64th Annual Meeting of Health Physics Society
Orlando, Florida, July 7 – 11, 2019
CEL-8 Download Code: **CEL8-54035**

The Importance of the Measurand in Health Physics

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Neology

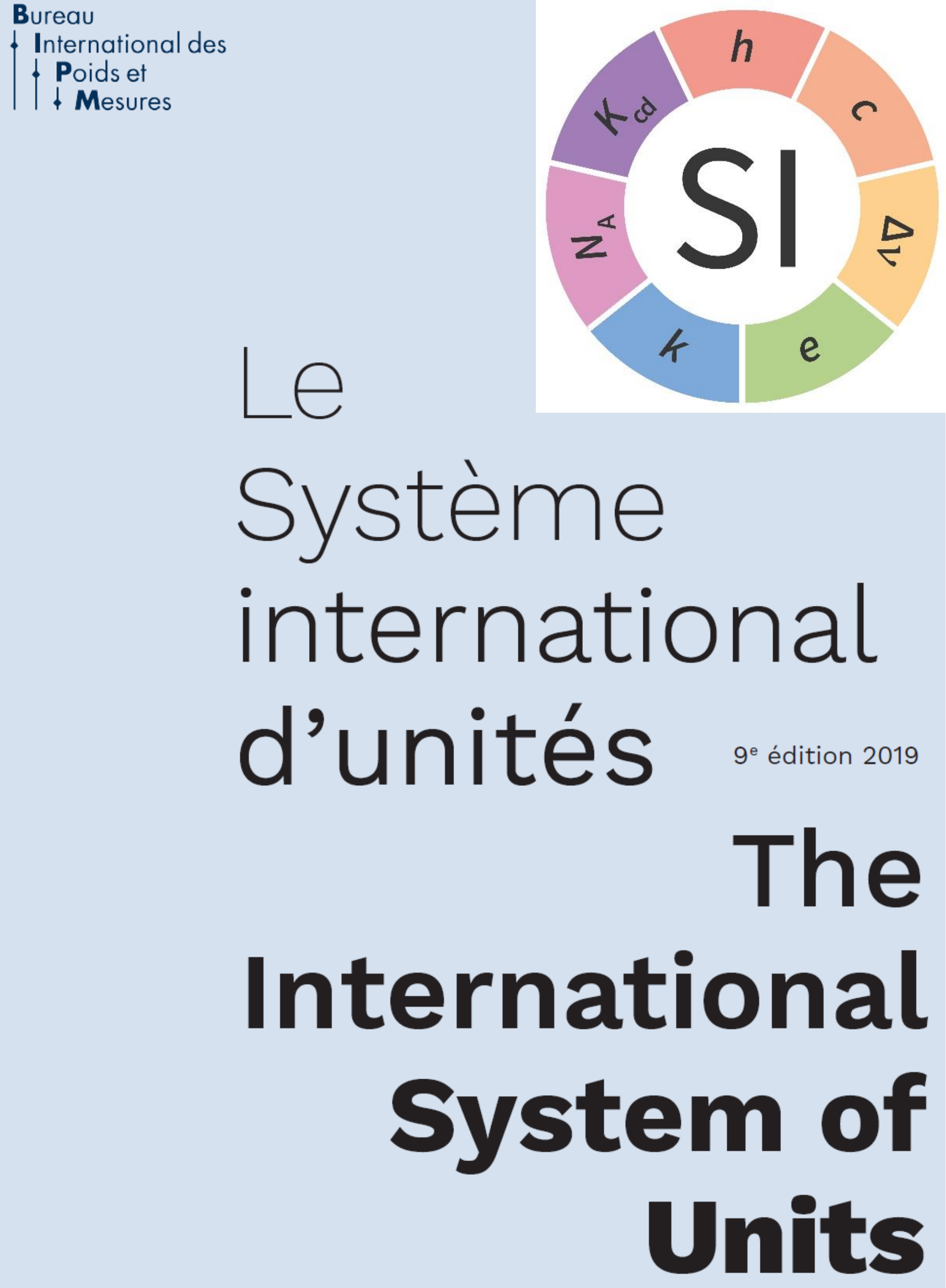
- Neology means “making up new words”
- Many of the greatest scientists in human history would not recognize so many of the words we use today
- Aristotle, Archimedes, Ptolemy, the folks who invented zero, Galileo, Copernicus, Newton, Darwin, Pasteur, and Maxwell wouldn’t have understood radioactivity, x-rays, relativity, quantum mechanics, contraception, black holes, refrigerators, or smart phones
- We need to do neology now and then when old words don’t suffice
- Sometime late in the last century, someone invented a new word: measurand

Measurand

- When making a measurement, what is "the quantity intended to be measured?"
- That phrase is the definition of "measurand" that appears in the latest version of the International Vocabulary of Metrology (the VIM).

Overview

- Measurand
- The VIM and the GUM
- From the *Error Model* to the *Uncertainty Model*
- Measurands and results of measurements
- Variability, uncertainty, bias, error, and blunder (throughout)
- Probabilistic statements about the possible values of the measurand given the measurement result(s)
- Example: Counting a long-lived radionuclide
- Is anything there? Decision rules like decision level *DL* (aka decision threshold *DT*
- The smallest usually detectable measurand (*SUDM*), formerly *MDA*

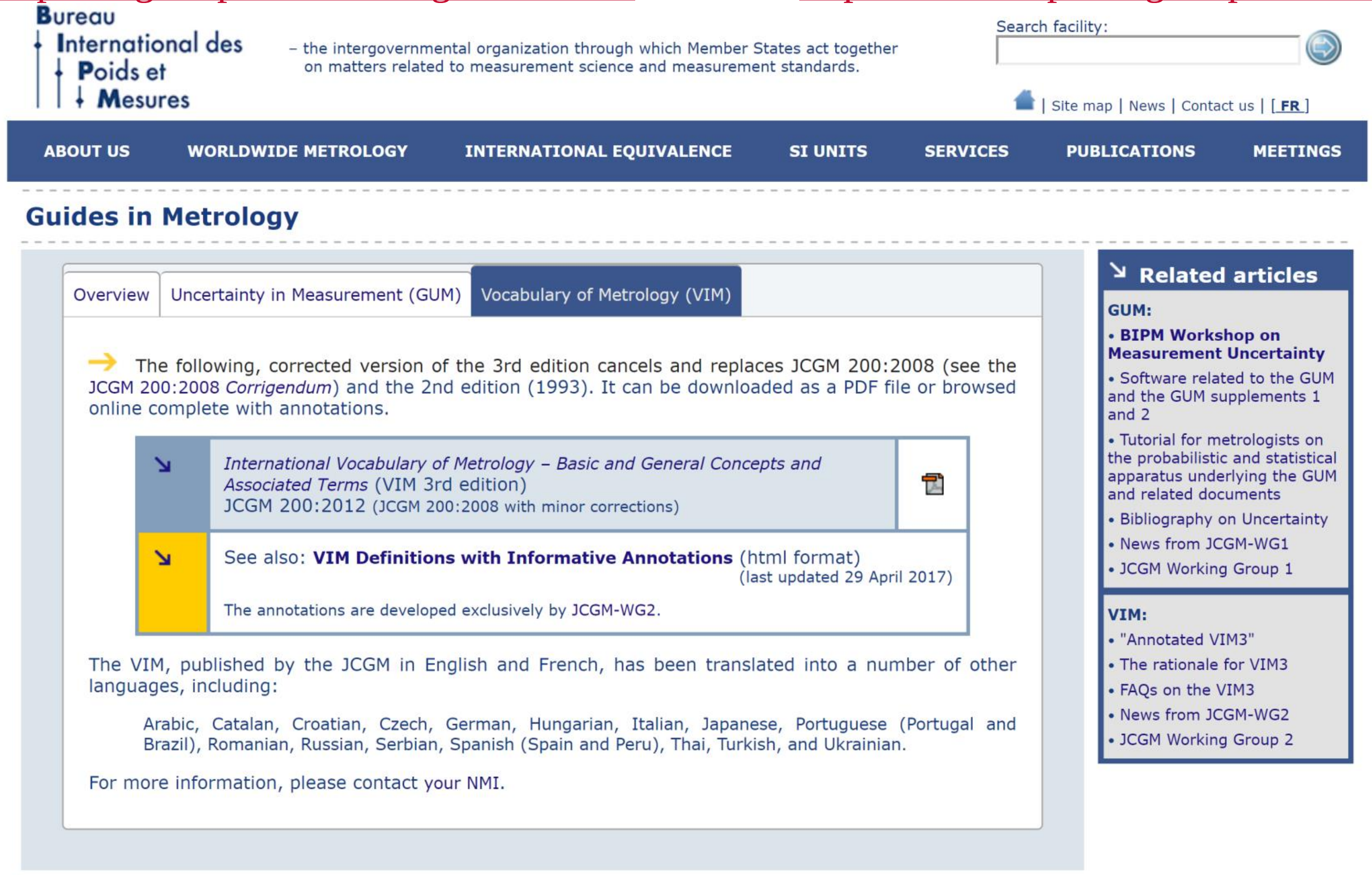


Quantities and Units

- The International System of Units (SI) is owned by the CGPM, of which USA, through NIST, is a partner
- The free 2019 SI "brochure" is available at https://www.bipm.org/en/publications/si-brochure/
- Relevant NIST documents are at physics.nist.gov/cuu
  - NIST is boss in the USA!
- As of 2019, all 7 fundamental quantities are based on physical constants, no longer on artifacts like the Pt-Ir kilogram in Paris

Free Downloads of the VIM and the GUM

- International Vocabulary of Metrology (VIM) https://www.bipm.org/en/publications/guides/#vim
- Guide to the Expression of Uncertainty in Measurement (GUM) https://www.bipm.org/en/publications/guides/#gum



2008 Guide to the Expression of Uncertainty in Measurement (GUM)

- https://www.bipm.org/en/publications/guides/#gum
  - https://www.nist.gov/pml/nist-technical-note-1297 (1994) is similar, but now out of date
- Extensive, well-thought-out framework for dealing with uncertainty in measurement
  - Clearly-defined concepts and terms
  - Practical approach
- The GUM doesn't cover
  - the use of measurements in models that have
    - uncertain assumptions
    - uncertain parameters
    - uncertain form
    - shared uncertainties
  - representativeness (e.g., of a breathing-zone air sample)
  - inference from measurements (e.g., dose-response relationship)

The Error Approach and the Uncertainty Approach

Welcome to the 21st Century!

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Old: The Error Approach (According to the VIM, before 1980)

- "The objective of measurement in the Error Approach is to determine an estimate of the true value that is as close as possible to that single true value.
- "The deviation from the true value is composed of random and systematic errors.
- "The two kinds of errors, assumed to be always distinguishable, have to be treated differently.
- "No rule can be derived on how they combine to form the total error of any given measurement result, usually taken as the estimate.
- "Usually, only an upper limit of the absolute value of the total error is estimated, sometimes loosely named 'uncertainty.'"

New: The Uncertainty Approach
(According to the VIM, since 1980)

- “The components of measurement uncertainty should be grouped into two categories, Type A and Type B, according to whether they were evaluated by statistical methods or otherwise, and that they be combined to yield a variance according to the rules of mathematical probability theory by also treating the Type B components in terms of variances.
- “The resulting standard deviation is an expression of a measurement uncertainty.
- “The Uncertainty Approach … focused on the mathematical treatment of measurement uncertainty through an explicit measurement model under the assumption that the measurand can be characterized by an essentially unique value.”

Uncertainty Approach 2

- "The objective of measurement in the Uncertainty Approach is not to determine a true value as closely as possible.
- "Rather, it is assumed that the information from measurement only permits assignment of an interval of reasonable values to the measurand.
- "…even the most refined measurement cannot reduce the interval to a single value because of the finite amount of detail in the definition of a measurand.
- "The objective of measurement is then to establish a probability that this essentially unique value [the measurand] lies within an interval of measured quantity values, based on the information available from measurement."
- "The interval of values offered to describe the measurand is the interval of values of measurement standards that would have given the same indications."

