

# Cryptography in the Presence of Quantum Computing --

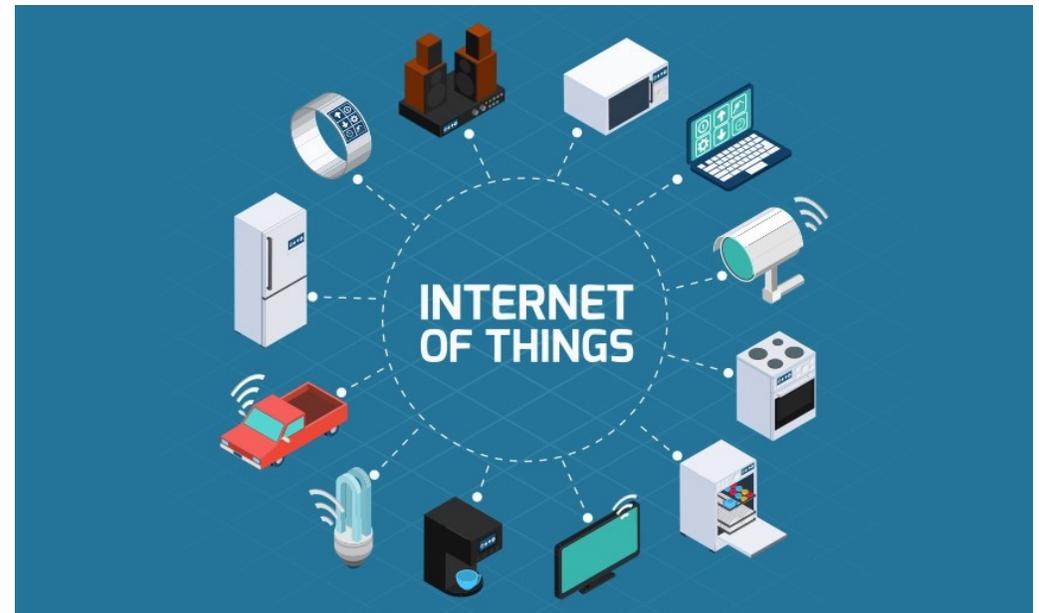
## New Opportunities and Research Directions

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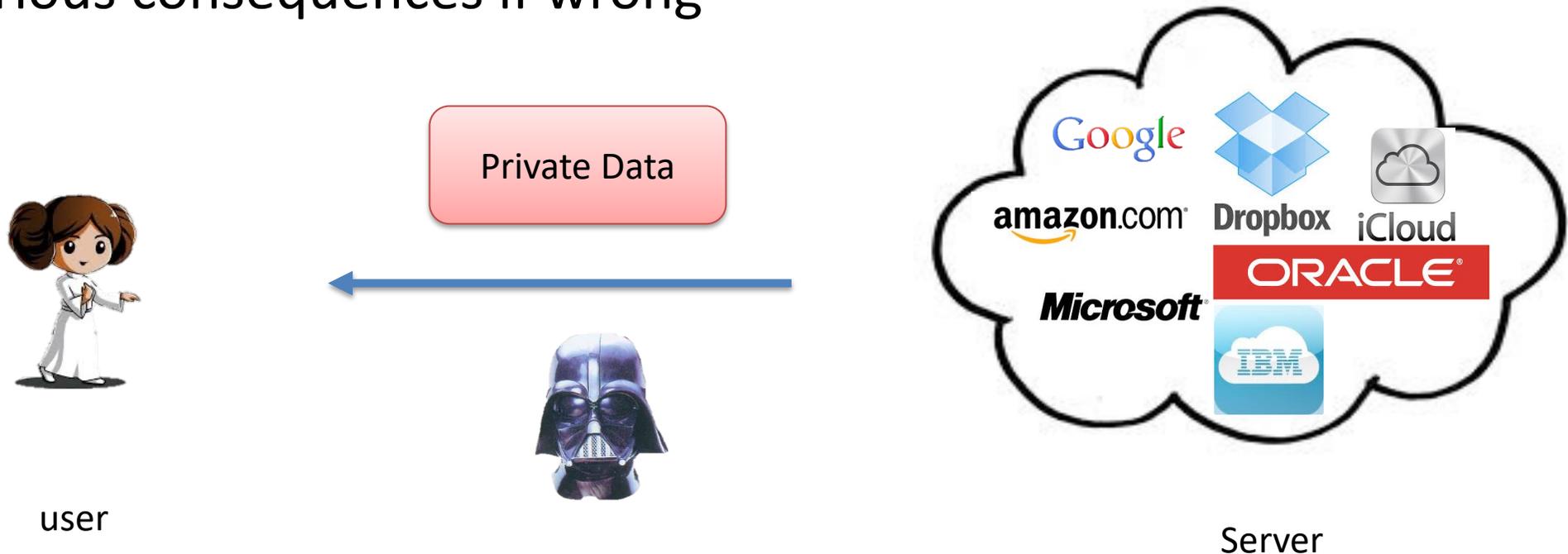
# Internet Technology

- Build a **connected** world
  - Online/mobile banking
  - Email
  - Social media
  - Online conference



