# POWER FLOW APPROXIMATIONS FOR MULTIPHASE DISTRIBUTION NETWORKS USING GAUSSIAN PROCESSES

Daniel Glover[1], Parikshit Pareek[2], Deepjyoti Deka[3], Anamika Dubey[1]

[1] Washington State University, Pullman, WA, [2] Indian Institute of Technology, Roorkee, Uttarakhand, [3] Massachusetts Institute of Technology, Cambridge, MA

Washington State University

## 1. Motivation & Challenges in ADNs

- ADNs growing more complex, managing bi-directional energy and data traffic
- Stochastic DERs and unpredictable load dynamics, combined with partial observability, create operational challenges for grid operators
- To simplify OPF problems, approximate power flow models have been proposed for unbalanced multiphase ADNs
- Learning based approaches increasingly leveraged for managing grid-edge resources
- DNNs data hungry, computationally inefficient, require constant retraining, inherently *black-box*
- Utilities need interpretable, fast, accurate power flow solutions when data is scarce or limited

## Distribution System Model

At node $i \in \mathcal{N}$ for a phase $\rho$, $(\rho_i \in \{a,b,c\})$

$V_i^\rho \angle \theta_i^\rho \in \mathbf{V}; \boldsymbol{\theta}$

$s_{Li}^\rho = S_{L,i}^\rho - S_{DER,i}^\rho, \;\; S_{Li}^\rho = P_{Li}^\rho + jQ_{Li}^\rho,$

$s_{Li}^\rho \in \mathbf{s}, \;\; \mathbf{s} = [\mathbf{p};\mathbf{q}]$

We represent the family of nonlinear power flow equations as the mapping between the vector of PF solutions[1]:

$$\mathbf{z} = [\mathbf{V};\boldsymbol{\theta}] \text{ and net loads } \mathbf{z} = F(\mathbf{s}).$$

The nonlinear equations $F(\cdot)$ are generally solved iteratively via Newton-Raphson or Backward-Forward Sweep, however, the goal here is to use GP regression to obtain an approximate PF solution to $F(\mathbf{s})$, mapping the relationship between the load vector $\mathbf{s}$ and the voltage magnitude solution $\mathbf{V}$ (or $\boldsymbol{\theta}$).

## 2. Gaussian Process Power Flow Approximation Model

### Gaussian Process Regression (GP)

$$f_i(x) \longrightarrow \mathcal{D} = \{(x_i, y_i) | i = 1, ..., n\} \;\; X = [x_1, ..., x_n] \;\; \mathbf{y} = [y_1, ..., y_n]$$

GP with *Squared Exponential* kernel[2]