More Vocabulary

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Definition of Measurand

- **measurand** - the quantity intended to be measured
- Its value is generally unknown (and unknowable)
  - Exception (in Strom's opinion!): something we can count
- A measurand is the "true" value of a well-defined physical quantity that can be characterized by an essentially unique value
  - If the phenomenon of interest can be represented only as a distribution of values or is dependent on one or more parameters, such as time, then the measurands required for its description are the set of quantities describing that distribution or that dependence

2008 GUM General Metrological Terms - 1

GUM Term	Meaning
(measurable) quantity	property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference
value (of a quantity)	magnitude of a particular quantity generally expressed as a unit of measurement multiplied by a number
value of a measurand	the quantity intended to be measured. [the unknown value of a physical quantity representing the “true state of Nature” This is sometimes called the “true value” or the “actual value”]
conventional true value (of a quantity)	value attributed to a particular quantity and accepted, sometimes by convention, as having an uncertainty appropriate for a given purpose
measurement	process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity

2008 GUM General Metrological Terms - 2

GUM Term	Meaning
result of a measurement	value attributed to a measurand, obtained by measurement
uncorrected result	result of a measurement before correction for systematic error *(i.e., bias)*
corrected result	result of a measurement after correction for systematic error *(i.e., bias)*
accuracy of measurement	closeness of the agreement between the result of a measurement and a true value of the measurand
repeatability (of results of measurements)	closeness of the agreement between the results of successive measurements of the same measurand carried out under the same conditions of measurement
reproducibility (of results of measurements)	closeness of agreement between the results of measurements of the same measurand carried out under changed conditions of measurement

2008 GUM General Metrological Terms - 3

GUM Term	Meaning
uncertainty (of measurement)	parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand. *It is a bound for the likely size of the measurement error.*
error (of measurement)	result of a measurement minus a true value of the measurand *(i.e., the [unknowable] difference between a measured result the actual value of the measurand.)* "Error is an idealized concept and errors cannot be known exactly" (Note 3.2.1)
relative error	error of measurement divided by a true value of the measurand
correction	value added algebraically to the uncorrected result of a measurement to compensate for systematic error
correction factor	numerical factor by which the uncorrected result of a measurement is multiplied to compensate for systematic error

Type A and Type B Uncertainty Evaluations

- Uncertainty that is evaluated by the statistical analysis of series of observations is called a "**Type A**" uncertainty evaluation.
- Uncertainty that is evaluated by means *other* than the statistical analysis of a series of observations is called a "**Type B**" uncertainty evaluation.

- Note that using $\sqrt{N}$ as an estimate of the standard deviation of N counts is a Type B uncertainty evaluation!

Uncertainty and Variability

- Uncertainty
  - stems from lack of knowledge, so it can be characterized and managed but not eliminated
  - can be reduced by the use of more or better data
- Variability
  - is an inherent characteristic of a population, inasmuch as people vary substantially in their exposures and their susceptibility to potentially harmful effects of the exposures
  - cannot be reduced, but it can be better characterized with improved information

-- National Research Council. 2008. Science and Decisions: Advancing Risk Assessment. http://www.nap.edu/catalog.php?record_id=12209, National Academies Press, Washington, DC

Terms: Error, Uncertainty, Variability

- “The difference between error and uncertainty should always be borne in mind.”
- “For example, the result of a measurement after correction can unknowably be very close to the unknown value of the measurand, and thus have negligible error, even though it may have a large uncertainty.”
- *If you accept the GUM definitions of error and uncertainty*
  - *there are no such things as “error bars” on a graph!*
  - *such bars are “uncertainty bars”*
- Variability is the range of values for different individuals in a population
  - e.g., height, weight, metabolism

Random and Systematic Uncertainty versus Type A and Type B Uncertainty Evaluation

- GUM: There is not always a simple correspondence between the classification of uncertainty components into categories A and B and the commonly used classification of uncertainty components as “random” and “systematic.”
- The nature of an uncertainty component is conditioned by the use made of the corresponding quantity, that is, on how that quantity appears in the mathematical model that describes the measurement process.
- When the corresponding quantity is used in a different way, a “random” component may become a “systematic” component and vice versa.

Random and Systematic "Errors"

GUM Term	Meaning
random error	result of a measurement minus the mean that would result from an infinite number of measurements of the measurand carried out under repeatability conditions
systematic error	mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions minus a true value of the measurand

- ***Uncertainty*** is our estimate of how large the ***error*** may be
- *We do not know how large the error actually is*

Classical and Bayesian Statistical Inference

Classical and Bayesian Statistical Inference

- Bayesian statistical inference has replaced classical inference in more and more areas of interest to health physicists, such as determining
  - whether activity is present in a sample
  - what a detection system can be relied on to detect
  - what can be inferred about intake and committed dose from bioassay data

The Two Counting Problems:
1. The "Forward Problem"

- Radioactive decay is a Bernoulli process described by a binomial or Poisson distribution
  - A Bernoulli process is one concerned with the count of the total number of independent events, each with the same probability, occurring in a specified number of trials
- The "forward problem"
  - from properties of the process, we predict the distribution of counting results (mean, standard deviation (SD))
  - measurand $\rightarrow$ distribution of possible observations
  - as seen later, this is the Bayesian *likelihood function*

The Two Counting Problems:
2. The "Reverse Problem"

- Measure a counting result: N counts
- From the counting result, we infer the parameters of the underlying binomial or Poisson distribution (μ, standard deviation = σ)

  see, e.g., Rainwater and Wu (1947)
- What range of values of the measurand likely gave rise to the measurement result(s)?
- This is the problem we're really interested in
- This is a Bayesian problem!

Applications of Probability Theory to Nuclear Particle Detection

By L. J. RAINWATER and C. S. WU
Pupin Physics Laboratory, Columbia University

October, 1947 - NUCLEONICS

Comparison of Two Kinds of Statistics

- Classical statistics
  - does the forward problem well
  - does not do the reverse problem at all
- Bayesian statistics does the reverse problem using
  - a prior probability distribution
  - the observed results
  - a likelihood function (a classical expression of the forward problem)

Bayes's Rule (Simple form)

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)} \qquad \text{Posterior} = \frac{\text{Likelihood} \times \text{Prior}}{\text{Normalizing Factor}}$$

$$\text{Probability that the measurand (true count rate) is } B$$
$$\text{given that we've observed a count rate of } A$$
$$= \frac{(\text{Likelihood of } A \text{ given } B) \times (\text{Prior probability of } B)}{\text{Normalizing Factor}}$$

- Bayes rule gives probabilistic descriptions of the values the measurand could plausibly have given
  - the measurement results
  - how the measurement result(s) depend on the measurand, and
  - what we knew about the measurand before we started

Bayesian Approach: The Prior Probability 1

$$P(B_i \mid A) = \frac{L(A \mid B_i)P(B_i)}{\sum_{\text{all } j} L(A \mid B_j)P(B_j)}$$

- Some form of prior probability is required!
- The prior probability is what you know before you start
- The prior can have more or less effect on the posterior, depending on the precision of the data
- The prior can be subjective
- The prior is sometimes the topic of unresolvable arguments

Bayesian Approach: The Prior Probability 2

$$P(B_i \mid A) = \frac{L(A \mid B_i)P(B_i)}{\sum_{\text{all } j} L(A \mid B_j)P(B_j)}$$

- The prior can be "nothing"
  - even "nothing" can take several forms
  - "uniform," "flat," or "uninformative" prior: all values of *B* are "equally probable"
  - "vague" prior: all values of ln(*B*) are equally probable…
- The prior can be other information – here are examples for intakes:
  - the CAM alarmed or there was facial or skin contamination or a positive nasal swab
  - the worker had a previous intake or a previous positive bioassay
- The prior can be hard to nail down
  - "small values of blank are more likely than large ones"

Philosophical Statement of Bayes's Rule

$$P(\text{measurand}|\text{evidence}) = \frac{L(\text{evidence}\,|\,\text{measurand})P(\text{measurand})}{\text{normalizing factor}}$$

- The measurand or "state of nature" (e.g., count rate from analyte) is what we want to know
- The "evidence" is what we have observed
- The likelihood of the "evidence" given the measurand is what we know about the way nature works
- The probability of the measurand is what we believed before we obtained the evidence

Bayes's Rule: Continuous Form

- Ps are probability densities

$$\mathrm{P}(\mu \mid N) = \frac{\mathrm{L}(N \mid \mu)\mathrm{P}(\mu)}{\int_0^\infty \mathrm{L}(N \mid \nu)\mathrm{P}(\nu)\,\mathrm{d}\nu}$$

$$\text{Posterior} = \frac{\text{Likelihood} \times \text{Prior}}{\text{Normalizing Factor}}$$

- We want to determine the posterior probability density
- *This is the probability of various values of the measurand μ given the measurement result(s) N*

Bayes's Rule for a Poisson Likelihood

- The *posterior* probability of any particular value of the measurand, μ given that we've observed N counts, is

$$\mathrm{P}(\mu \mid N) = \frac{\mathrm{L}(N \mid \mu)\mathrm{P}(\mu)}{\int_0^\infty \mathrm{L}(N \mid \nu)\mathrm{P}(\nu)\,\mathrm{d}\nu} = \frac{\frac{\mu^N e^{-\mu}}{N!}k}{\int_0^\infty \frac{\nu^N e^{-\nu}}{N!}k\,\mathrm{d}\nu} = \frac{\mu^N e^{-\mu}}{N!}$$

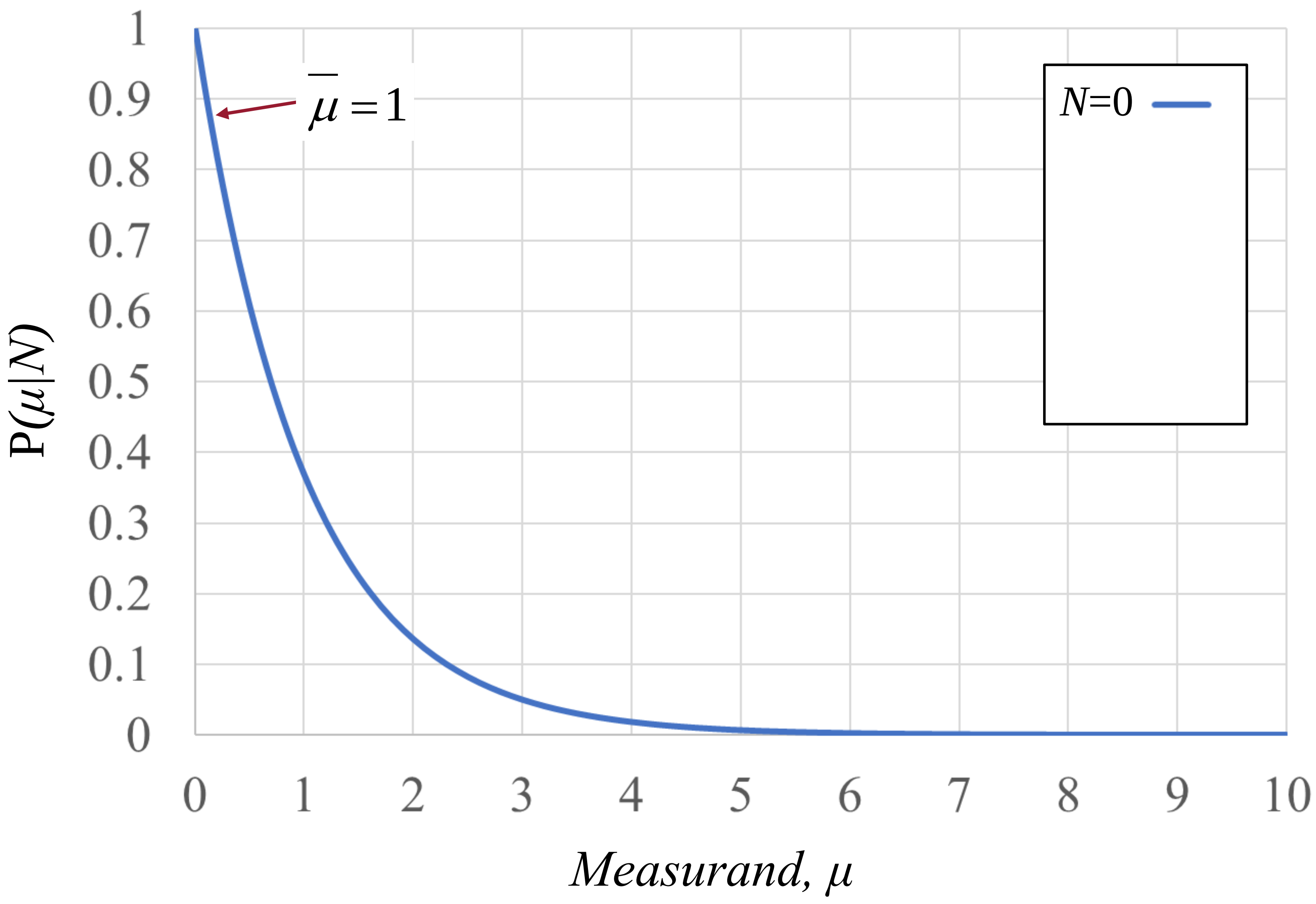
- In this case, the *posterior probability density function* is just the *likelihood function* with the dependent and independent variables reversed
- *This is the probability of various values of the measurand μ given the measurement result(s) N*

Bayesian Posterior Probabilities for a Poisson Likelihood

	N=0	N=1	N=2	N=3	N=4	N=5
Bayesian posterior probability distribution $P(\mu \mid N) = \frac{\mu^N e^{-\mu}}{N!}$	$e^{-\mu}$	$\mu e^{-\mu}$	$\frac{\mu^2 e^{-\mu}}{2}$	$\frac{\mu^3 e^{-\mu}}{6}$	$\frac{\mu^4 e^{-\mu}}{24}$	$\frac{\mu^5 e^{-\mu}}{120}$
Average* of P(μ|N)	1	2	3	4	5	6
*also known as *mean*, *arithmetic mean*, *expectation*, *and expectation value*						

- when you observe N counts, you should
  - record N
  - use N+1 in your calculations if using Bayes's theorem with a uniform (ignorant) prior

Bayesian Posterior for $N = 0$



Wait! What?

