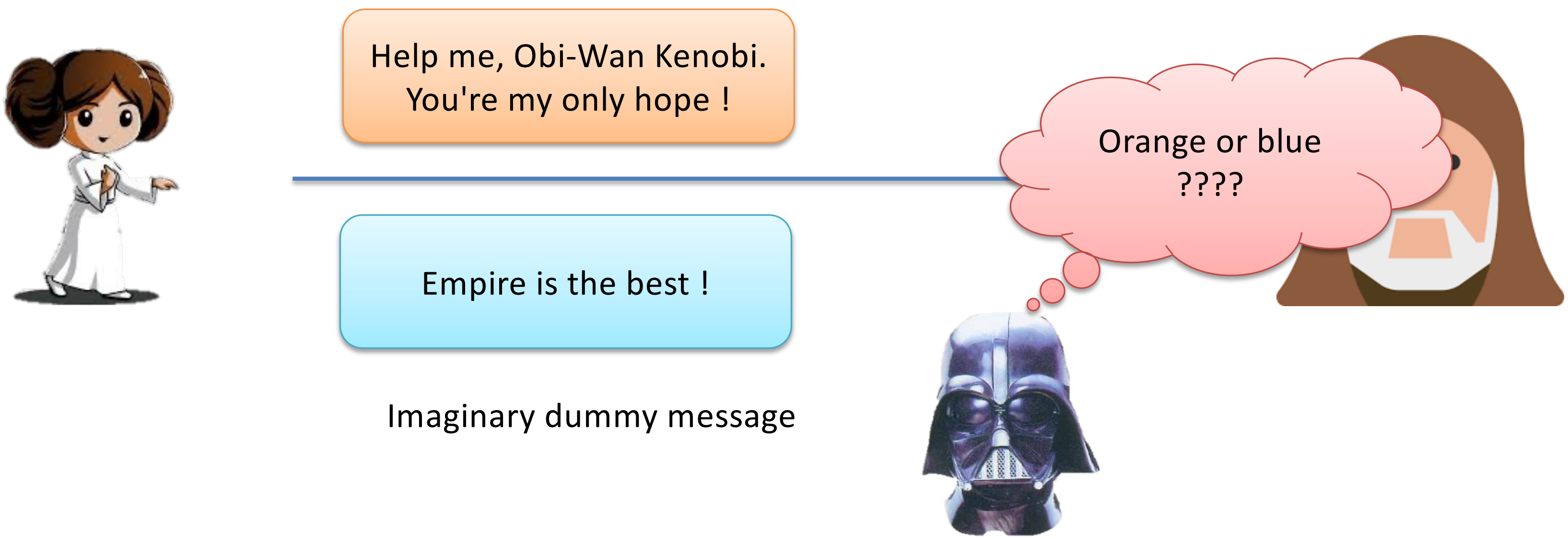
Modern Cryptography

- Define Security
  - Formulate a notion that captures “secure” channel
    - Not able to recover the whole plaintext?
    - We need: “attacker cannot learn anything” [Goldwasser-Micali82]