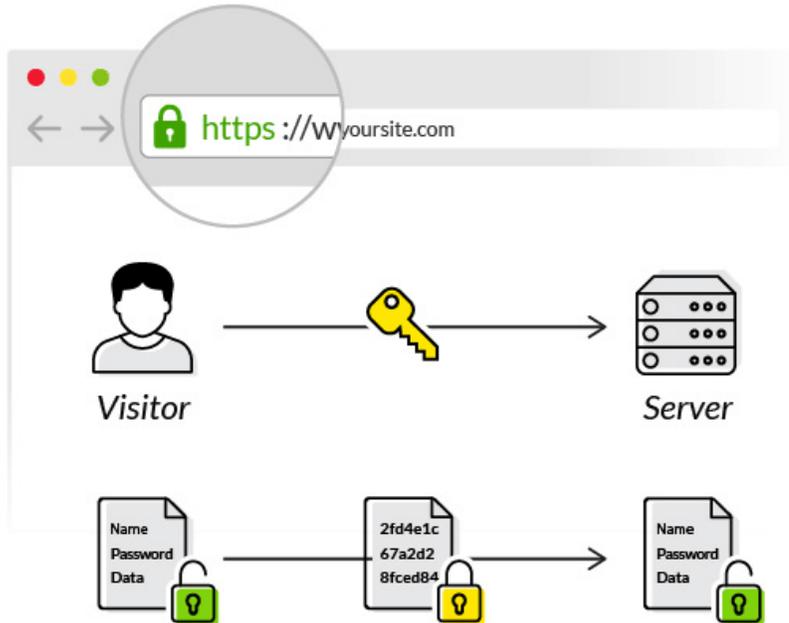
# Security of Cyberspace

- **Privacy** and **Authentication** are important!
  - **Sensitive** data in cyberspace
  - Serious consequences if wrong



# Important Technology

- Public-key cryptography (PKC)
  - Foundation of https
  - Email, secure payment, social media logins, etc.



# Foundation of PKC

- Need math problems **not** solvable by even super computers
  - Only **a few** candidates

Factoring:

Secret key:  $(p, q)$ , Public key:  $N=pq$   
RSA crypto systems

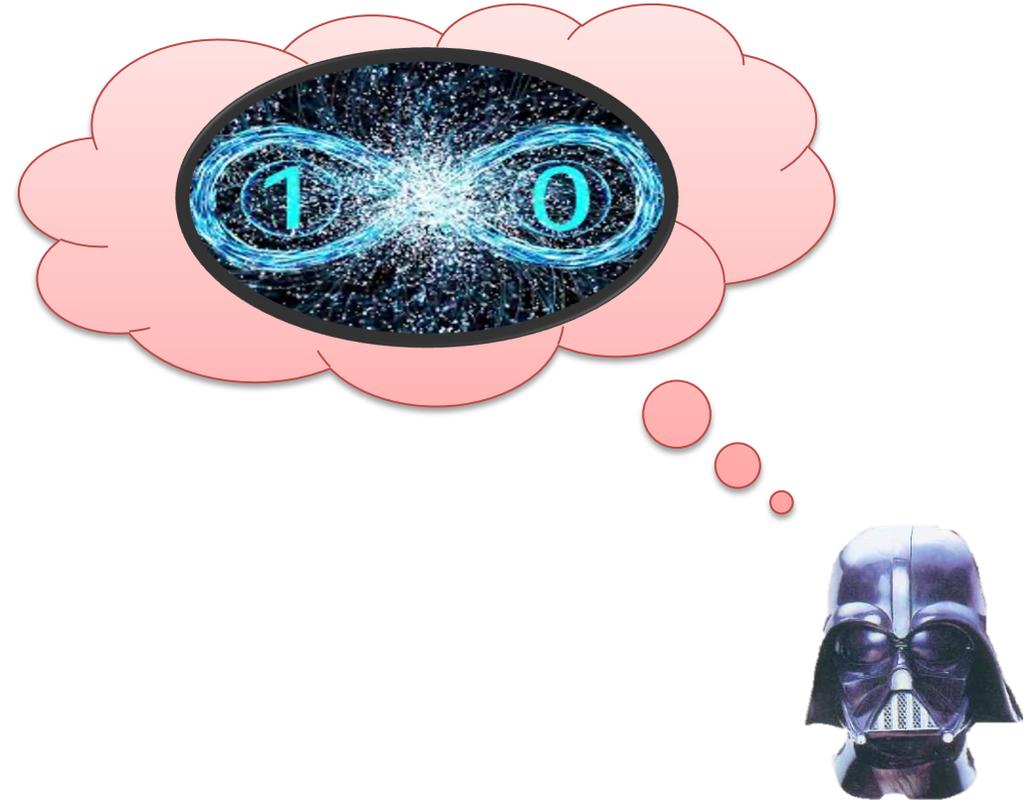
Discrete Log:

Secret key  $x$ , Public key:  $(g, g^x)$   
Diffie-Hellman Key Exchange



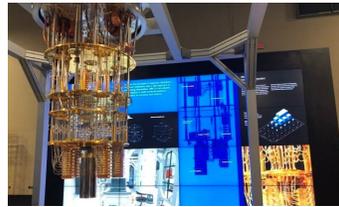
# Foundation is Challenged!