- You're telling me that when I observe zero counts, the expected value of the mean of the number of counts is one?
- Yes.

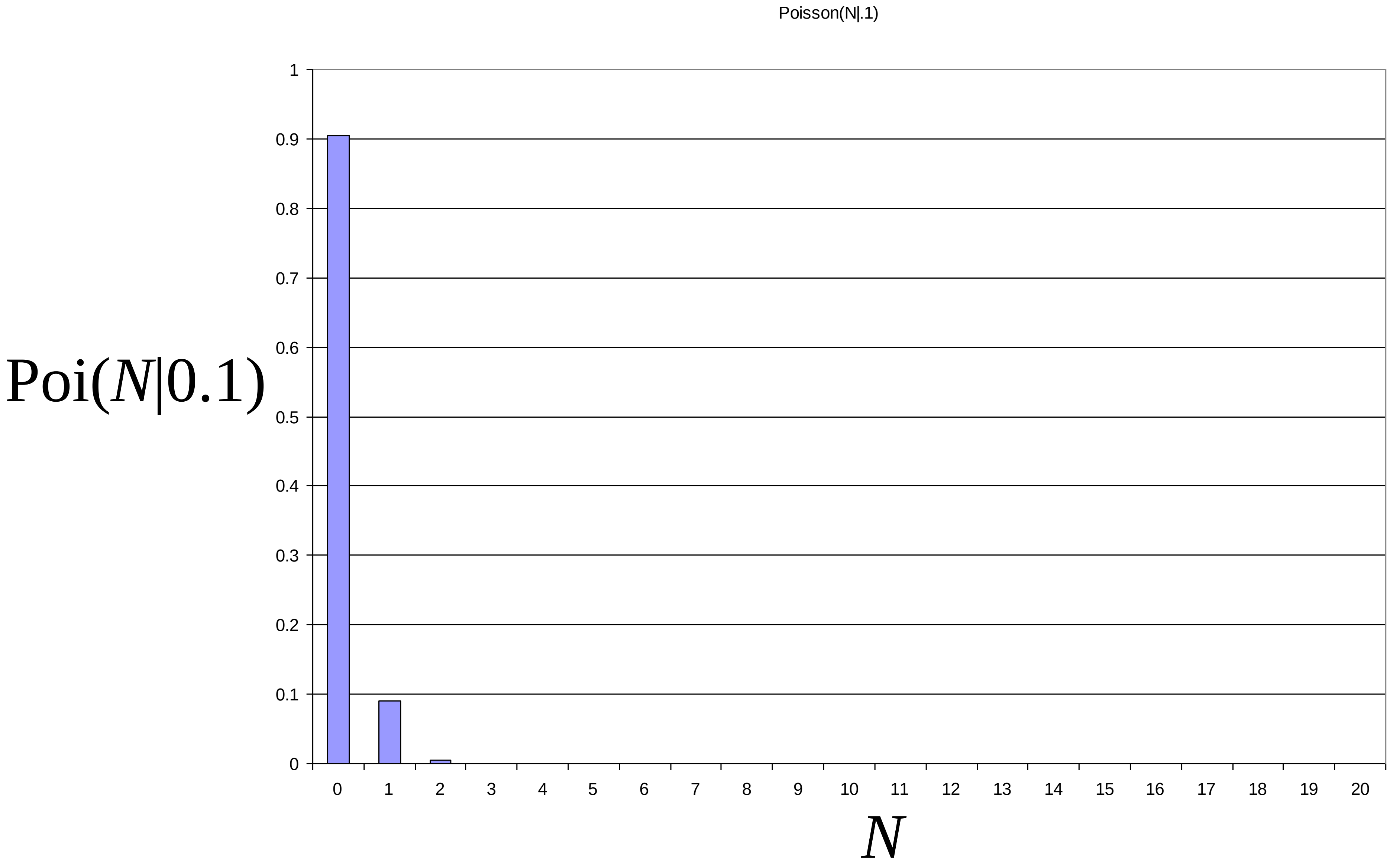
This result (that the mean expected value for u is larger than n) is unexpected at first but may be easily understood from a simple example. Thus, if $n = 0$, it is clear that u is not necessarily also zero; therefore u_{av} cannot be zero. Note, by contrast, that if $u_{av} = 0$, then $n = 0$ is the only possible result.

Rainwater & Wu (1947)

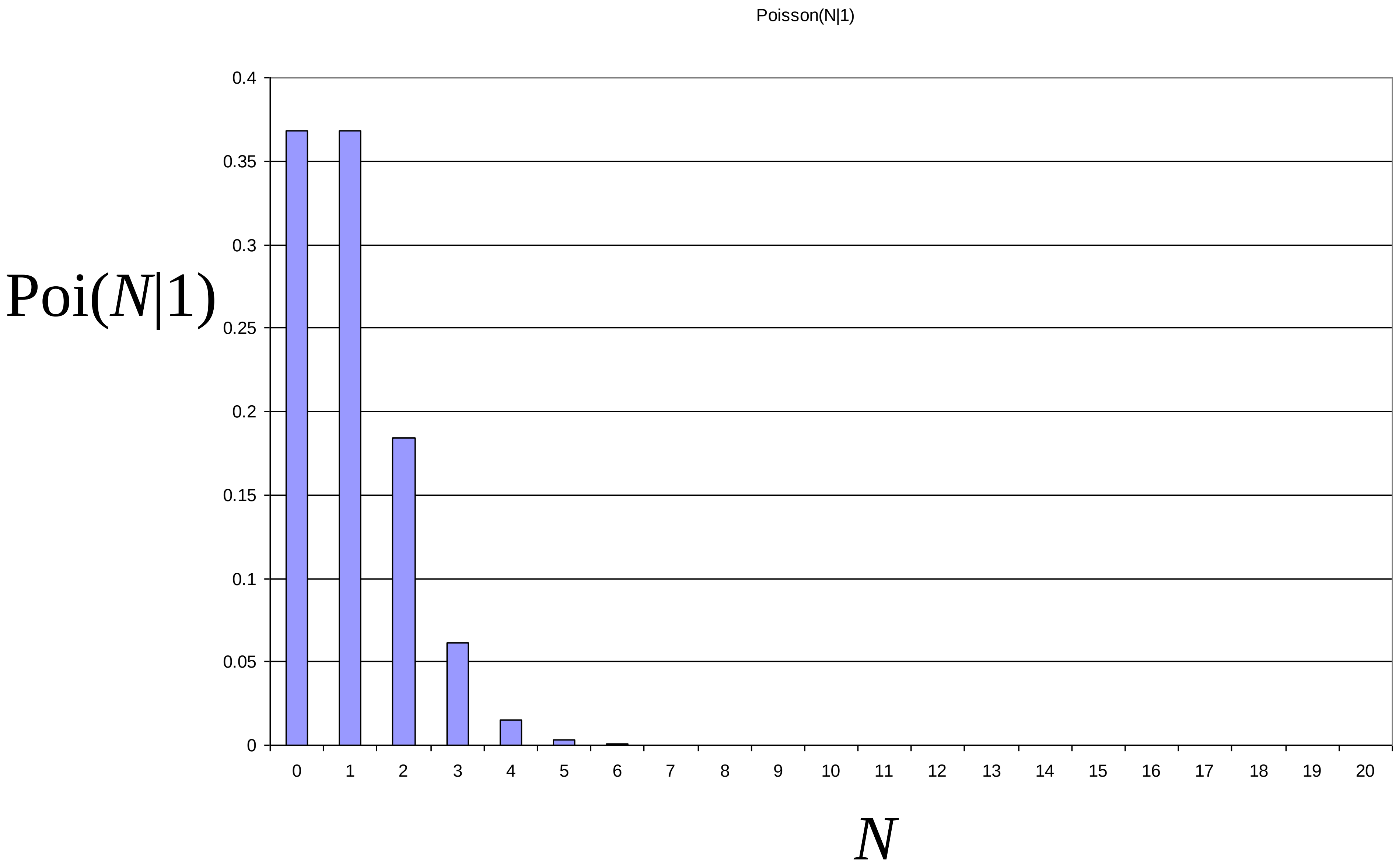
- The closer the true, nonzero, positive mean μ of a Poisson distribution is to zero, the more likely a sample from that distribution will be zero

μ	$P(0\|\mu)$
0	1
0.1	0.905
0.5	0.607
1	0.368
2	0.135
3	0.050
5	0.007
10	0.00005

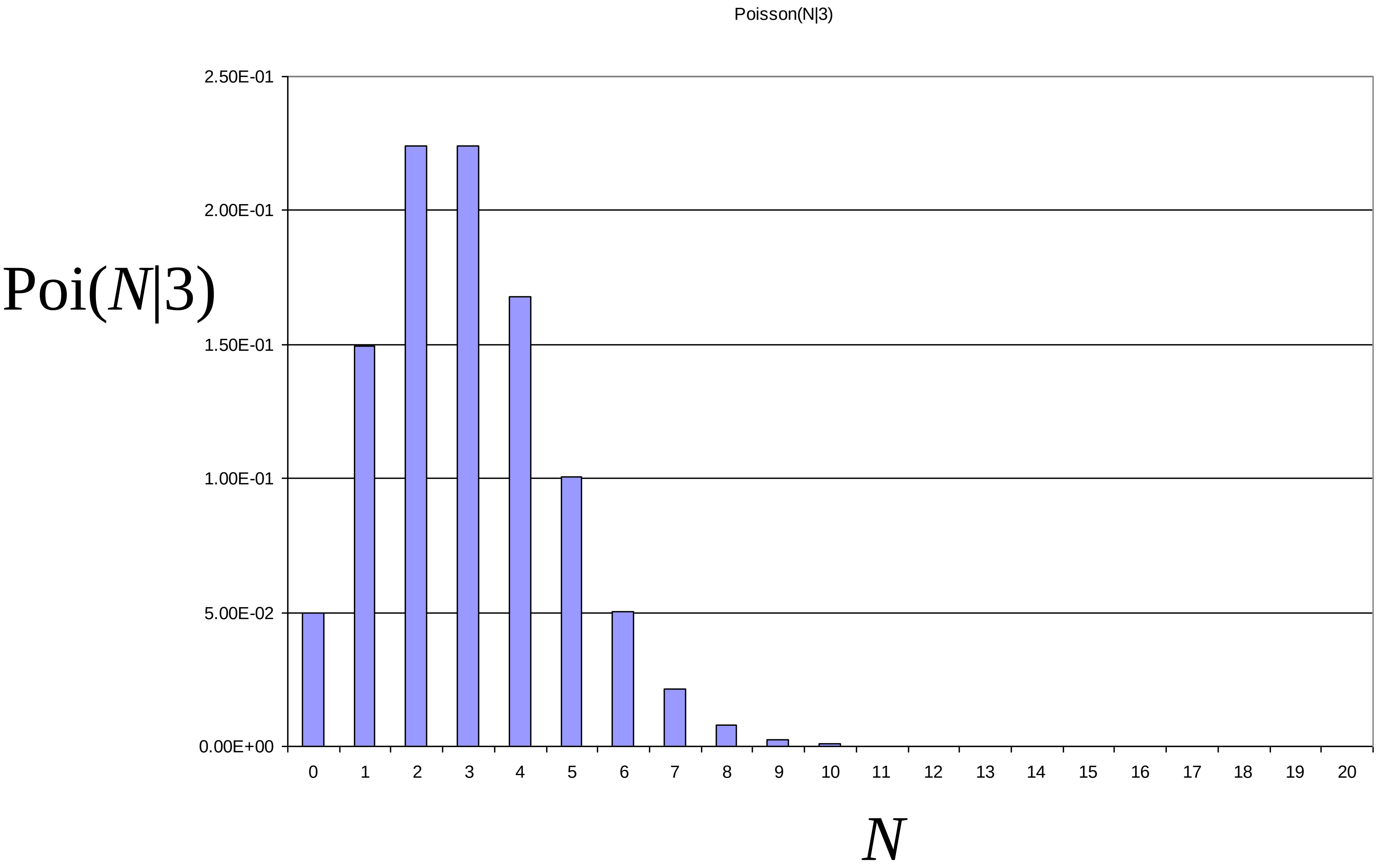
Poisson Distribution, $\mu = \rho t = 0.1$