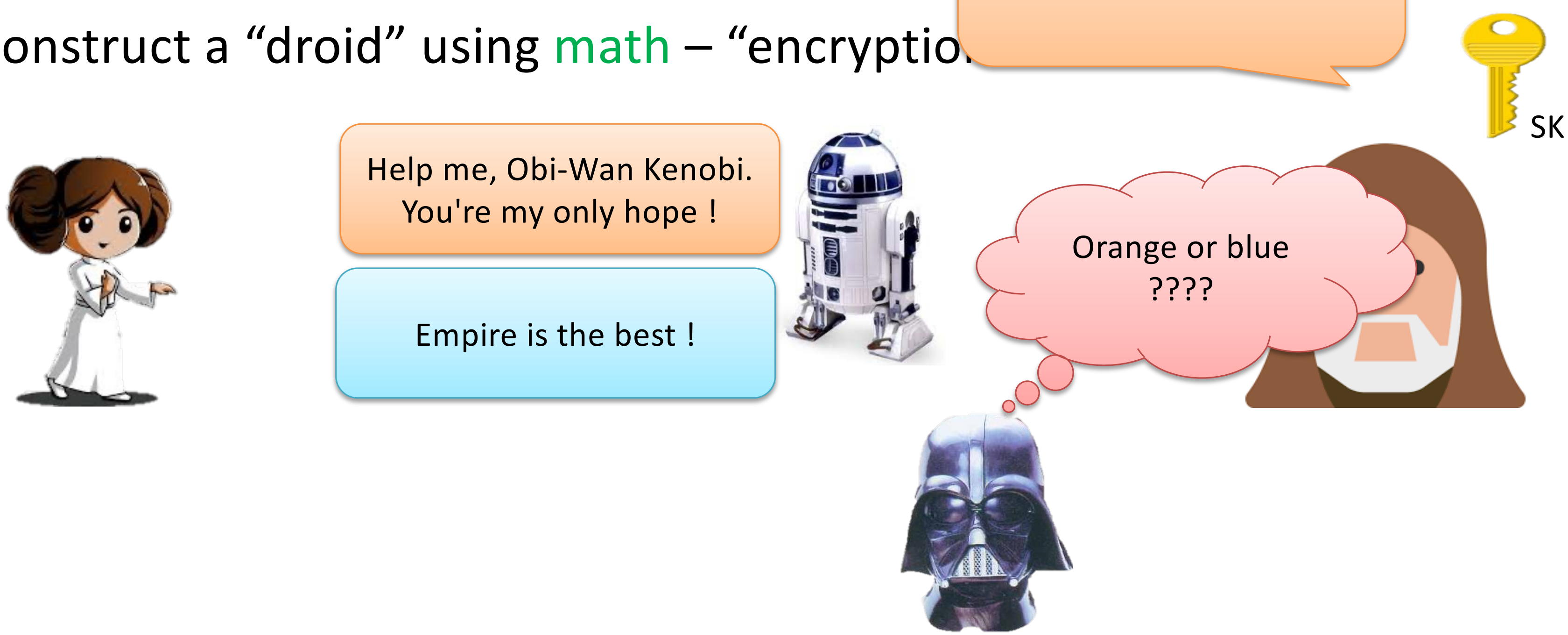
Modern Cryptography

- Define Security
  - Formulate a notion that captures "secure" channel
    - Not able to recover the whole plaintext?
    - We need: "attacker cannot learn anything" [Goldwasser-Micali82]
  - Explicitly requested in the NIST PQC call



Modern Cryptography

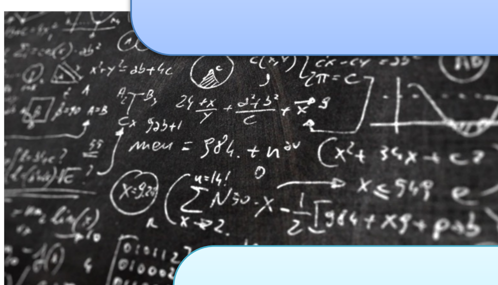
- How to realize the secure goal?
  - No real physical secure channel
  - Construct a “droid” using math – “encryptio



Modern Cryptography

- How do we prove security against infinitely many attacks?
  - The re
    - Har

If an adversary can break the crypto system, then there exists a reduction (that uses the adversary) that can solve the math problem.



Hard

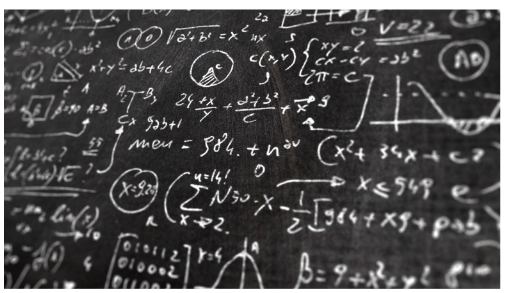
$\leq$

Reduction

If the math problem is not solvable, then no adversary can break the crypto system. => Crypto system is secure

Modern Cryptography

- We have some candidates
  - e.g., RSA, Discrete Log => Secure Crypto



Hard Math Problem

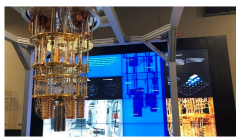
RSA, DL

$\leq$

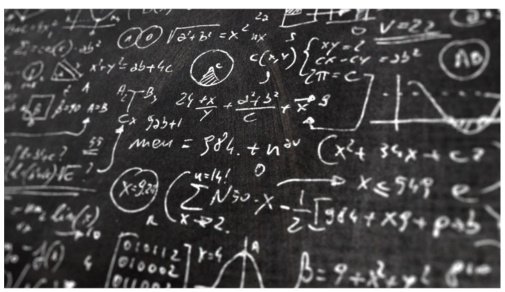
Reduction

Crypto System

In the Quantum Era

Quantum Computing



Hard Math Problem

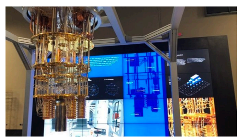
RSA, DL

$\leq$

Reduction

Crypto System

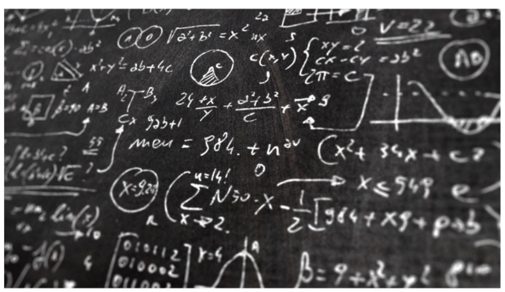
In the Quantum Era



Quan

What we need:

- New hard math problem against quantum
- New design and proof of security



$\leq$

Reduction

Crypto System

Hard Math Problem

~~RSA, DL~~

Roadmap

- Background
- Recent Progress
- Challenges and Vision

NIST's PQC Call

- Aim to standardize future PQC [2016 - now]
  - Take more than 20 years for migration
- Challenges
  - Hard to find plausible math problems
  - Setting specific parameters for efficiency + security
  - Implementation-level details
  - Real-world deployment

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NIST’s PQC Call

- New math hard problems and crypto designs
  - Code-based
  - Lattice-based
  - Hash-based
  - Isogeny-based
  - Multivariate-based
  - More…

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NIST Current Progress

- 3rd Round:
  - Public-key encryption: Kyber
  - Signature: Dilithium, FALCON, SPHINCS+
  - Selected for standardization
- 4th Round:
  - Ongoing
  - A lot going on

Lattice-based

Hash-based

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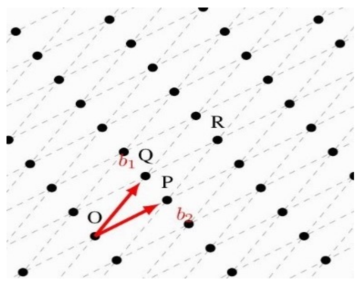
Sleepless Cryptographers

- Many candidates were broken
  - Rainbow (multivariate) [Crypto 2022]
  - SIKE (isogeny) [Eurocrypt 2023]
  - More …
- Still plausible
  - Lattice
  - Hash
  - Code
  - Perhaps isogeny ???

New Hope: Lattice-based Cryptography



- Advantages of Lattices:
  - Efficient operations
  - Resistance to quantum attacks (plausible)
  - Foundation of advanced crypto systems for richer crypto capabilities and applications

New Hard Problem

- Learning with errors (LWE) [Regev 2005]
  - Theory [Peikert's survey 16]
  - Practice [NIST ongoing PQC comp]

- New PQ candidates in theory!
  - Public-key encryption
  - Signatures
  - Key exchange
  - Three variants are going to be standardized by NIST