- [Shor94] **poly-time quantum** algorithm to **solve** factoring and DLog
  - Quantum is **more powerful**
- Technology was **not** there.
  - E.g., “ $15 = 3 * 5$ ” (2001)
  - Not practical yet



# Technology Advances!

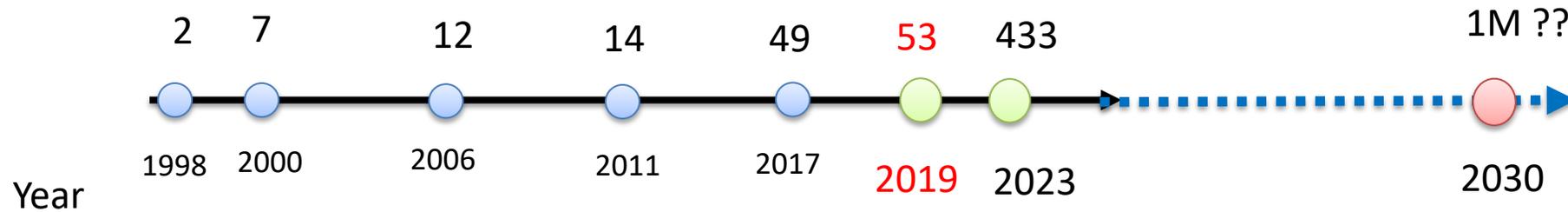
- Towards building larger quantum computers



IBM



# of qubits



First Quantum Supremacy  
for specific task (Google)

# Facing New Reality

- NIST: Post Quantum (PQ) PKC standardization call [2016 - ongoing]
- Industry: Evaluate performances of PQ candidates



Tales  
from the  
Crypto  
Team



[AWS Security Blog](#)

## Post-quantum TLS now supported in AWS KMS

by Andrew Hopkins | on 04 NOV 2019 | in [Advanced \(300\)](#), [AWS Key Management Service](#), [Security, Identity, & Compliance](#) | [Permalink](#) | [Comments](#) | [Share](#)

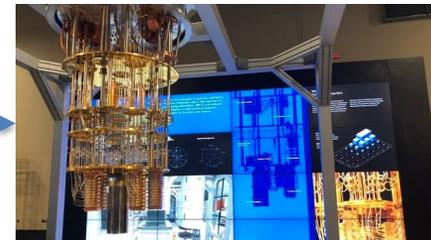
**NIST**  
National Institute of  
Standards and Technology

# Traditional PKC Crisis

- **Obsolete** eventually at some point...
  - New security infrastructure
- Critical time to develop new sciences
  - New **foundation** for PQ Crypto
  - New advanced Crypto **capabilities**



Cryptography



Quantum Computing

# My Research

- Basic Mission: **Rebuild** basic crypto tools **against** quantum computing
  - Post-quantum (PQ) cryptography [NIST current efforts]
  - Future security of internet applications



- Vision: Enable **efficient richer** crypto capabilities
  - Computing on encrypted data, **Fully homomorphic encryption (FHE)**
  - Applications to **private ML and data analytics**
  - Numerous **advanced** crypto designs

# Roadmap



## **Background**

Crypto Basics



## **My Work**

High Level Overview

Applications



## **Vision**

Future Opportunities

# Cryptography in General

- **What** is “security”?
  - No attacker can “break” the system
  - What does that mean?
- **How** to achieve “security”?
  - How to defend against infinitely many possible attacks?

# Modern Cryptography

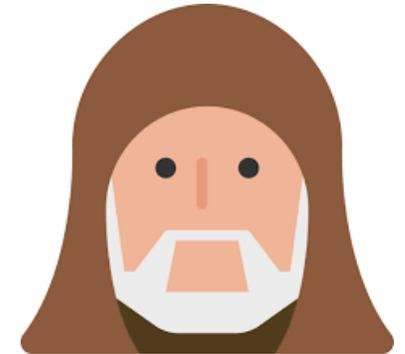
- Define a Clear Security Goal
  - E.g., Secure Channel



Send private messages

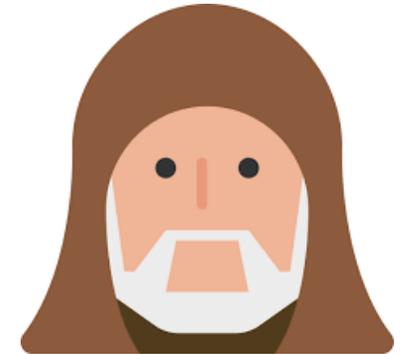
Secure

?????



# Modern Cryptography

- Define Security
  - Formulate a **notion** that captures “secure” channel
    - Not able to **recover** the whole plaintext?
    - We need: “attacker **cannot learn** anything” [Goldwasser-Micali82]



# Modern Cryptography

- Define Security
  - Formulate a **notion** that captures “secure” channel
    - Not able to **recover** the whole plaintext?
    - We need: “attacker **cannot learn** anything” [Goldwasser-Micali82]
  - Explicitly requested in the **NIST PQC** call

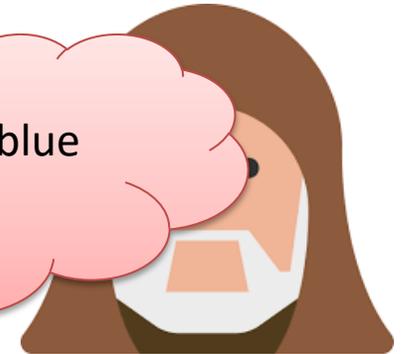


Help me, Obi-Wan Kenobi.  
You're my only hope !

Empire is the best !

Imaginary dummy message

Orange or blue  
????



# Modern Cryptography

- How to realize the secure goal?
  - **No real** physical secure channel
  - Construct a “droid” using **math** – “encryption.”

Help me, Obi-Wan Kenobi.  
You're my only hope !

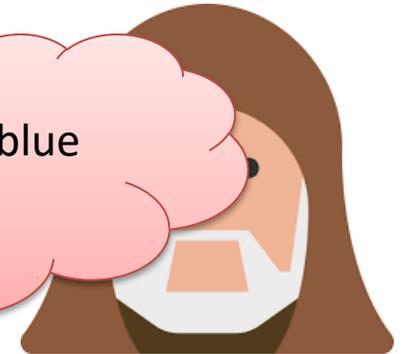


Help me, Obi-Wan Kenobi.  
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Orange or blue  
????