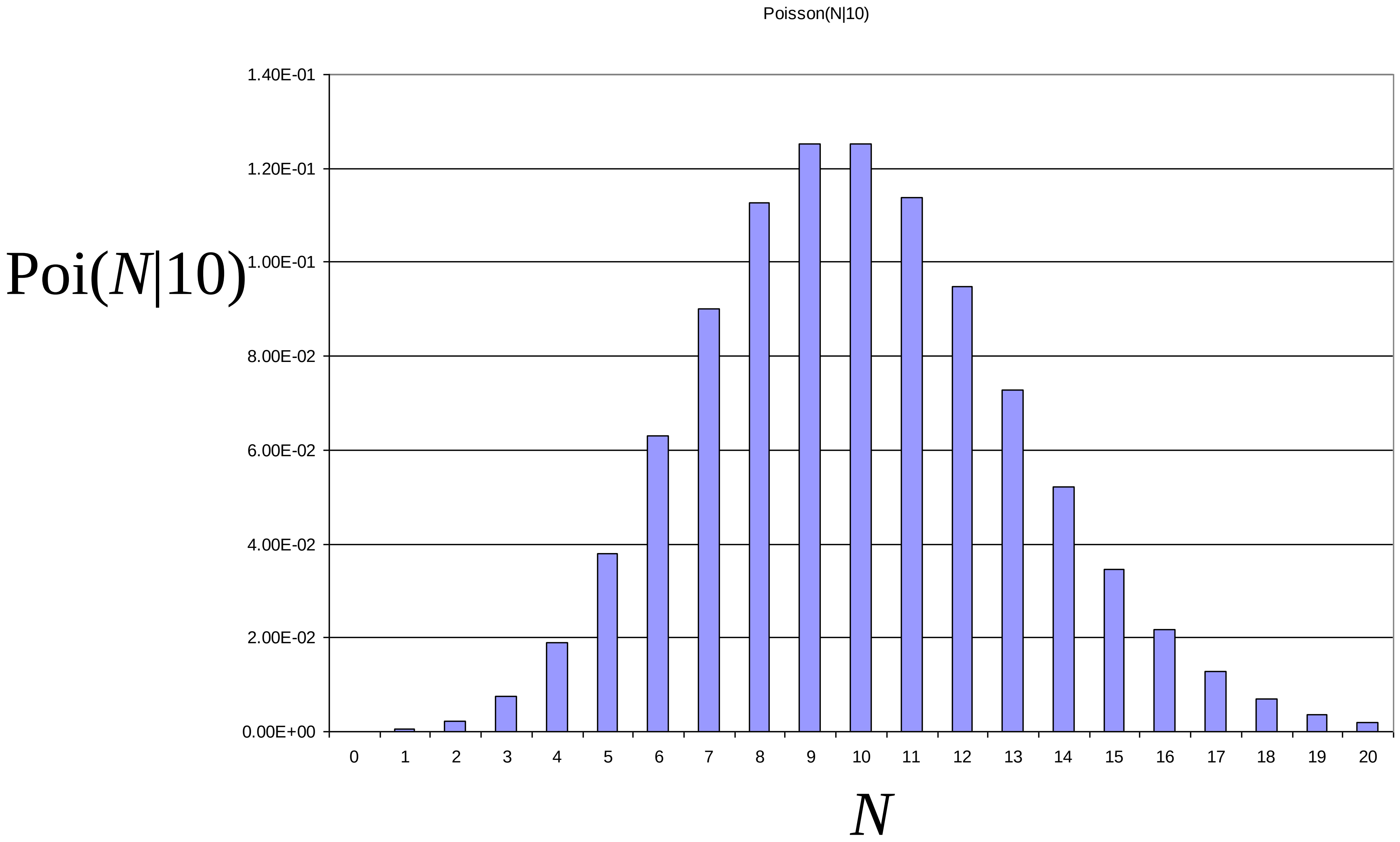
Poisson Distribution, $\mu = \rho t = 1$



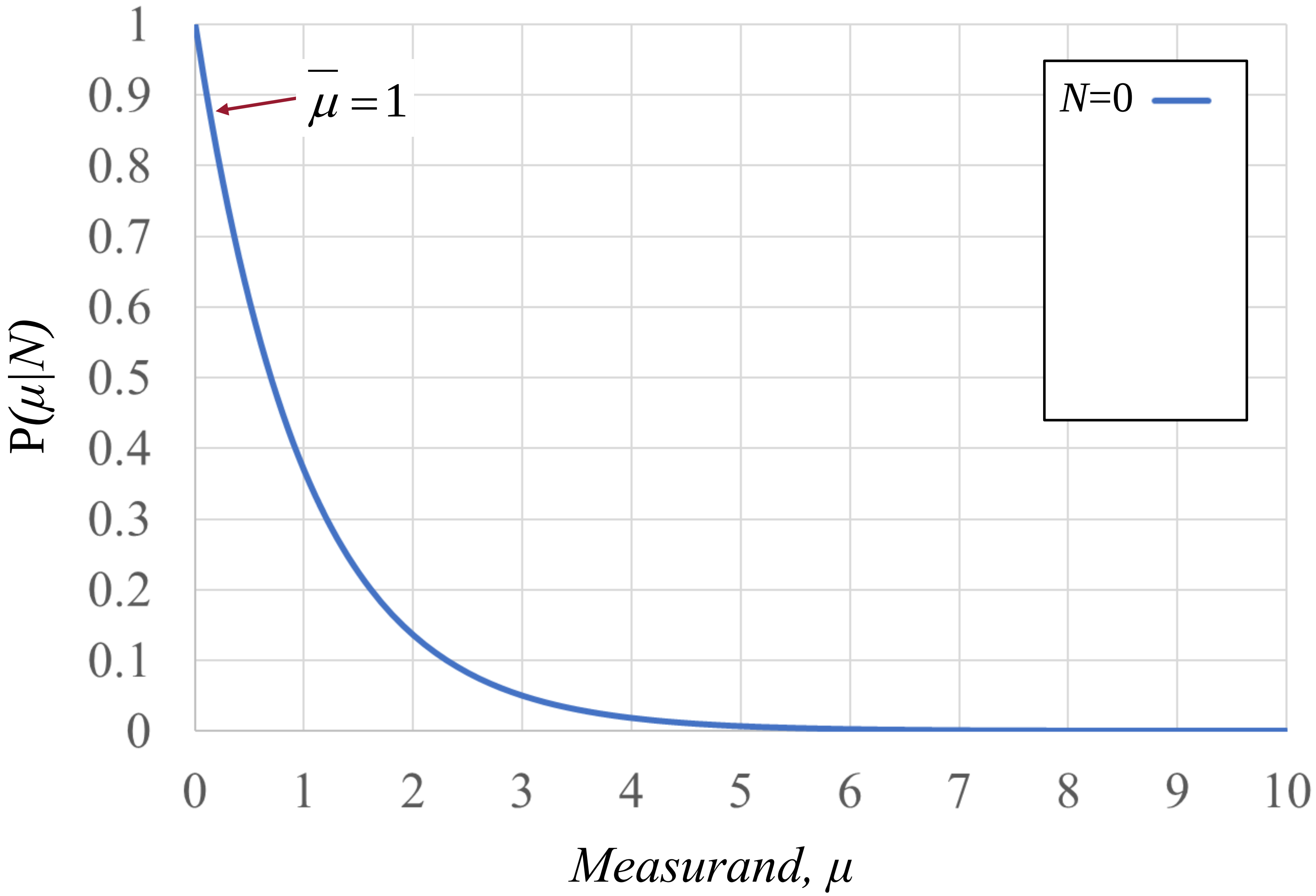
Poisson Distribution, $\mu = \rho t = 3$