$$f(x) \sim \mathcal{GP}(m(x), k(X,X))$$

$$k(x^i, x^j) = \sigma_s^2 \exp\left[\frac{-\|x^i - x^j\|^2}{2l}\right]$$

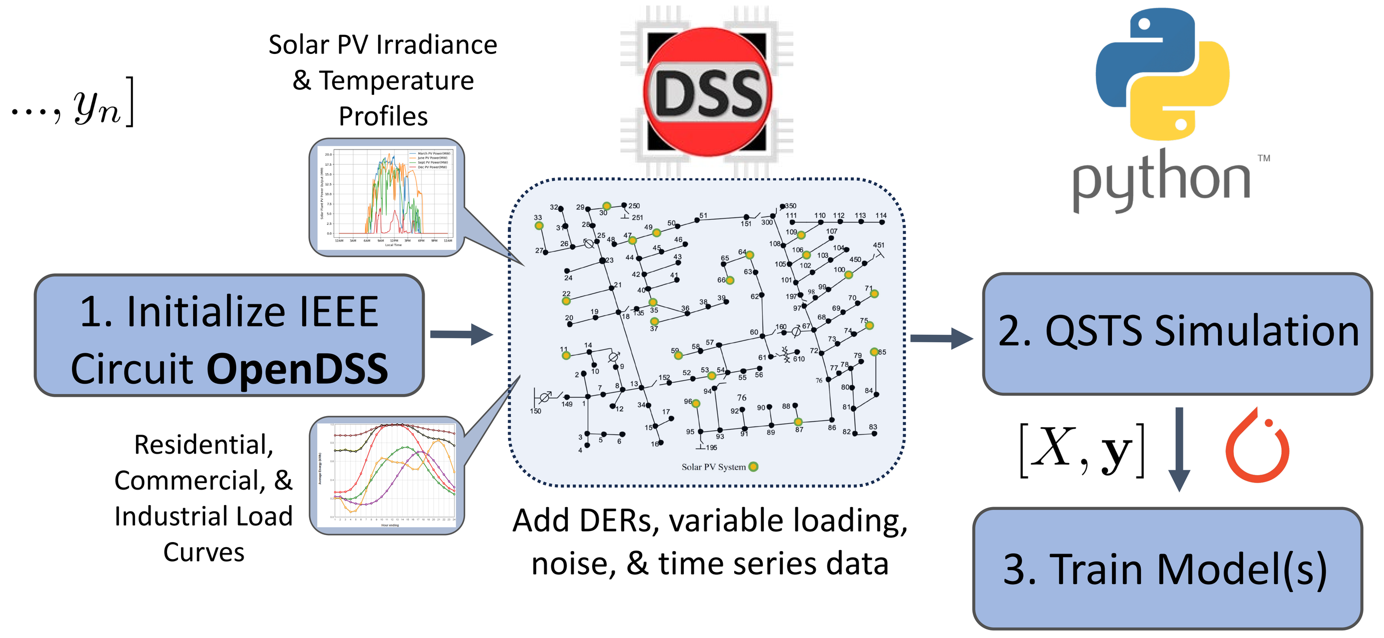
$$\text{MLL} = \Theta^*(\sigma_s^2, l) = arg\max_\Theta \log pr(\mathbf{y}|X,\Theta)$$

For a new input $x^*$, the mean multiphase PF prediction $\hat{f}$ at $x^*$ is given as:

$$\hat{f}(x^*)|X, \mathbf{y}, x^*, \Theta^* \sim \mathcal{N}(\hat{f}(x^*), cov(f^*(x^*)))$$

$$\hat{f}(x^*) = \mathbf{k}^T (K + \sigma_\epsilon^2 I)^{-1} \mathbf{y}$$

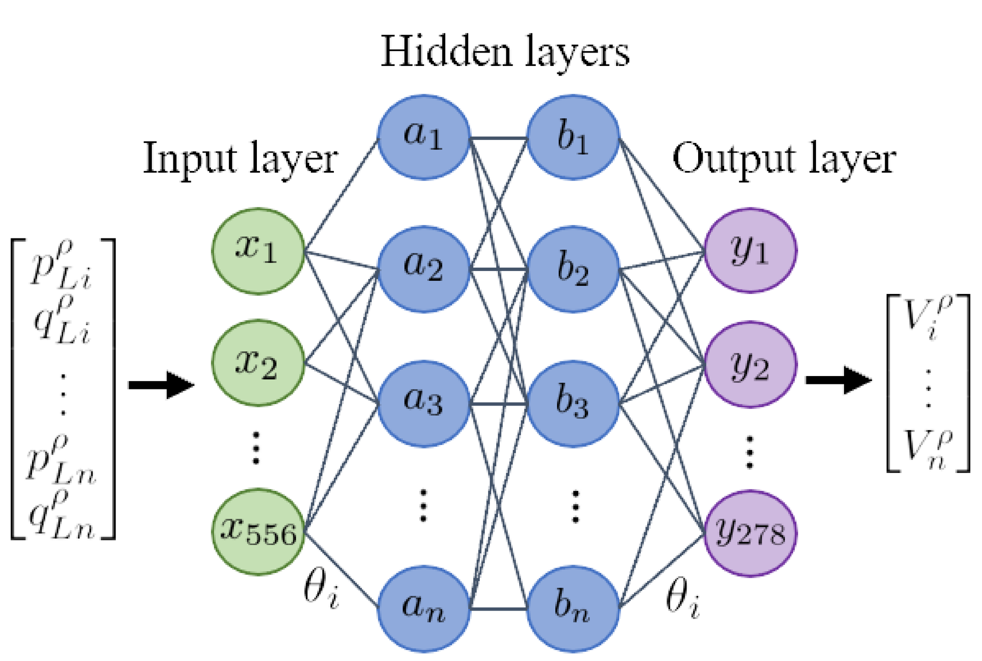
### QSTS Data Generation & Acquisition




*Design matrix* $X = [s_{Li}^\rho, ..., s_{Ln}^\rho] = [p_{Li}^\rho, q_{Li}^\rho, ..., p_{Ln}^\rho, q_{Ln}^\rho]$ and the corresponding nonlinear PF solution per phase of voltage magnitudes $\mathbf{y} = [V_i^\rho, ..., V_n^\rho]$ (or angles $\theta_i^\rho$).

