Disaggregating measurement uncertainty from population variability and Bayesian treatment of uncensored results

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In making low-level radioactivity measurements of populations, it is commonly observed that a substantial portion of net results is negative. Furthermore, the observed variance of the measurement results arises from a combination of measurement uncertainty and population variability. This paper presents a method for disaggregating measurement uncertainty from population variability to produce a probability density function (PDF) of possibly true results. To do this, simple, justifiable and reasonable assumptions are made about the relationship of the measurements to the measurands (the 'true values'). The measurements are assumed to be unbiased, that is, that their average value is the average of the measurands. Using traditional estimates of each measurement's uncertainty, a likelihood PDF for each individual's measurand is produced. Then using the same assumptions and all the data from the population of individuals, a prior PDF of measurands for the population is produced. The prior PDF is non-negative, and the average is equal to the average of the measurement results for the population. Using Bayes's theorem, posterior PDFs of each individual measurand are calculated. The uncertainty in these Bayesian posterior PDFs appears to be all Berkson with no remaining classical component. The method is applied to baseline bioassay data from the Hanford site. The data include ^{90}Sr urinalysis measurements of 128 people, ^{137}Cs in vivo measurements of 5337 people and ^{239}Pu urinalysis measurements of 3270 people. The method produces excellent results for the ^{90}Sr and ^{137}Cs measurements, since there are non-zero concentrations of these global fallout radionuclides in people who have not been occupationally exposed. The method does not work for the ^{239}Pu measurements in non-occupationally exposed people because the population average is essentially zero relative to the sensitivity of the measurement technique. The method is shown to give results similar to classical statistical inference when the measurements have relatively small uncertainty.

USTUR-0424A-16