Poisson Distribution, $\mu = \rho t = 10$

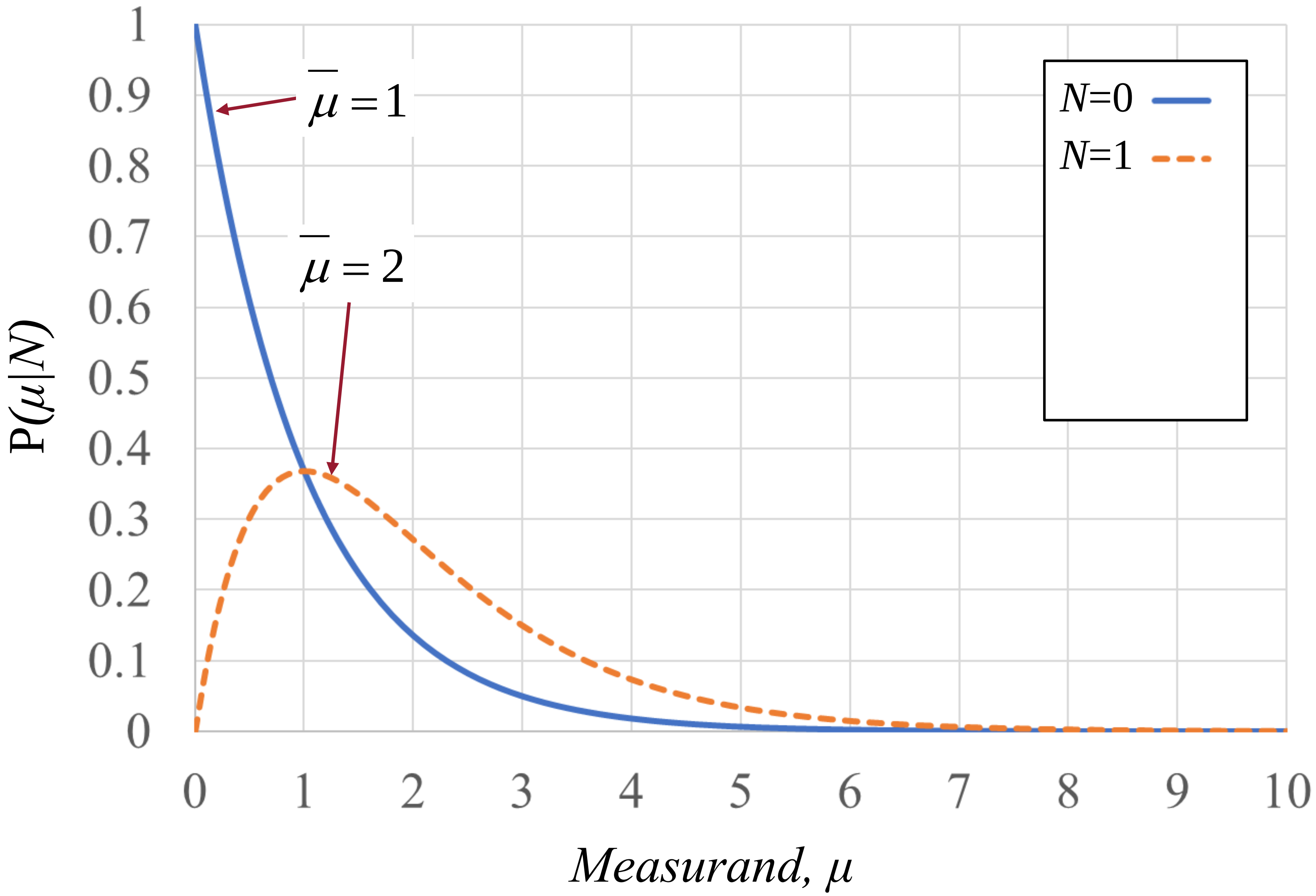


Bayesian Posterior for $N = 0$



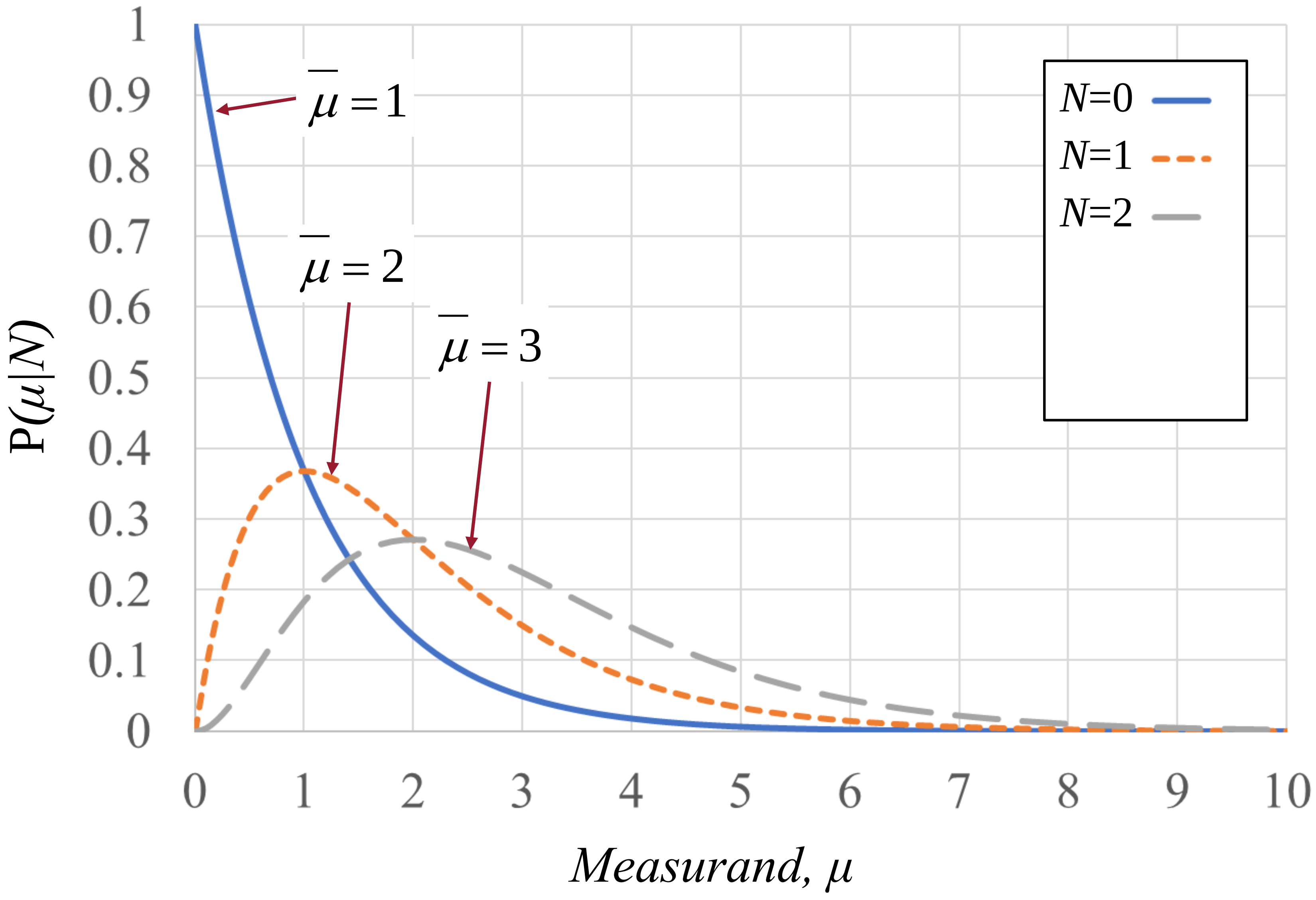
μ	$P(0\|\mu)$
0	1
0.1	0.905
0.5	0.607
1	0.368
2	0.135
3	0.050
5	0.007
10	0.00005

Bayesian Posteriors for $N = 0$ and 1



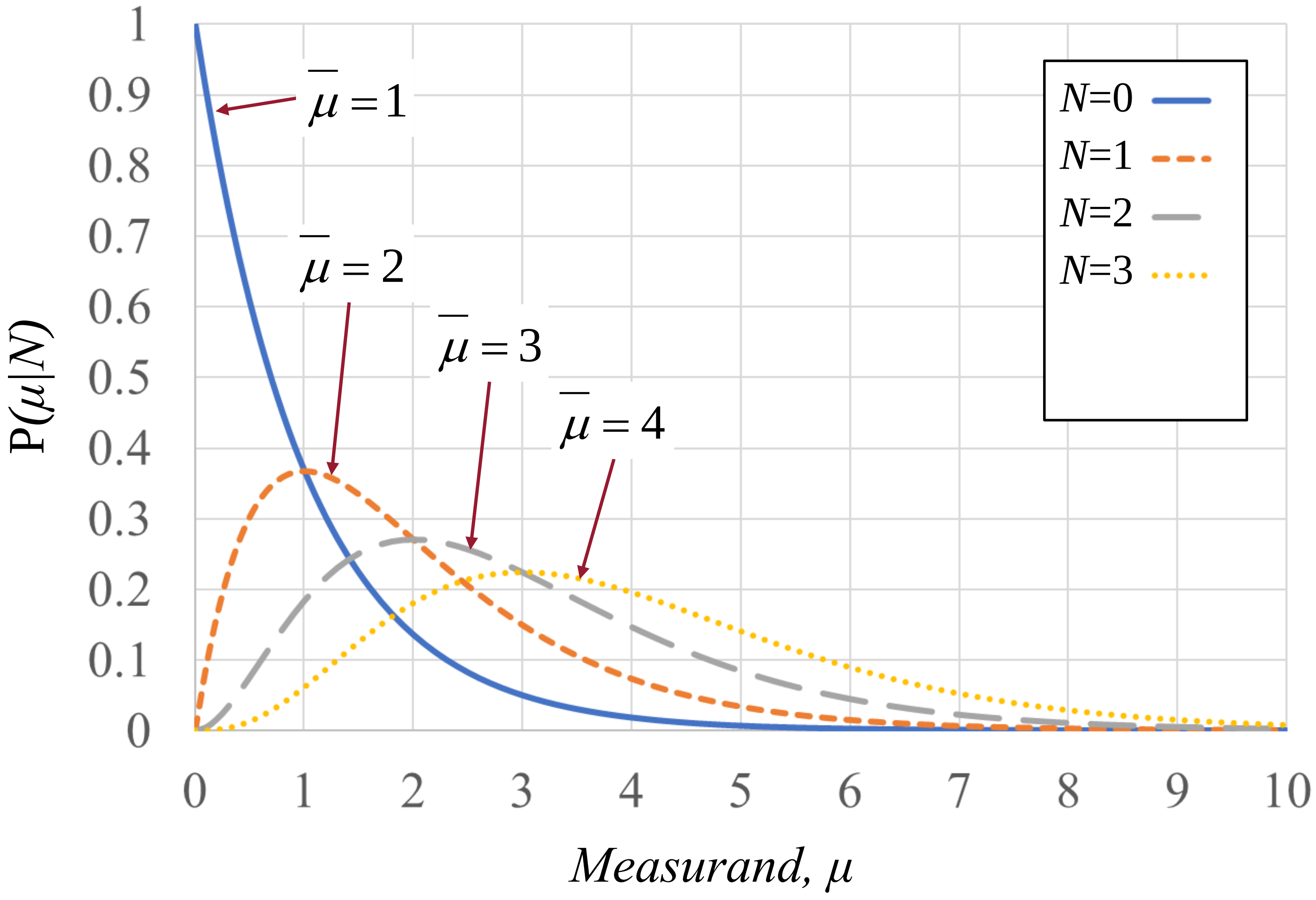
μ	$P(0\|\mu)$
0	1
0.1	0.905
0.5	0.607
1	0.368
2	0.135
3	0.050
5	0.007
10	0.00005

Bayesian Posteriors for N = 0, 1, and 2



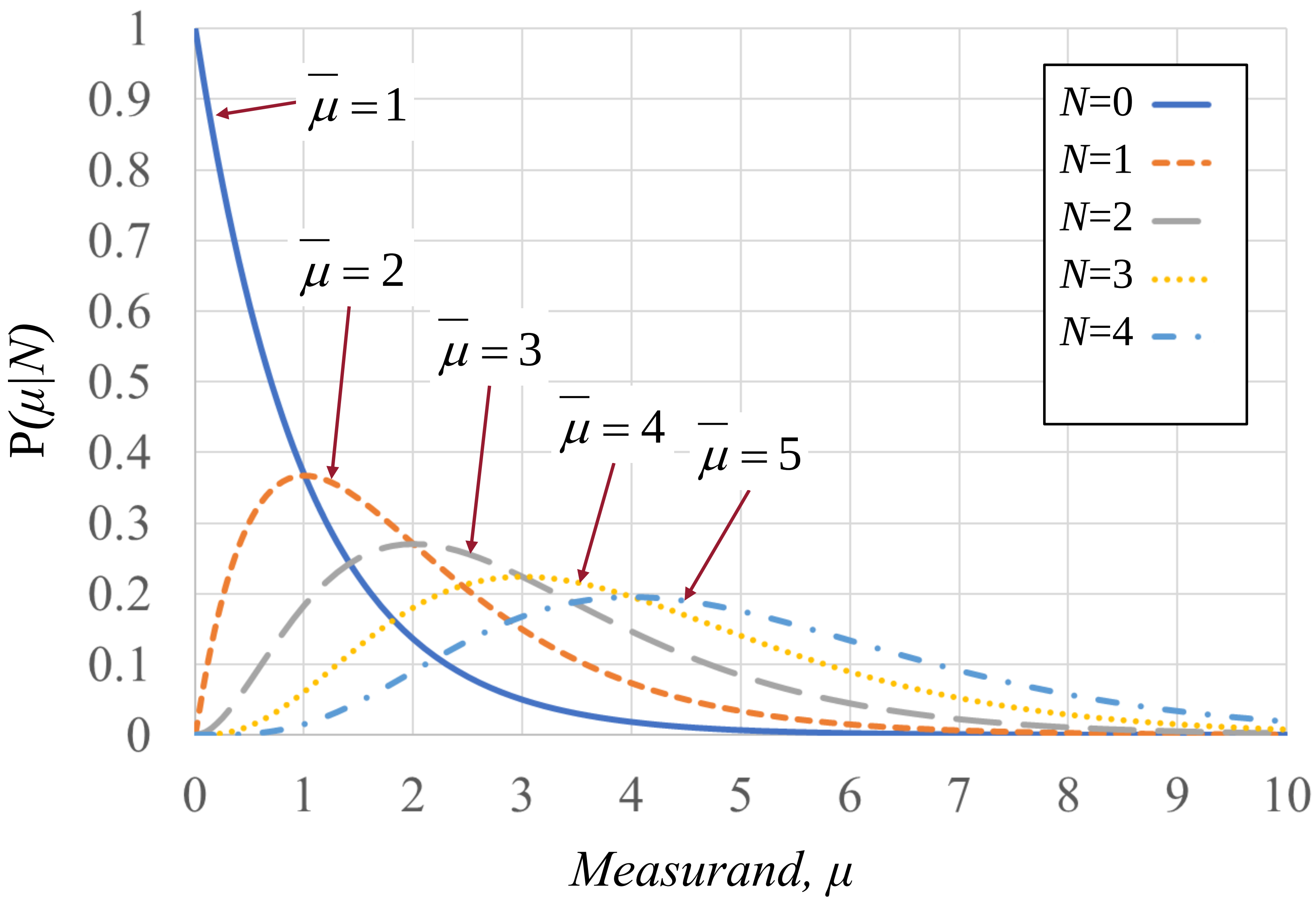
μ	$P(0\|\mu)$
0	1
0.1	0.905
0.5	0.607
1	0.368
2	0.135
3	0.050
5	0.007
10	0.00005

