GEOMETRICAL OPTICS

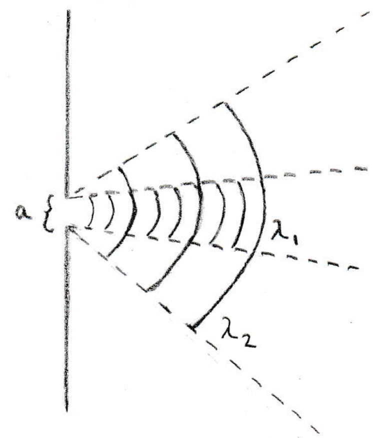
GEOMETRICAL OPTICS, OR RAY OPTICS, IS A USEFUL APPROXIMATION OF WAVE OPTICS USED TO DESCRIBE THE PROPAGATION OF LIGHT IN TERMS OF RAYS (TO BE DESCRIBED).

THIS IS A USEFUL ABSTRACTION TO APPROXIMATE THE PATHS IN WHICH A LIGHT WAVE PROPAGATES IN CERTAIN CIRCUMSTANCES.

THIS APPROXIMATION IS USED, FOR EXAMPLE, IN THE PRACTICAL DESIGN OF MANY OPTICAL SYSTEMS AND INSTRUMENTS.

DRAWING #2: SINGLE-APERATURE DIFFRACTION

CONSIDER TWO WAVES WITH DIFFERENT WAVELENGTHS PASSING THROUGH AN APERATURE OF SIZE a:



NOTE: THE DOTTED LINES INDICATE THE POSITION TOWARDS THE FIRST DARK FRINGE.

IT CAN BE SEEN THAT THE EXTENT TO WHICH THE WAVE SPREADS OUT BEHIND THE APERATURE IS PROPORTIONAL TO λ (FOR A FIXED a)...

... MORE GENERALLY, THE FACTOR THAT DETERMINES HOW MUCH A WAVE SPREADS OUT BEHIND THE APERATURE IS λ/a.

EXAMPLE: SINGLE-SLIT DIFFRACTION

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right)$$

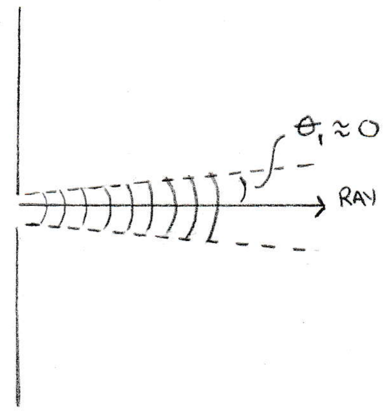
$$\lim_{\lambda/a \to 0} \theta_1 = 0$$

NOTE: THIS STATEMENT IS TRUE IRRESPECTIVE OF THE GEOMETRIC SHAPE OF THE APERATURE...

... IT IS ALSO TRUE FOR ANY STRUCTURE WHICH THE WAVE INTERACTS WITH

DRAWING #3: THE RAY MODEL

IF $\lambda \ll a$, THEN THE WAVE WILL BE BE DIFFRACTED VERY LITTLE BY THE APERATURE; IT WILL REMAIN IN A WELL-DEFINED BEAM:



IT CAN BE USEFUL (FOR CALCULATIONS, ETC.) TO DESCRIBE THIS BEAM BY A RAY: A LINE DRAWN PERPENDICULAR TO THE WAVEFRONTS WITH AN ARROW THAT POINTS IN THE DIRECTION OF THE WAVES PROPAGATION.

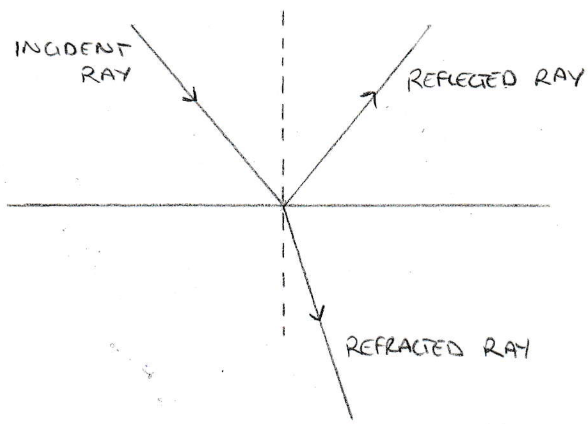
NOTE: IN THIS DESCRIPTION, IT IS NOT POSSIBLE TO DESCRIBE CERTAIN EFFECTS CENTRAL TO WAVES, SUCH AS INTERFERENCE (AND OBVIOUSLY DIFFRACTION).