# Modern Cryptography

- How do we prove security against infinitely many attacks?

– The re

- Hard

If an **adversary** can break the crypto system, then there exists a **reduction** (that uses the adversary) that can solve the math problem.



Hard



Reduction



If the math problem is not solvable, then no **adversary** can break the crypto system. => Crypto system is secure



# Modern Cryptography

- We have some candidates
  - e.g., RSA, Discrete Log => Secure Crypto



Hard Math Problem

RSA, DL



Reduction



Crypto System



# In the Quantum Era



Quantum Computing



Hard Math Problem

~~RSA, DL~~

$\nabla$

Reduction



Crypto System



# In the Quantum Era



Quan



What we need:

- New hard math problem against quantum
- New design and proof of security



Hard Math Problem

~~RSA, DL~~

∩

Reduction



Crypto System



# NIST's PQC Call

- Aim to standardize future PQC [2016 - now]
  - Take **more than** 20 years for migration
- Challenges
  - Hard to find plausible math problems
  - Setting specific parameters for efficiency + security
  - Implementation-level details
  - Real-world deployment

# NIST's PQC Call

- New math hard problems and crypto designs
  - Code-based
  - Lattice-based
  - Hash-based
  - Isogeny-based
  - Multivariate-based
  - More...

# NIST Current Progress

- 3<sup>rd</sup> Round: Lattice-based
  - Public-key encryption: Kyber
  - Signature Dilithium, FALCON, SPHINCS+
  - Selected for standardizationHash-based
- 4<sup>th</sup> Round:
  - Ongoing
  - A lot of exciting (heartbreaking) news

# The Nature of Science

- Many candidates were broken
  - Rainbow (multivariate) [Crypto 2022]
  - SIKE (isogeny) [Eurocrypt 2023]
  - More ...
- Still plausible
  - Lattice
  - Hash
  - Code
  - Perhaps isogeny ???

# Roadmap



## Background

Crypto Basics



## My Work

High Level Overview

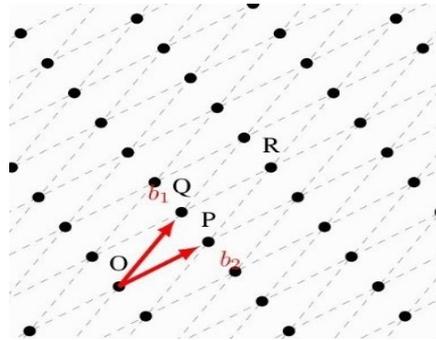
Applications



## Vision

Future Opportunities

# New Hope: Lattice-based Cryptography



- Advantages of Lattices:

- Efficient operations
- Resistance to quantum attacks (plausible)
- Foundation of advanced crypto systems for richer crypto capabilities and applications

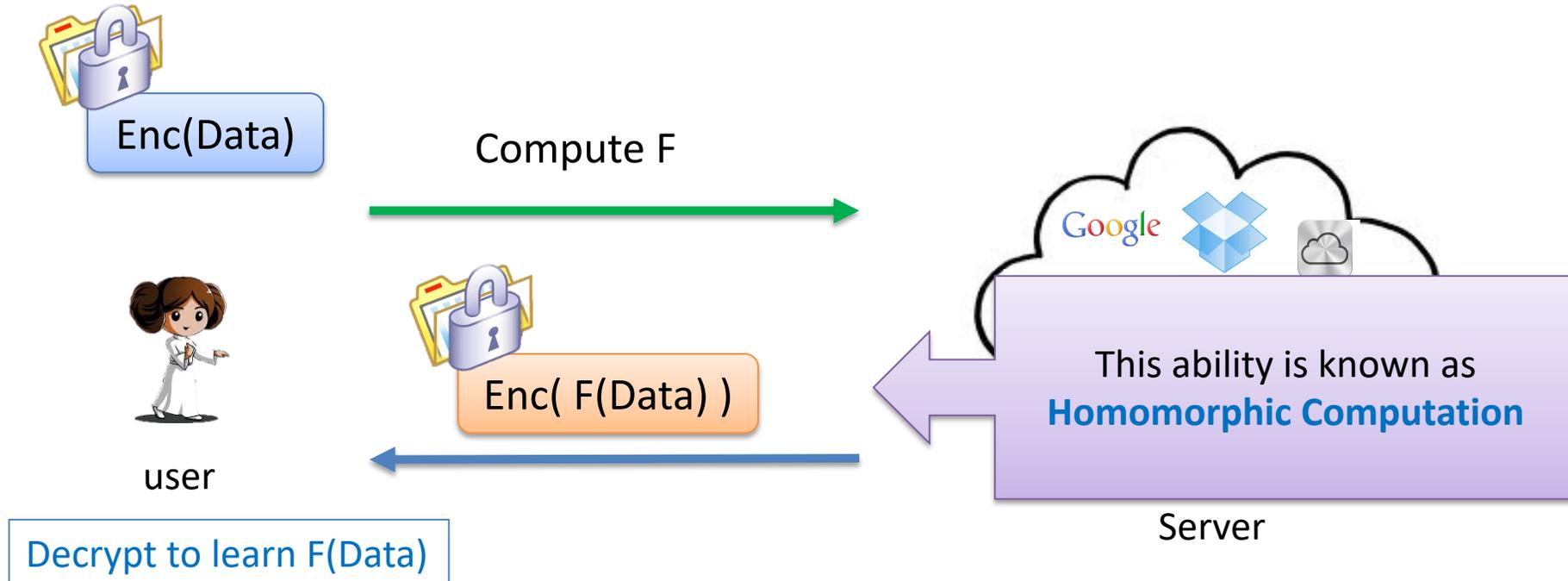
# New Hard Problem

- Learning with errors (LWE) [Regev 2005]
  - Theory [Peikert's survey 16]
  - Practice [NIST ongoing PQC comp]
- New PQ candidates in theory!
  - Public-key encryption
  - Signatures
  - Key exchange
  - Three variants are going to be standardized by NIST