Bayesian Posteriors for N = 0, 1, 2, and 3



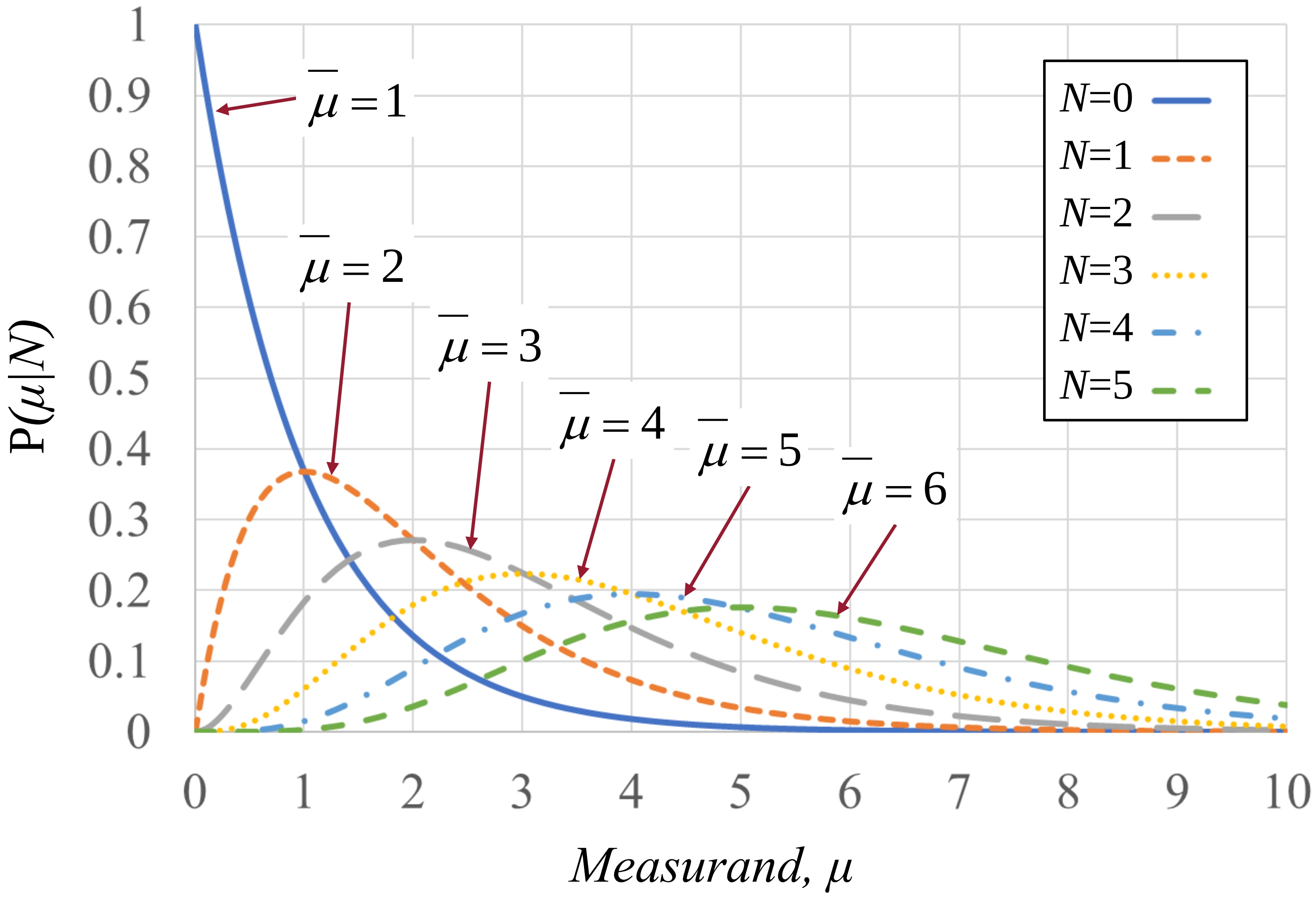
μ	$P(0\|\mu)$
0	1
0.1	0.905
0.5	0.607
1	0.368
2	0.135
3	0.050
5	0.007
10	0.00005

Bayesian Posteriors for $N = 0, 1, 2, 3$, and 4



μ	$P(0\|\mu)$
0	1
0.1	0.905
0.5	0.607
1	0.368
2	0.135
3	0.050
5	0.007
10	0.00005

Bayesian Posteriors for N = 0, 1, 2, 3, 4, and 5



μ	$P(0\|\mu)$
0	1
0.1	0.905
0.5	0.607
1	0.368
2	0.135
3	0.050
5	0.007
10	0.00005

Observe *N*, Use *N*+1 (1)

- The use of $N + 1$ has been around since the fall of 1945, if not before
- In 2005, I asked Gerhardt Friedlander about his reference for $N + 1$ in *Radiochemistry*, the 1949 book by Friedlander & Kennedy
- Friedlander had referenced a personal communication from R.W. Dodson
- Friedlander emailed me about Richard W. Dodson, who would later head chemistry at Brookhaven http://www.chemistry.bnl.gov/dodson/dodson.htm:
  - "I first heard [Dodson] derive (indeed based on Bayes' theorem) the result that we quoted in our 1949 book in a set of three lectures on counting statistics that he gave at Los Alamos in the fall of 1945 in the framework of a course on radiochemistry given by Joe Kennedy and me. It was the lecture notes from that course that became the basis of our 1949 textbook. To the best of my knowledge Dodson never published his notes on counting statistics (although I remember urging him to do so) because he felt that they didn't really contain anything new or original."

Observe *N*, Use *N*+1 (2)

- *N*+1 has been in
  - Rainwater & Wu. 1947. Nucleonics 1:60-69
  - Every edition of Friedlander & Kennedy's *Nuclear and Radiochemistry* since 1949
  - Thomas J. 1963. Risø Report 70. Danish AEC
  - Stevenson PC. 1966. NAS-NS-3109. The National Academy of Sciences
  - Many more recent works
- When are we going to wake up and smell the coffee?

Classical Statistics:
Traditional Relationships Among Observed Quantities

$$R_b = \frac{N_b}{t_b}\,;\; R_g = \frac{N_g}{t_g}$$

$$R_n = R_g - R_b = \frac{N_g}{t_g} - \frac{N_b}{t_b}$$

$$s(R_n) = \sqrt{\frac{s^2(N_g)}{t_g^2} + \frac{s^2(N_b)}{t_b^2}} \approx \sqrt{\frac{N_g}{t_g^2} + \frac{N_b}{t_b^2}}$$

This is an ISO Type B uncertainty evaluation

What’s Wrong with this Picture?

Among other things, it gives a lot of biased (wrong) answers

Bayesian Statistics with Uniform Prior:
Relationships Among Observed Quantities

$$R_b = \frac{N_b + 1}{t_b} \;;\; R_g = \frac{N_g + 1}{t_g}$$

$$R_n = R_g - R_b = \frac{N_g + 1}{t_g} - \frac{N_b + 1}{t_b}$$

If $t_g = t_b$, this is identical to the classical result!

$$s(R_n) = \sqrt{\frac{s^2(N_g + 1)}{t_g^2} + \frac{s^2(N_b + 1)}{t_b^2}} \approx \sqrt{\frac{N_g + 1}{t_g^2} + \frac{N_b + 1}{t_b^2}}$$

This is an ISO Type B uncertainty evaluation

Decision Rules:
Is There Any Activity There?
(Is the Measurand Greater than Zero?)

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Error Terminology

Type I Error

Type II Error

Warning: “Error” has other meanings outside of metrology!

uEnglish Lesson!

- Baseball: “error” means “blunder” or “mistake”
- Too many statisticians use “error” loosely and non-specifically:
  - “uncertainty” in the metrology sense
  - “error” in the metrology sense
  - “mistake” = “wrong decision”

Error Terminology

- A **Type I error** (wrong decision) is falsely concluding there's activity present when no activity is present
- A **Type II error** is falsely concluding there's no activity present when activity is present
- The **probability of a Type I** error is called α
- The **probability of a Type II** error is called β
- The number of standard deviations above zero on the standard normal distribution having a probability of α or β of being higher is known as the "**standard normal deviate**," k_α or k_β
  - these are $k_{1-\alpha}$ or $k_{1-\beta}$ in ISO notation
- For $\alpha = 0.05$ (a 5% chance of making a Type I error), $k_\alpha = 1.645$
- For $\beta = 0.05$ (a 5% chance of making a Type II error), $k_\beta = 1.645$

Calculating k_α

- $3.29 = 2 \times 1.645 = 2 \times k_\alpha$
- In Excel, calculate k_α using for $\alpha = 0.05$

  =NORMSINV(1 – 0.05) *(Excel 2003)*

  =NORM.S.INV(1 – 0.05) *(Excel 2007 and later)*

  returns 1.645

Notation

- N_b, N_g, N_n, observed numbers of background, gross, and net counts
- t_b, t_g, observed background and gross count times
- R_b, R_g, R_n, observed background, gross, and net count rates
- ρ_b, true (but unknown) background count rate
- $\mu_b = \rho_b t_b$, expectation value of number of background counts in time t_b
- α, a priori false positive rate
- α', actual false positive rate
- k_α, z, standard normal deviates
- *DL*, decision level (can be for counts or count rate)
  - I prefer decision threshold, *DT*, which is the self-defining international usage

The Commonly Used Decision Rule

- Nicholson's (1963) D_2 rule, Currie's (1968) rule, ANSI/HPS N13.30-1996, MARSSIM, even MARLAP for large numbers of counts

$$DL_{\mathrm{N13.30}}(N_{\mathrm{b}},\alpha) = k_{\alpha}\sqrt{2N_{\mathrm{b}}}$$

$$DL_{\mathrm{N13.30}}(N_{\mathrm{b}},0.05) = 1.645\sqrt{2N_{\mathrm{b}}}$$

$$= 2.329\sqrt{N_{\mathrm{b}}}$$

$$DL_{\mathrm{N13.30}}(R_{\mathrm{n}},\alpha) = k_{\alpha}\sqrt{\frac{N_{\mathrm{b}}}{t_{\mathrm{b}}}\left(\frac{1}{t_{\mathrm{b}}}+\frac{1}{t_{\mathrm{g}}}\right)}$$

Evaluating uncertainty for a single measurement

- When only one observation of a number of counts N is made, a Type A uncertainty evaluation is not possible
- By assuming that measurement result N is the measurand μ (that is, by assuming that N is the mean of the Poisson distribution from which we are randomly sampling), we also assume that
  - N is the variance, $s^2(N)$, of that Poisson distribution
  - $\sqrt{N}$ is the standard deviation, $s(N)$, of that Poisson distribution
- Thus,

$$s(N) = \sqrt{N}$$

This is an ISO Type B uncertainty evaluation

- This is a poor assumption for small numbers of counts

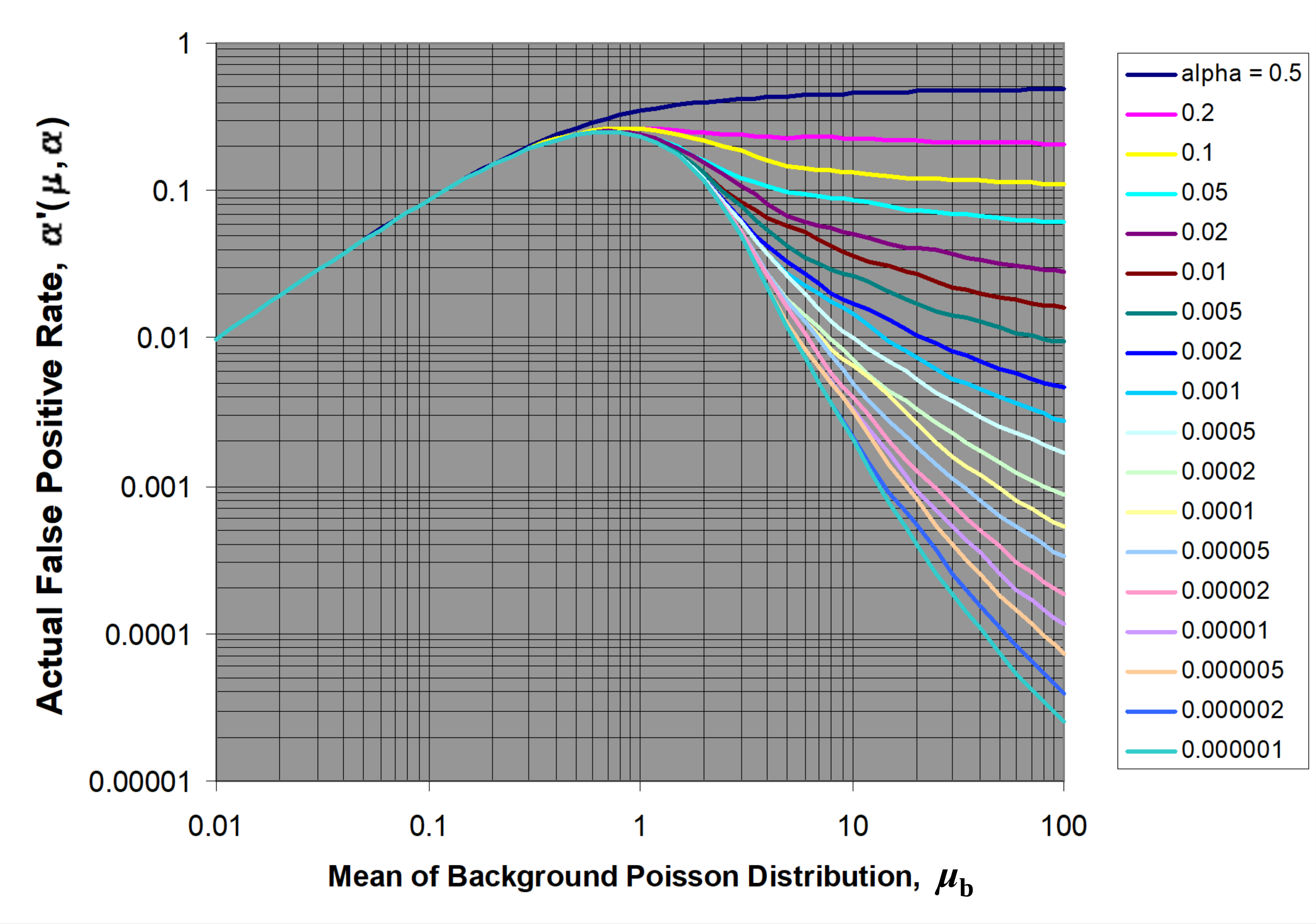
Problems with the Current Decision Rule

- Actual false positive rate α' is *independent* of α at very small numbers of counts