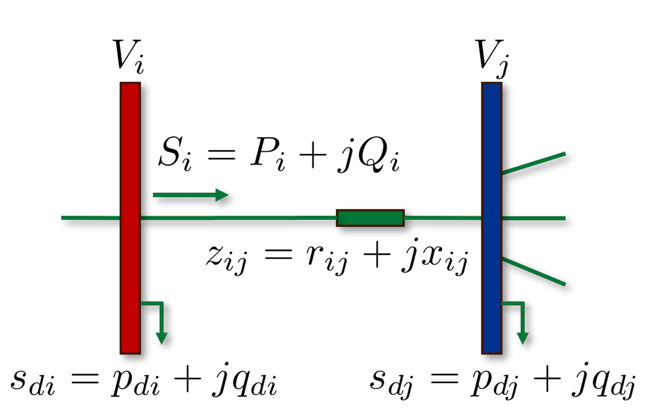
## 3. Benchmark Approximation Methods

### Deep Neural Network (DNN)




$$\mathcal{L}_{MSE} \leftarrow \frac{1}{n}\sum_{i=1}^n (\hat{y}_i - y_i)^2 \text{ to update } f(\boldsymbol{\theta})$$

### Linearized Distribution Power Flow (LDF)




$$P_{ij}^{\rho\rho} - p_{L,j}^\rho = \sum_{k:j\to k} P_{jk}^{\rho\rho} \quad \rho \in \{a,b,c\}$$

$$Q_{ij}^{\rho\rho} - q_{L,j}^\rho = \sum_{k:j\to k} Q_{jk}^{\rho\rho} \quad \rho \in \{a,b,c\}$$

$$v_i^\rho - v_j^\rho = \sum_{q\in\phi_j} 2\mathbb{R}[S_{ij}^{pq}(z_{ij}^{pq})^*] p,q \in \{a,b,c\}$$

## 4. Multiphase Power Flow Comparison Case Study

### Training Efficiency & Accuracy

**CASE 1: 1 Day, 24 Hours Data**

| PF | MSE | MAE | TIME |
|---|---|---|---|
| LDF | 0.0013 | 0.053 | 00:00:20 |
| DNN | 0.02 | 0.1197 | 00:00:14 |
| GP | **1.67e-09** | **2.9e-05** | **00:00:01** |

**CASE 2: 7 Days, 168 Hours Data**

| PF | MSE | MAE | TIME |
|---|---|---|---|
| LDF | 0.00013 | 0.0069 | 00:03:30 |
| DNN | 0.0103 | 0.0514 | 00:00:25 |
| GP | **3.14e-10** | **1.3e-05** | **00:00:04** |