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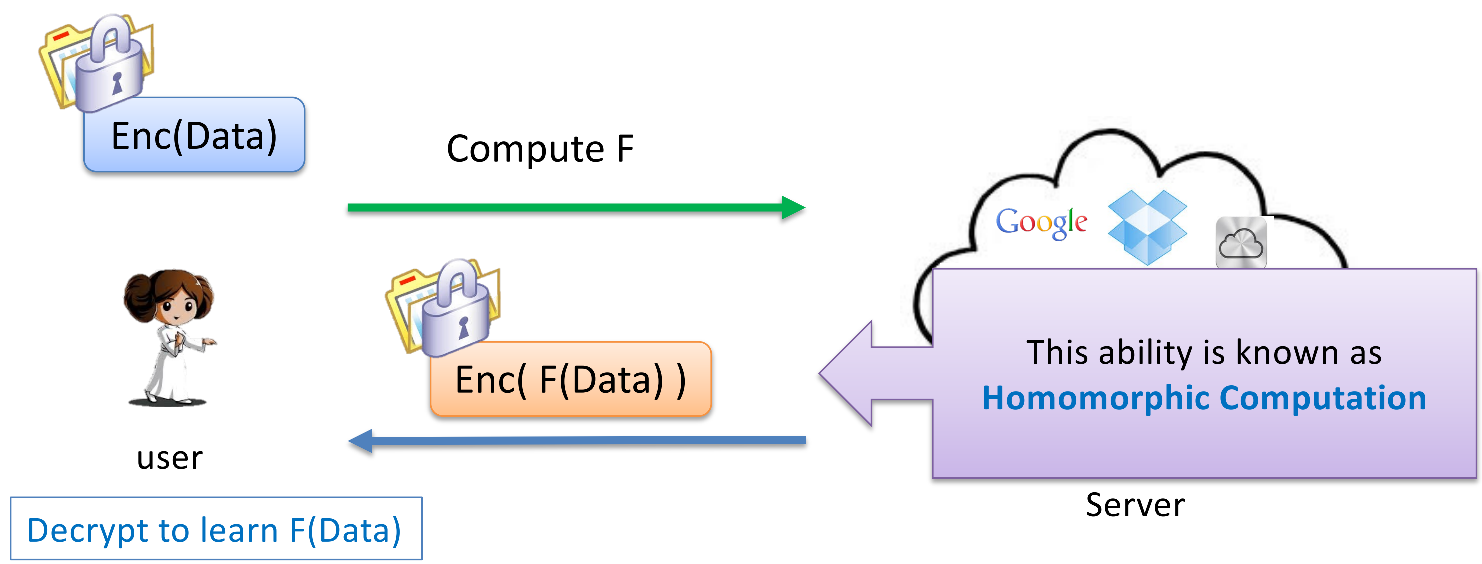
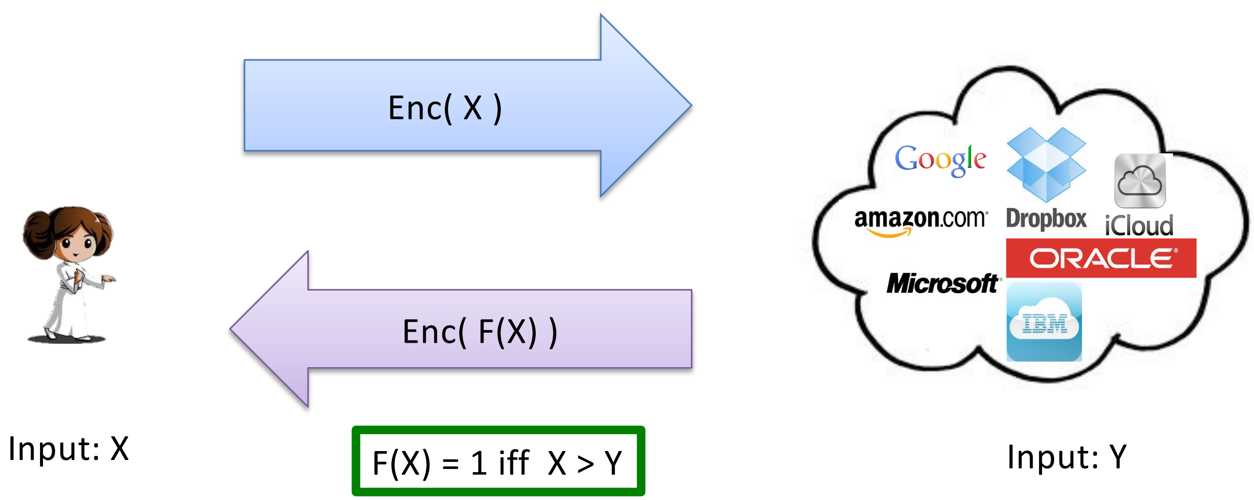
Advanced Capabilities

- Computation on encrypted data
  - Fully Homomorphic Encryption (FHE) [Gentry, BV, GSW]
  - Outsource computation
  - Holy grail to keep data secure while in use [DARPA DPRIVE]



Application to MPC

- An elegant solution to classic Yao's Millionaire Problem [1980's]
  - Two parties hold private inputs X and Y. Determine which is larger without revealing what they are
  - E-finance, e.g., compare numbers that are confidential



New Applications – Private MLaaS

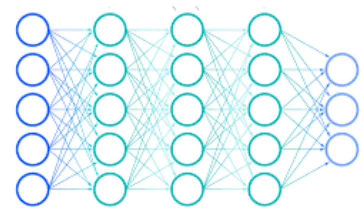
- New solutions to private ML as a Service [2020's]
  - Cloud holds a private ML Model Y
  - User has private data X
  - User wants to outsource analysis of private X without revealing X
  - Cloud does not want to reveal Y

Enc(X)

Enc(F(X))

Input: X

F(X) = analysis using param Y



Input: Y

Triumph of Crypto Theory

- Theorem 1 [Regev]
  - Under hardness LWE, there exists a PQ Public-key Encryption (PKE)
- Theorem 2 [Gentry, BV, BGV, GSW, AP…]
  - Under hardness of LWE, there exists a PQ fully homomorphic encryption (FHE) for any arbitrary function of homomorphic computation
- Theorems 3, 4, 5….
  - Under hardness of LWE, there exists a wide array of advanced PQ cryptosystems, e.g., identity-based encryption, attributed-based encryption, functional encryption, and more…