$$\mu_b = \rho_b t_b << 1$$

- Way too many false positives at $\mu_b \approx 0.7$
- Even at $\mu_b = 10$, only asymptotically approaches α for larger values
- For very small α, no good even at $\mu_b = 100$!

$$DL\,(0.05,\,N_b) = 2.33\sqrt{N_b}$$



Why the Currie/N13.30 Decision Rule Fails at Very Low Background Rates

- The traditional decision rule is based on 2 false assumptions:
  1. "The observed value N_b is a good estimate of the measurand μ_b" (it's not)
  2. "The observed value N_b is a good estimate of the variance of μ_b" (it's not)
     - that is, that "$N_b^{1/2}$ is a good estimate of the standard deviation of μ_b" (it's not)
- Both assumptions have been made by many authors

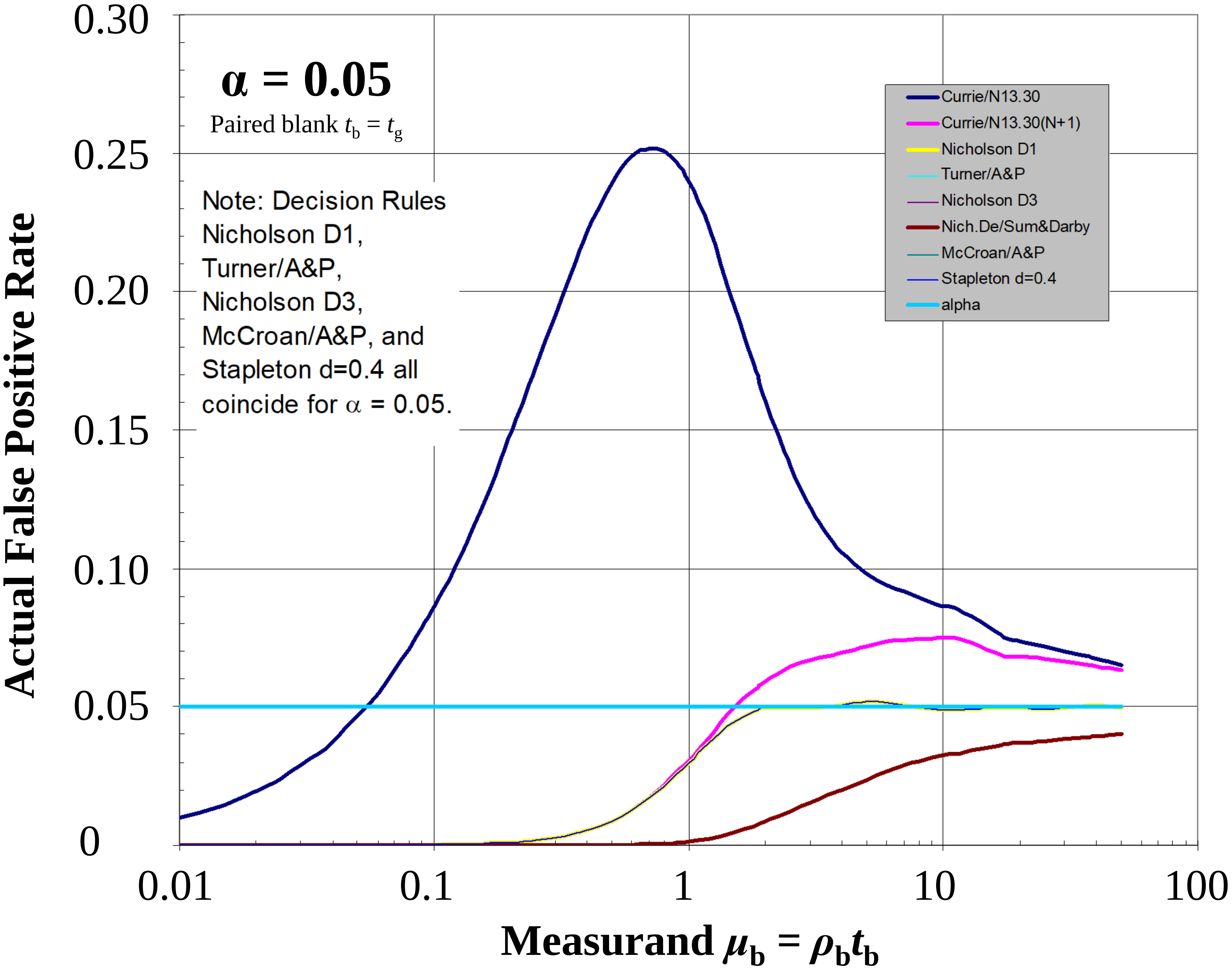
“N_b + 1” Decision Rule

- Bayesian inference of background rate
- Question: If one observes N_b counts, what is the ***expectation value*** of the background distribution that gave rise to this observation ?
- Bayesian answer (uniform prior): $\mu_b = N_b + 1$

$$DL_{N+1}(N_b, \alpha) = k_\alpha \sqrt{2(N_b + 1)}$$

- Idea: Friedlander & Kennedy1949; Friedlander et al. 1955, 1963; Stevenson 1966; Little 1982

$$DL_{N+1}(R_n, \alpha) = k_\alpha \sqrt{\frac{(N_b + 1)}{t_b}\left(\frac{1}{t_b} + \frac{1}{t_g}\right)}$$



Adapted from Strom and MacLellan (2001)

What Is the Smallest Measurand that One Can Reliably Detect?

What Value of the Measurand Would Usually Give a Measurement Result above the Decision Level?

Classical Statistics Predicts the Probability of a Range of Values of Observed Counts Based on a Hypothetical Value of the Measurand

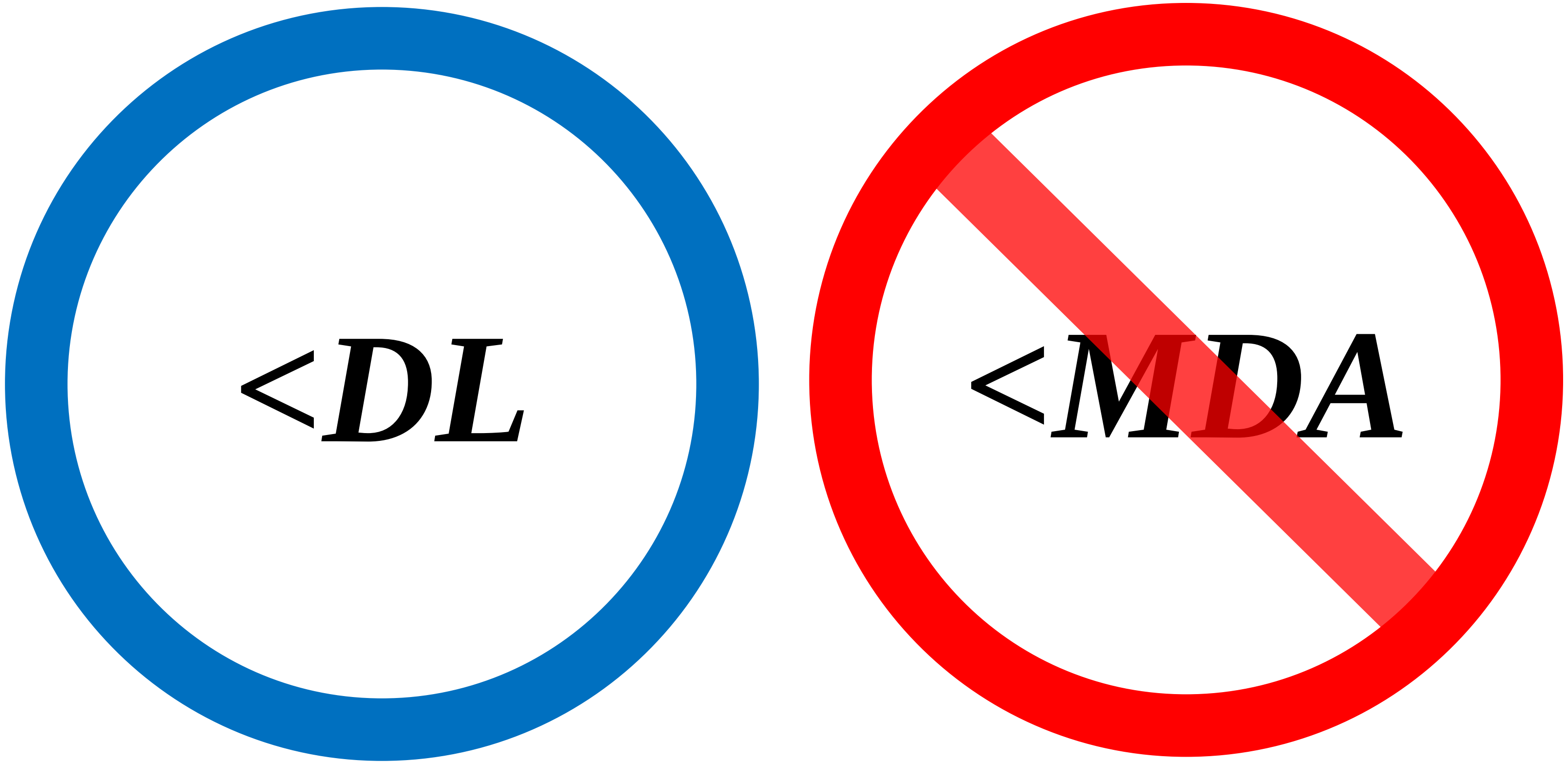
- What is the probability that a particular value of the measurand would result in an observation greater than the decision level (*DL*)?
- What value of the measurand would give a count rate greater than the *DL* 95% of the time?
  - that is, what measurand would have a false-negative probability $\beta = 0.05$?
  - let's define "usually" as "95% of the time"
- This quantity is the "smallest usually detectable measurand," *SUDM*
- This quantity is incorrectly called by a lot of stupid and confusing names, like
  - "detection level," L_D
  - "minimum detectable amount," *MDA*
  - "limit of detection," *LOD*
  - and on and on…

The "Smallest Usually Detectable Measurand" *SUDM*

- The measurand (true amount) that will usually give a counting result above the decision level *DL* (or, better, the decision threshold, *DT*)
  - ✓ "usually" means (1-β), where β is the acceptable probability of not detecting
  - ✓ typically, we choose $\beta = 0.05$, that is, only 1 time in 20 will we fail to detect an unknown whose true activity = *SUDM(A)*

$$SUDM_{N+1}(A) = \frac{3 + (k_\alpha + k_\beta)\sqrt{\frac{(N_b + 1)}{t_b} t_g \left(1 + \frac{t_g}{t_b}\right)}}{Y t_g} = \frac{\frac{3}{t_g} + 3.29\sqrt{\frac{(N_b + 1)}{t_b}\left(\frac{1}{t_g} + \frac{1}{t_b}\right)}}{Y}$$