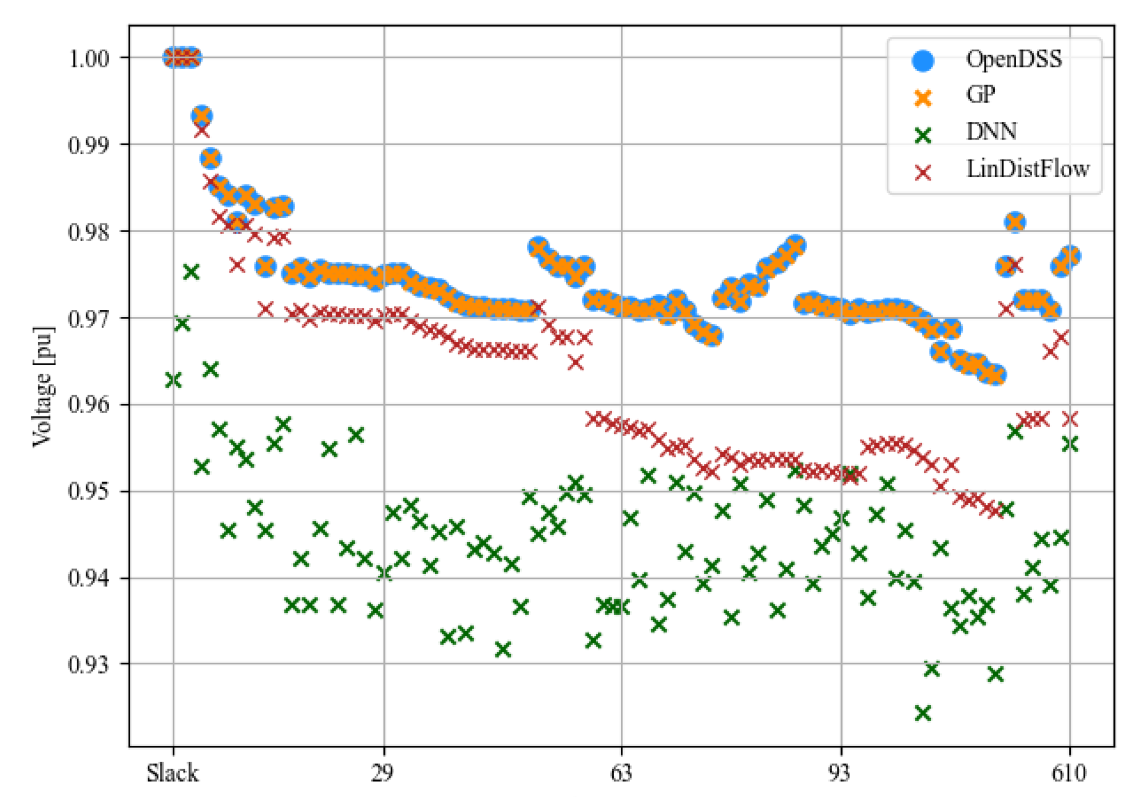
**CASE 3: 30 Days, 720 Hours Data**

| PF | MSE | MAE | TIME |
|---|---|---|---|
| LDF | 3.1e-05 | 0.0016 | 00:15:02 |
| DNN | 7.14e-06 | 0.0023 | **00:00:43** |
| GP | **6.53e-11** | **5.44e-06** | 00:00:44 |