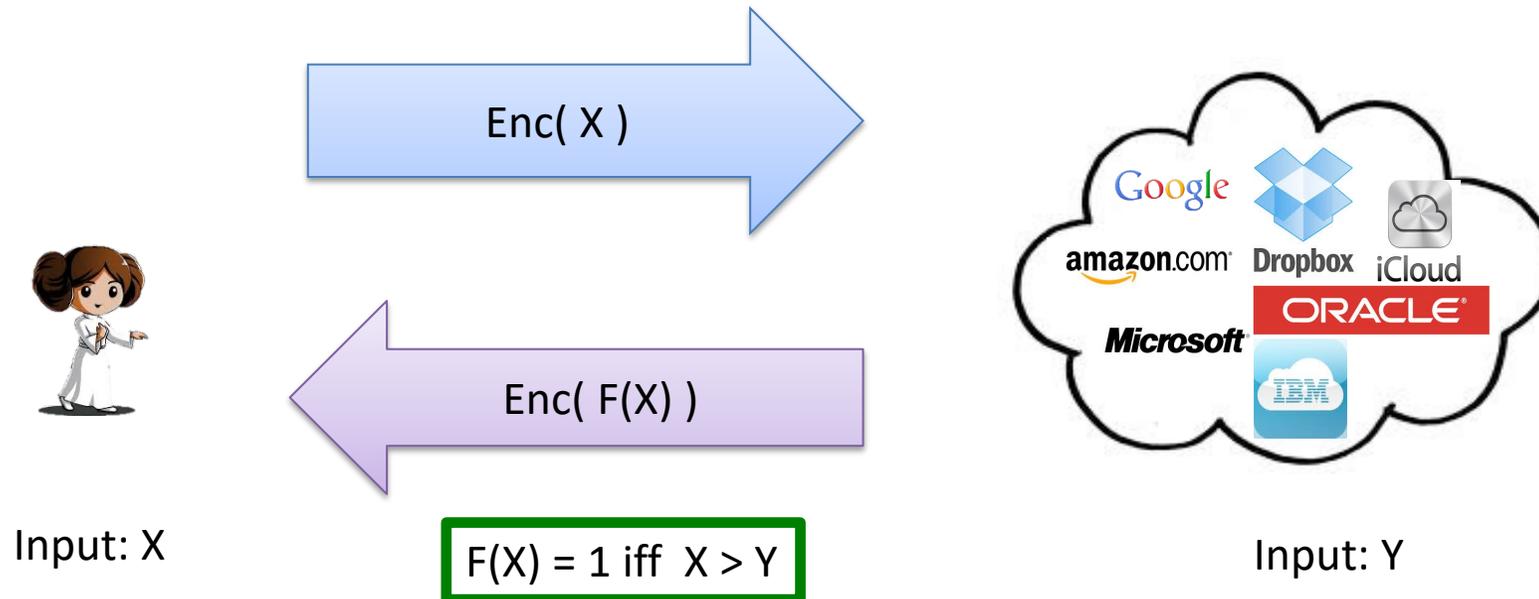
# Advanced Capabilities

- Computation on **encrypted** data
  - Fully Homomorphic Encryption (**FHE**) [Gentry, BV, GSW]
  - Outsource computation
  - **Holy grail** to keep data secure while in use [DARPA DPRIVE]



# Application to MPC

- An **elegant** solution to classic **Yao's Millionaire Problem** [1980's]
  - Two parties hold private inputs  $X$  and  $Y$ . Determine which is larger **without** revealing what they are
  - E-finance, e.g., compare numbers that are confidential

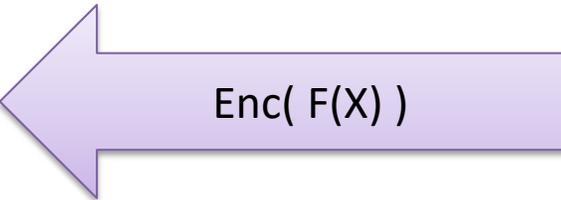
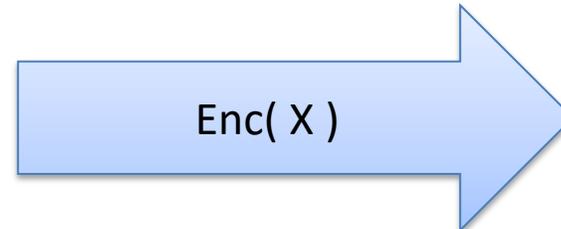