DRAWING #4: GEOMETRICAL OPTICS

THE DESCRIPTION OF LIGHT PROPAGATION IN TERMS OF RAYS IS CALLED GEOMETRICAL OPTICS, OR RAY OPTICS,

THIS DESCRIPTION IS USEFUL FOR GEOMETRICALLY DESCRIBING THE PATHS ALONG WHICH LIGHT PROPAGATES IN VARIOUS CIRCUMSTANCES.

EXAMPLE: THE REFLECTION AND REFRACTION OF LIGHT



DRAWING #5: THE WAVE VS RAY MODELS OF LIGHT

WE CAN IDENTIFY TWO IMPORTANT REGIMES REGARDING THE PROPAGATION OF LIGHT:

- IF $\lambda \approx a$, THEN LIGHT IS DIFFRACTED BY A SIGNIFICANT EXTENT BY THE APERTURE. SINCE DIFFRACTION, AND THUS INTERFERENCE, ARE IMPORTANT, WE SHOULD USE WAVE OPTICS.

- IF $\lambda \ll a$, DIFFRACTION IS NEGLIGIBLE. WE CAN THEREFORE USE GEOMETRICAL OPTICS.

DRAWING #6: FERMAT'S PRINCIPLE OF LEAST TIME

SUPPOSE THAT THERE ARE TWO POINTS A AND B:

... AND WE WISH TO KNOW THE PATH IN WHICH LIGHT TAKES IN GOING FROM A TO B.

THE GENERALIZED PRINCIPLE* THAT DESCRIBES THE PATH TAKEN BY LIGHT BETWEEN TWO POINTS WAS FORMULATED BY PIERRE de FERMAT IN 1662:

<u>FERMAT'S PRINCIPLE</u> (OR <u>PRINCIPLE OF LEAST TIME</u>): OUT OF ALL POSSIBLE PATHS WHICH LIGHT MIGHT TAKE TO GET FROM ONE POINT TO ANOTHER, IT TAKES THAT WHICH REQUIRES THE LEAST TIME.

*NOTE: THIS VERSION OF THE PRINCIPLE IS ACTUALLY NOT GENERAL. THERE IS A MORE MODERN STATEMENT, BUT IT IS BEYOND THE SCOPE OF THIS COURSE.

NOTE: FERMAT'S PRINCIPLE (ABOVE) CAN BE TAKEN AS THE DEFINITION OF A LIGHT RAY.

NOTE: EARLY VERSIONS OF THE PRINCIPLE OF LEAST TIME WERE FORMULATED PRIOR TO FERMAT (EVEN AS FAR BACK AS PLATO ~400 BC).

RETURNING TO OUR QUESTION FROM EARLIER:

CONSIDER THREE PATHS (LABELED 1, 2, AND 3).

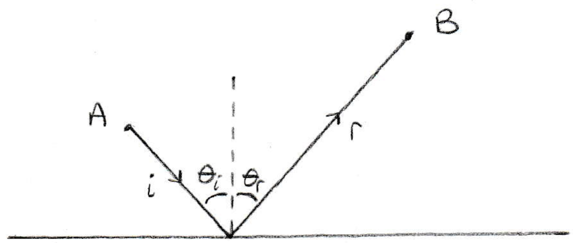
THE WAY TO GET FROM A TO B IN THE LEAST TIME IS (CLEARLY) PATH 2 (THE STRAIGHT LINE)...

... AND ACCORDING TO FERMAT'S PRINCIPLE, THIS IS THE PATH WHICH LIGHT WILL TAKE.

(NOTE: FOR LIGHT OF CONSTANT VELOCITY, THE TIME TAKEN TO TRAVEL BETWEEN TWO POINTS IS PROPORTIONAL TO THE DISTANCE BETWEEN THE POINTS.)