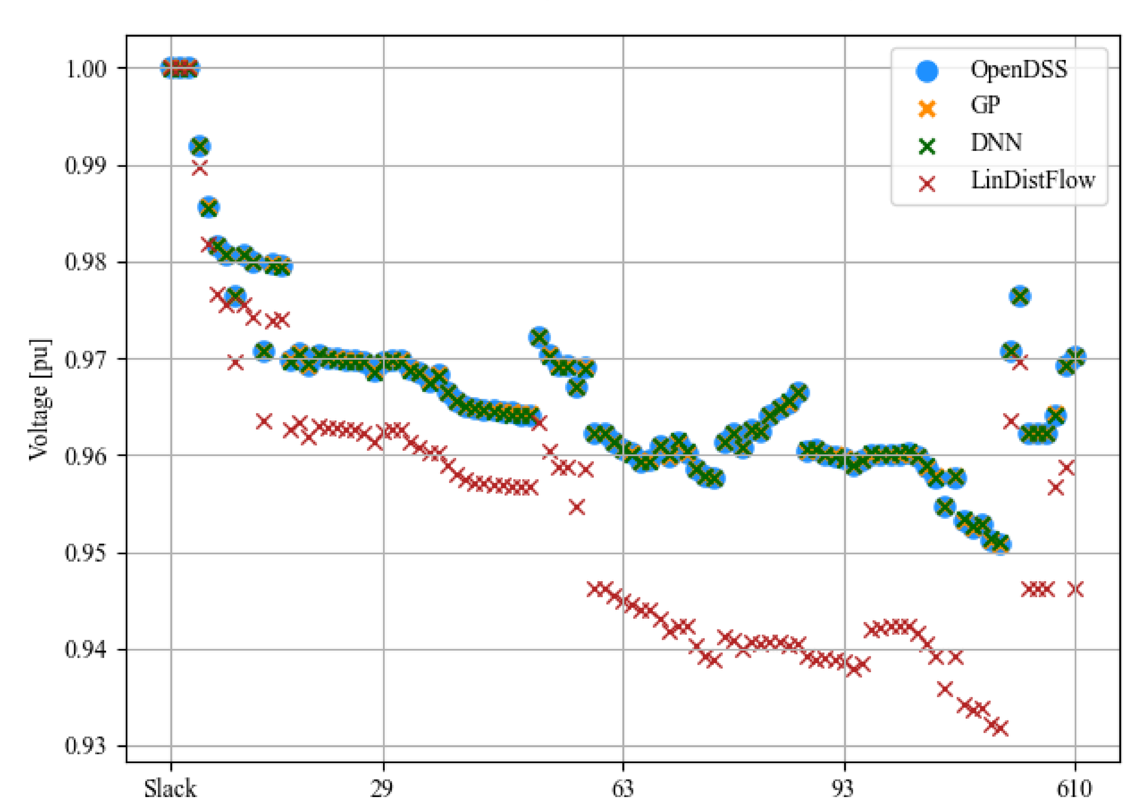
**CASE 4: 90 Days, 2160 Hours Data**

| PF | MSE | MAE | TIME |
|---|---|---|---|
| LDF | 1.1e-05 | 5.6e-04 | 00:55:44 |
| DNN | 4.12e-08 | 1.48e-04 | **00:01:51** |
| GP | **2.22e-11** | **2.93e-06** | 00:27:21 |

*Simulations run on Intel® Core i7-7500 CPU, 16GB RAM




**Case 1** (above) and **Case 4** (below) training validation and model comparison. The GP, DNN, and LDF vs OpenDSS nonlinear power flow solution predictions. IEEE 123-Bus Test System with multiple DERs.