# New Applications – Private MLaaS

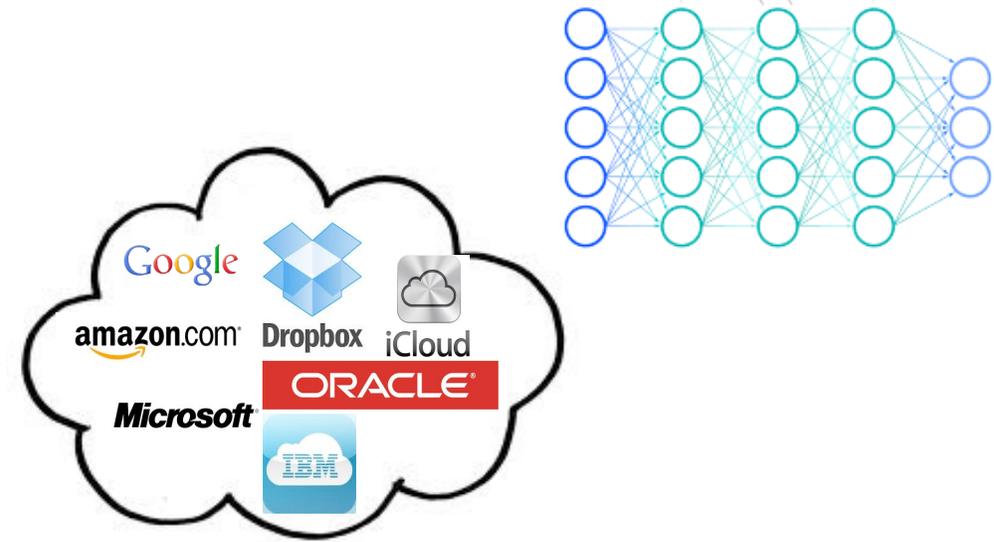
- New solutions to **private ML as a Service** [2020's]
  - Cloud holds a private ML Model  $Y$
  - User has private data  $X$
  - User wants to outsource analysis of private  $X$  **without** revealing  $X$
  - Cloud does **not** want to reveal  $Y$



Input:  $X$



$F(X) = \text{analysis using param } Y$



Input:  $Y$

# Triumph of Crypto Theory

- Theorem 1 [Regev]
  - Under hardness LWE, there exists a **PQ Public-key Encryption** (PKE)
- Theorem 2 [Gentry, BV, BGV, GSW, AP...]
  - Under hardness of LWE, there exists a **PQ fully homomorphic encryption** (FHE) for **any arbitrary** function of homomorphic computation
- Theorems 3, 4, 5....
  - Under hardness of LWE, there exists **a wide array** of advanced **PQ** cryptosystems, e.g., identity-based encryption, attributed-based encryption, functional encryption, and more...

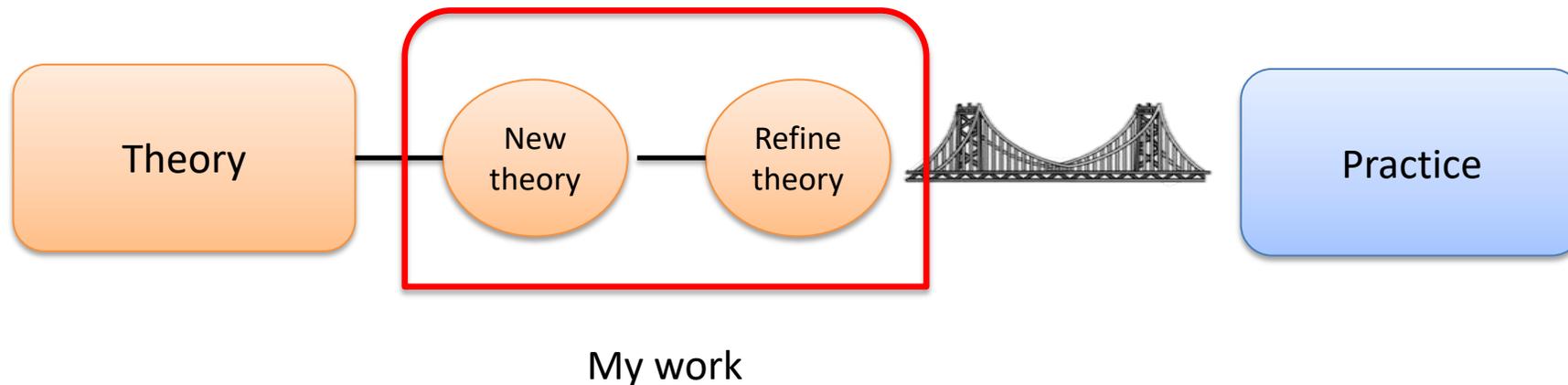
# In Praise of LWE



- Gödel Prize 2018 to Regev
- Citation
  - Served as the foundation for **countless** subsequent works
  - **Revolution** in cryptography in both theory and practice
  - A simple and yet amazingly versatile foundation for **nearly every kind** of cryptographic object
    - along with many that were **unimaginable** until recently, and which still have no known constructions without LWE

# Problem Solved?

- LWE offers solutions in **theory**
- **Gap** between theory and practice
  - Beyond only engineering efforts
- Need to **expand** and **refine** the theory



# Determine the Drawbacks

- Fact: Plain-LWE is **not** efficient
  - Why and How?
- Why: large concrete parameters/computation
  - **Large** keys in general
  - **Cumbersome** noise sampling procedure
- Consequence: Plain-LWE based Frodo **not** selected as **finalist** by NIST 😞

# To Improve Efficiency

- Other **variants** of LWE
  - Algebraic Rings
  - Other form of errors
- Research questions
  - Are these variants **hard**?
  - Can we do **more** for the **advanced** capabilities?