In Praise of LWE

- Gödel Prize 2018 to Regev

- Citation
  - Served as the foundation for countless subsequent works
  - Revolution in cryptography in both theory and practice
  - A simple and yet amazingly versatile foundation for nearly every kind of cryptographic object
    - along with many that were unimaginable until recently, and which still have no known constructions without LWE

Learning with Errors (LWE) [Regev]

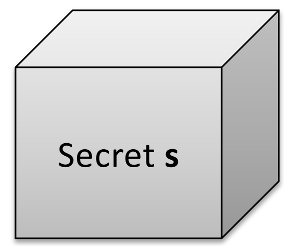
- Parameters: $n, m, q \in \mathbb{N}$ and error distribution χ over $\mathbb{Z}$.
- **Task:** learn secret $\boldsymbol{s} \in \mathbb{Z}_q^n$ given many "noisy inner products"

$$\mathbf{a}_1 \xleftarrow{\$} \mathbb{Z}_q^n, \quad b_1 = \langle \mathbf{a}_1, \mathbf{s} \rangle + e_1 \in \mathbb{Z}_q$$

$$\mathbf{a}_2 \xleftarrow{\$} \mathbb{Z}_q^n, \quad b_2 = \langle \mathbf{a}_2, \mathbf{s} \rangle + e_2 \in \mathbb{Z}_q$$

$$\vdots$$

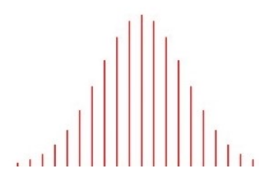
$$\mathbf{a}_m \xleftarrow{\$} \mathbb{Z}_q^n, \quad b_m = \langle \mathbf{a}_m, \mathbf{s} \rangle + e_m \in \mathbb{Z}_q$$



$\mathbf{b} = \mathbf{A} \cdot \mathbf{s} + \mathbf{e} \in \mathbb{Z}_q^m$.

nd **s**

Even powerful (quantum) adversary cannot learn the secret



$e_i \sim \chi$ (Gaussian)

However…

- LWE constructions are not efficient
  - Large Key
  - Large Ciphertext
  - Slow in computation…

How Inefficient?

- E.g., the original PKE of [Re
  – Later FHEs [BV11, GSW13] b

Research Goal

Improve LWE, leading to practical solutions

- Future cybersecurity infrastructure
- Advanced crypto capabilities

PK Length	1MB
Noise Sampling in Encryption	• 100 cycles per sample • Need 1000 samples • 30 – 40% of the process

- Large efficiency loss compared with pre-quantum RSA
  – E.g., 3072 or 4096 bits of PK length
- No NIST candidate really uses this [exact] sampling

To Improve Efficiency

- More algebraic structures [SSTX, LPR]
  - Use (polynomial, algebraic) rings
  - Called (Module) Ring LWE

- Use rounding [BPR]
  - Use elementary integer (INT) operation
  - Called (Module) Ring LWR [Rounding]
  - Imply pseudorandom functions (PRF)

Fundamental Questions

- Are the variants of LWE hard?
  - Would additional structures (ring and rounding) affect security?
  - The adversary might take advantage of …

- Crypto is subtle and sensitive …
  - Insecurity comes from nuances

Some of my Research

- Show formally Ring + Rounding variant is as hard as Ring

- Prior work
  - Ring LWE is as hard as some ideal lattice problem

- Ring + rounding is as hard as some ideal lattice problem
  - Currently no known algorithm for ideal lattice problem

Roadmap

- Background
- Recent Progress
- Challenges and Vision

Challenges

- On Apr 10, 2024, Yilei Chen claimed a new quantum poly-time algorithm that solves LWE
  - 9-step algorithm
  - eprint/2024/555
- Implications
  - PKE/Sig submitted to NIST are not affected under parameter range of the attack
  - Several variants of FHEs are not post-quantum (only 3rd gen remains secure)
  - LWE might not be post-quantum at all ….
- A lot of discussions….
  - https://scottaaronson.blog/?p=7946
  - https://nigelsmart.github.io/LWE.html
- On Apr 18, 2024, a fatal mistake was discovered
  - LWE and their variants remain post-quantum secure

Vision

- LWE +
  - New algebraic techniques to improve speed/security for PKE, FHE, etc
  - Analysis of the security foundation
  - Software + hardware acceleration

- LWE –
  - Make the algorithm great again !!!

Silvio Micali
Turing Award 2012

Science wins either way

Thank You!