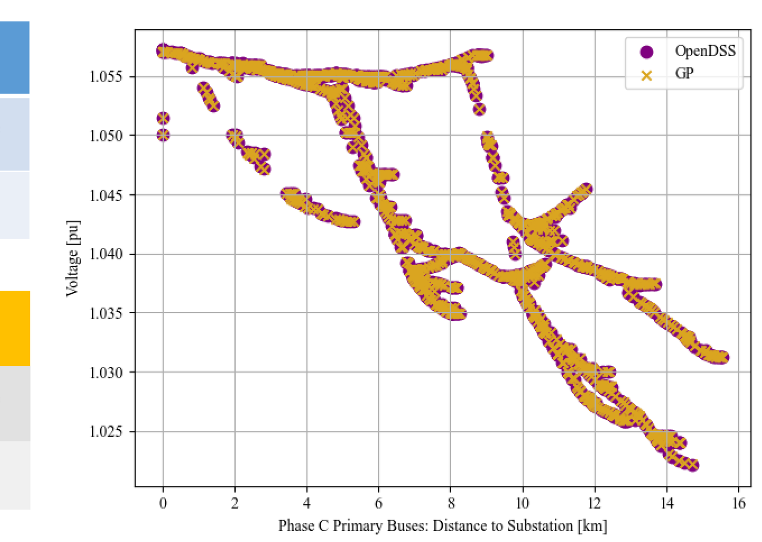

## 5. 24-Hour GP Trained Model: Six Day Evaluation

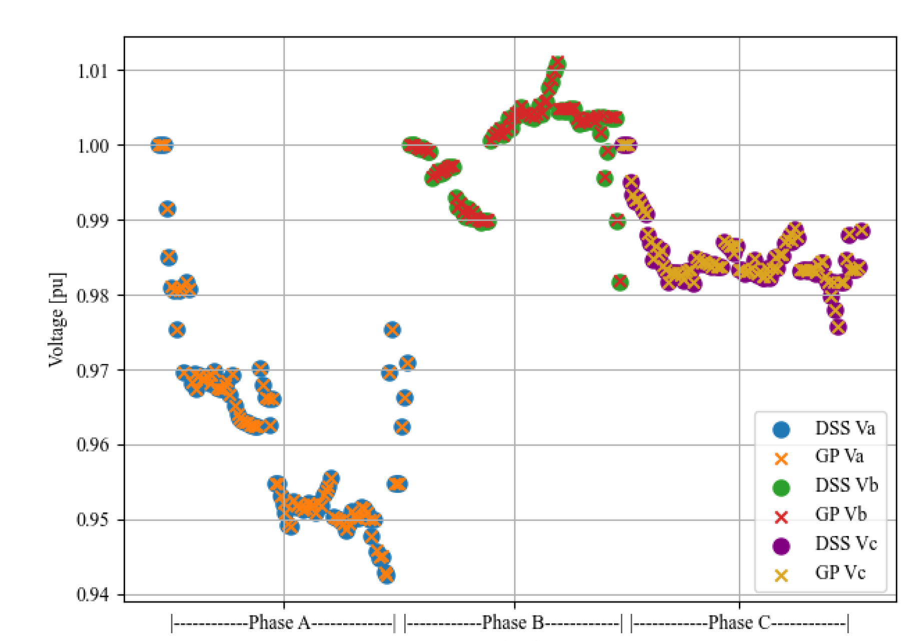
**IEEE 123-Bus Test System**

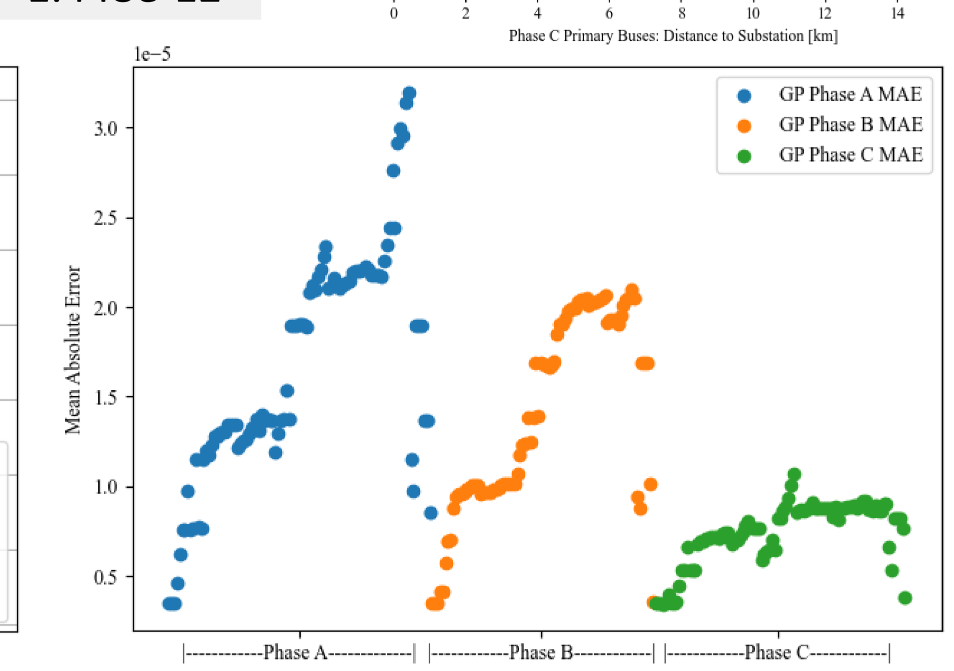
| Avg MAE | Avg MSE | Max Error | Min Error |
|---|---|---|---|
| 2.869e-04 | 4.255e-06 | 0.000595 | 7.914e-11 |

**IEEE 8500-Node Test System: Primary Feeder**

| Avg MAE | Avg MSE | Max Error | Min Error |
|---|---|---|---|
| 5.833e-05 | 8.472e-09 | 0.000308 | 1.448e-12 |




## 6. Concluding Key Remarks & References

- Results showed the trained GP can correctly map net load injections to the unbalanced multiphase nonlinear PF solution considering high DER penetrations and variable loading fluctuations
- An **85%** reduction in training sample size yields a **92.8%** reduction improvement in training time, corresponding to a **99%** relative reduction in MAE compared to baseline DNNs
- A GP PF approximator can reliably scale to larger feeder laterals with minimal data under uncertainty
- GPs offer utility operators a cost prohibitive solution without the need for retraining (non-parametric)
- GPs are superior in terms of training efficiency and accuracy under limited measurements compared to DNN PF models.

1. Pareek, Parikshit, and Hung D. Nguyen. "A framework for analytical power flow solution using Gaussian process learning." *IEEE Transactions on Sustainable Energy* 13, no. 1 (2021): 452-463.
2. C. K. Williams and C. E. Rasmussen, Gaussian processes for machine learning. MIT press Cambridge, MA, 2006, vol. 2, no. 3.

Contacts:
Daniel Glover (daniel.glover1@wsu.edu), Parikshit Pareek (pareek@ee.iitr.ac.in), Deepjyoti Deka (deepj87@mit.edu), Anamika Dubey (anamika.dubey@wsu.edu)

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