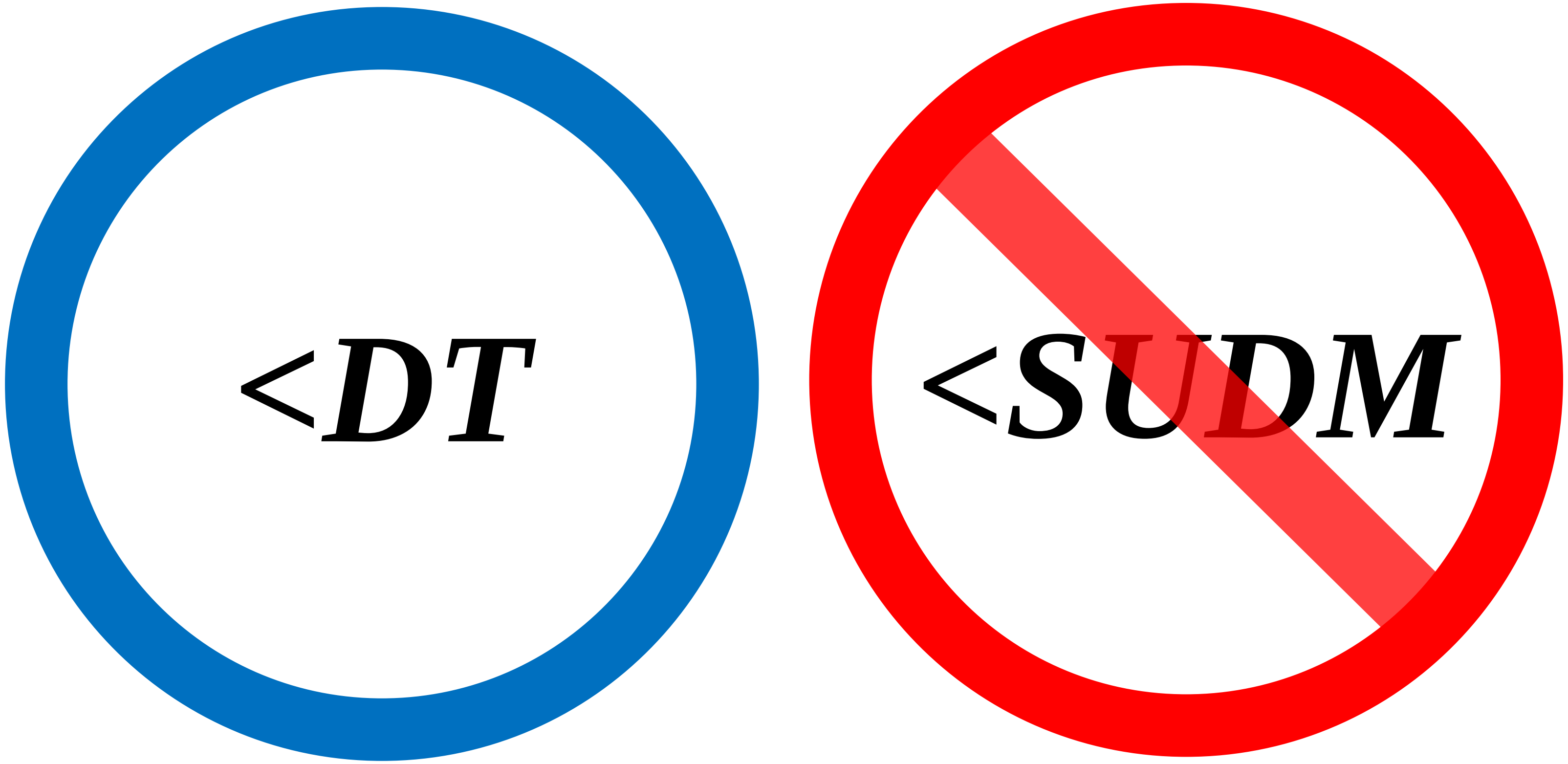
where *Y* is the counting yield (counts per second per becquerel) or (counts per nuclear transition)



Always compare a result with the decision level.

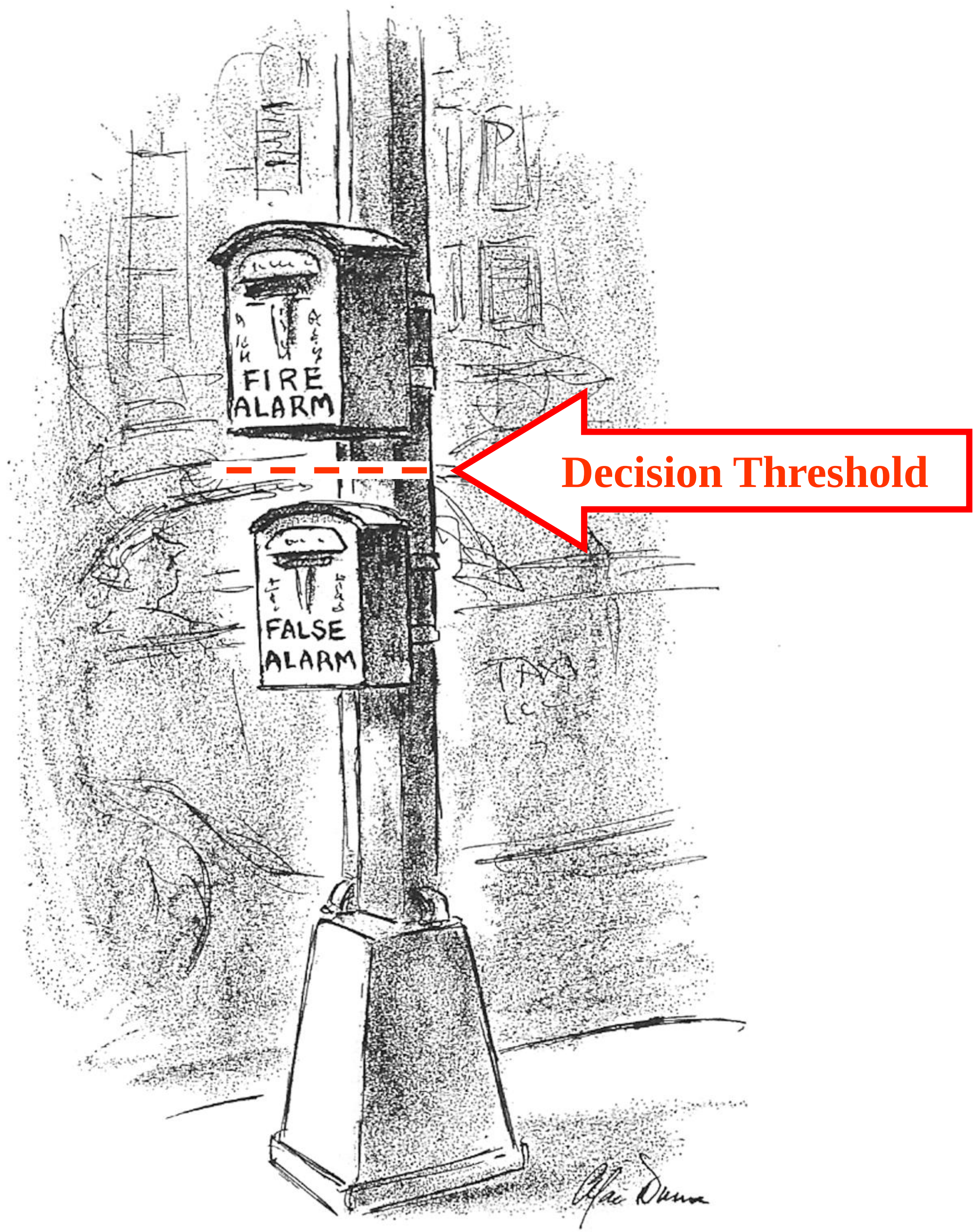
Never compare a result with the minimum detectable amount!

A shout out to Rick Brake of LANL, who drew this on a cocktail napkin at a bar in Santa Fe at the 1992 BAER conference

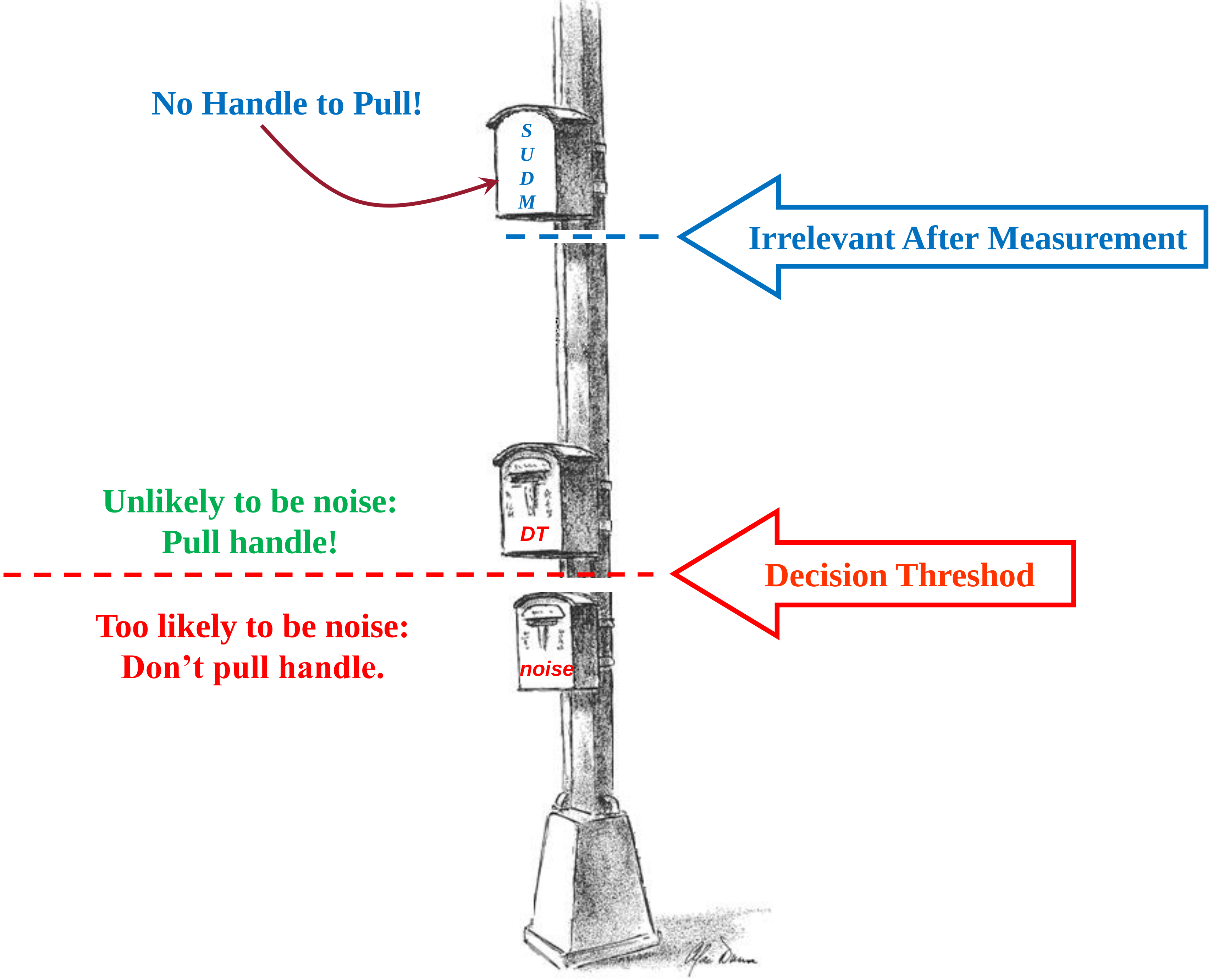


Always compare a result with the decision threshold.

Never compare a result with the smallest usually detectable measurand!



Alan Dunn in *The New Yorker* (1972)


No Handle to Pull!
S
U
D
M
Irrelevant After Measurement
Unlikely to be noise:
Pull handle!
DT
Decision Threshod
Too likely to be noise:
Don’t pull handle.
noise

Conclusions

- The measurand is
  - “the quantity intended to be measured”
  - the unknown, and usually unknowable, “true state of Nature”
- Measurement results are correctly used to describe an interval in which the measurand probably falls
- Bayes’s theorem allows us to make probabilistic statements about the measurand
- Traditional formulas for decision level and so-called minimum detectable activity perform poorly
- Even the most simplistic Bayesian solutions perform better than traditional formulas