# My Work

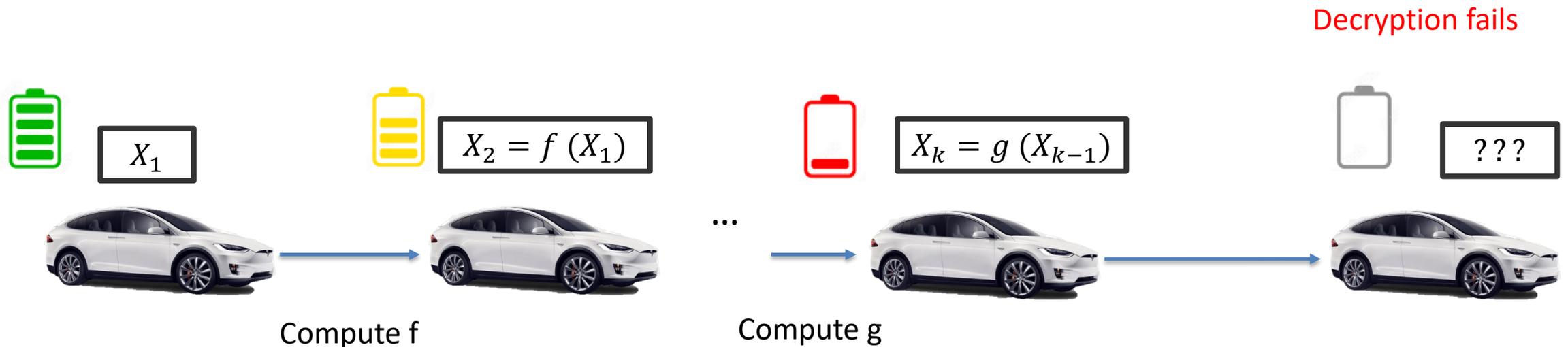
- New algebraic techniques
  - Proving some useful variants of LWE is **plausibly** hard
  - More **efficient** FHE methods

# More Efficient FHE

- Fact: "F"-HE relies on a core technique called **bootstrapping**
  - Important but slow
- Algebraic techniques => More efficient bootstrapping  
[LiuWang23a, LiuWang23b]

# FHE Computation

- Basic facts:
  - All known FHE ciphertexts contain “**noise**”
  - Basic operations (e.g., add, mult, NAND) are rather **fast**
  - Computation **increases** noise
  - Noise becomes too large => **cannot** proceed computation



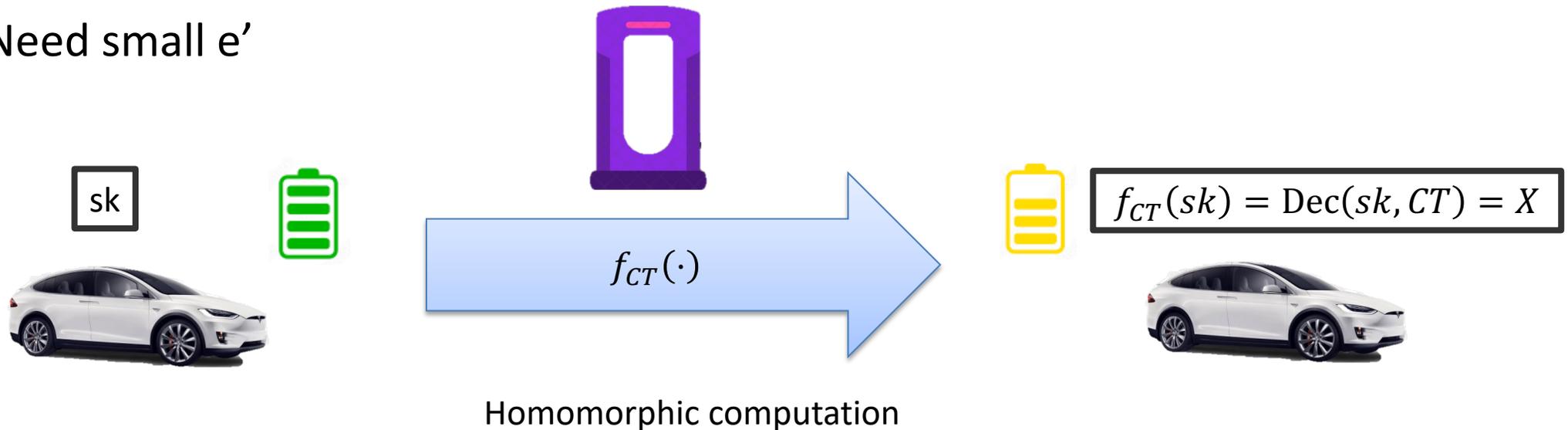
# Bottleneck of FHE Computation

- To further compute on ciphertext, need to “clean” noise
  - This is called **Bootstrapping**
- Bootstrapping is **significantly slower** than other basic operations
  - The bottleneck



# Bootstrapping Framework [Gentry]

- Need Bootstrapping Key, i.e.,  $BK = \text{FHE.Enc}(sk)$
- Input  $CT = \text{FHE.Enc}(X)$ , which might be somewhat noisy
  - Define  $f_{CT}(\cdot) = \text{Dec}(\cdot, CT)$
- Eval also incurs noise  $e'$ 
  - Need small  $e'$



# Bootstrapping is Bottleneck

- Was slow: 30 mins to bootstrap one CT
- Significant improvements:
  - **Large** params/space (10 GB) + **SIMD** (Single Instruction Multiple Data)
    - 20 seconds to bootstrap 10000 CTs
  - **Small** params/space (10 MB) + fast bootstrapping
    - 1 sec [FHEW15]
    - 0.1 sec [TFHE16]
    - 30 ms [further optimizations]
    - **No SIMD**

# My Work [LiuWang23a, LiuWang23b]

- A **new** mathematical framework
  - Small space
  - SIMD
- Open question:
  - Optimize the framework, more refined math techniques?
  - Determine the concrete parameters
  - Implementation and Deployment

# Roadmap



## Background

Crypto Basics



## My Work

High Level Overview

Applications



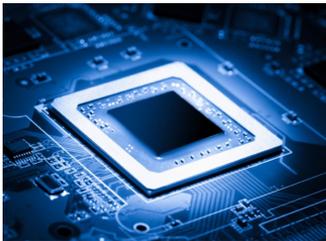
## Vision

Future Opportunities

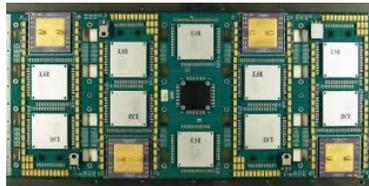
# Future Direction 1 – Core FHE

- New Foundation of FHE
  - Confirm and **optimize** the theoretic framework
  - Determine the **concrete** improvements over existing solutions
  - **Expand** the existing FHE libraries

ASIC



Multi-core CPU

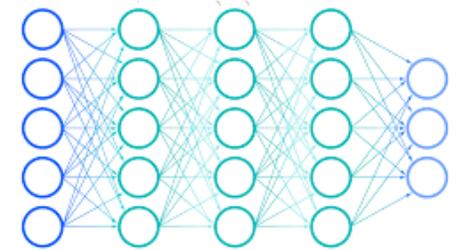


GPU



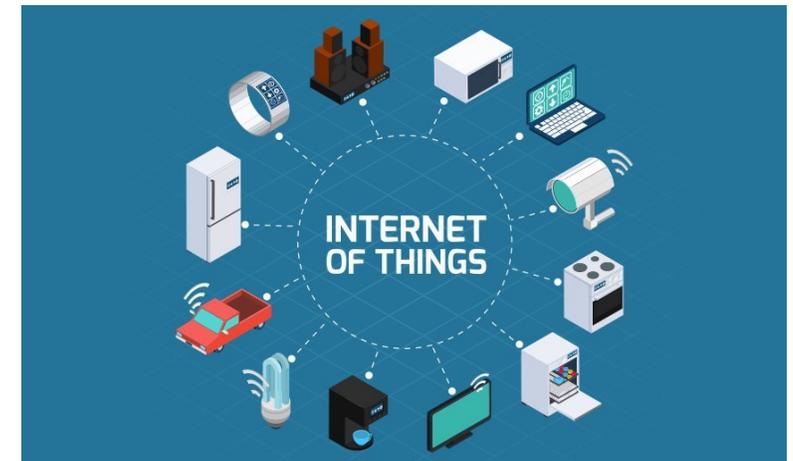
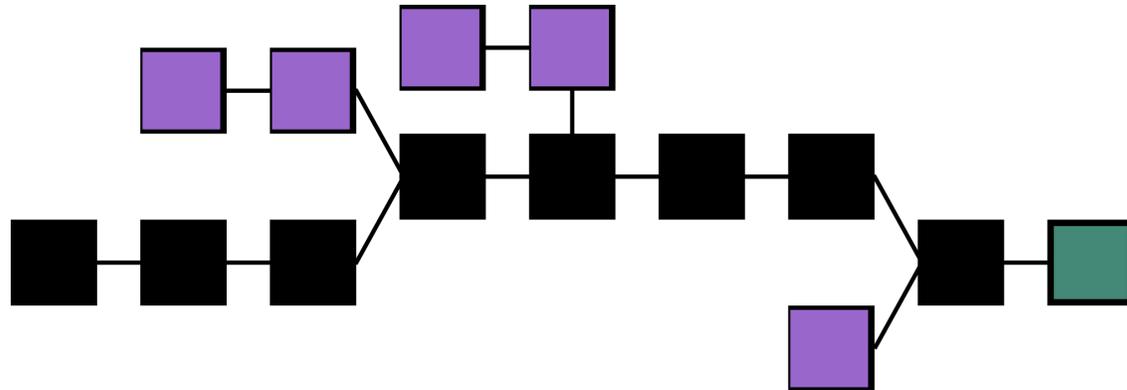
# Future Directions 2 – Applications to ML

- Applications in private ML and data analytics
  - New ML-friendly FHE computation architecture
  - New FHE-friendly ML models
- New collaborative opportunities !



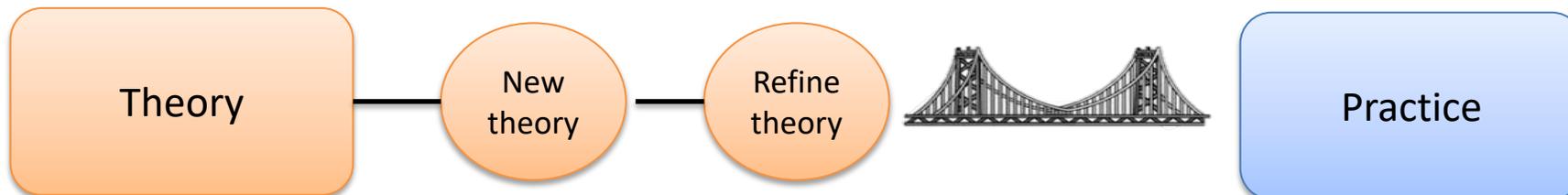
# Future Direction 3 – For Future Applications

- Efficient Advanced Crypto Capabilities
  - PQ zero-knowledge proofs
  - PQ anonymous credentials
  - Efficient and scalable MPC over large datasets
  - More ...
- Efficient PQ **privacy enhancing technologies** for the future



# Vision

- **2005 – 2023**: LWE implies nearly **every** kind of cryptographic object imaginable
  - Extremely successful theory
- **2023 and after**: New techniques to **make theory a reality**
  - Develop new theory
  - Refine existing theory
  - Continue bridging the gap between theory and practice





Thank You!