REFLECTION

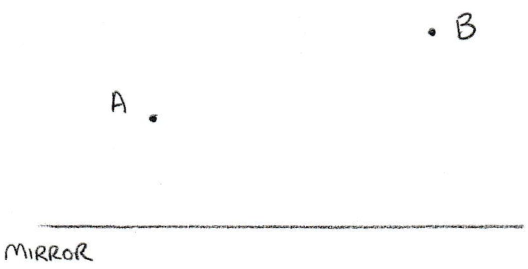
AS A FIRST (NON-TRIVIAL) EXAMPLE OF FERMAT'S PRINCIPLE, WE'LL SEE HOW IT GIVES RISE TO THE LAWS OF REFLECTION:



- THE INCIDENT RAY (i), THE REFLECTED RAY (r), AND THE NORMAL TO THE REFLECTION SURFACE (DOTTED LINE) AT THE POINT OF INCIDENCE LIE IN THE SAME PLANE
- THE ANGLE WHICH THE INCIDENT RAY MAKES WITH THE NORMAL (θ_i) IS EQUAL TO THE ANGLE WHICH THE REFLECTED RAY MAKES TO THE SAME NORMAL (θ_r)
- THE REFLECTED RAY AND THE INCIDENT RAY ARE ON OPPOSITE SIDES OF THE NORMAL.

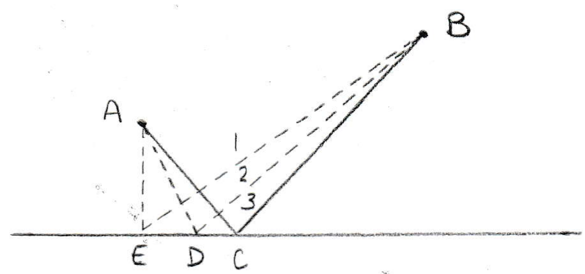
DRAWING # 7: REFLECTION 1

SUPPOSE NOW THAT THE POINTS A AND B ARE ABOVE A MIRROR WHICH PERFECTLY REFLECTS LIGHT:



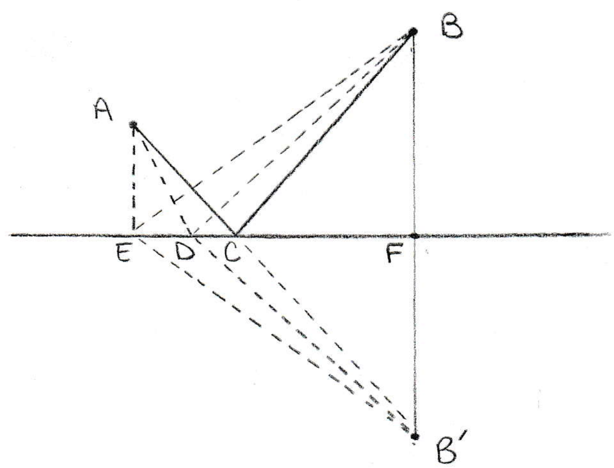
... AND WE ARE INTERESTED IN THE PATH WHICH LIGHT TAKES GOING FROM A TO B, ALSO WITH THE CONDITION THAT IT FIRST STRIKES THE MIRROR.

IN ORDER TO FIND THIS, WE NEED TO DETERMINE THE PATH, OUT OF ALL PATHS, REQUIRES THE LEAST TIME:



NOTE: AGAIN, FOR CONSTANT-VELOCITY LIGHT, THIS IS ALSO THE PATH OF LEAST DISTANCE.

WE CAN FIND POINT C USING A GEOMETRICAL TRICK:



... WE CAN INSERT AN IMAGE POINT B' OF B, THE SAME DISTANCE BELOW THE PLANE AS B IS ABOVE IT.

FOR ANY POINT P ON THE PLANE, THE DISTANCE OF THE LINE PB IS THEN EQUAL TO PB'; e.g.:

$$CB = CB'$$
$$DB = DB'$$
$$EB = EB'$$

DRAWING #8: REFLECTION 2

THE DISTANCE BETWEEN POINTS A AND B, WITH THE REQUIREMENT THAT IT MUST TOUCH A POINT P ON THE MIRROR, IS THEN:

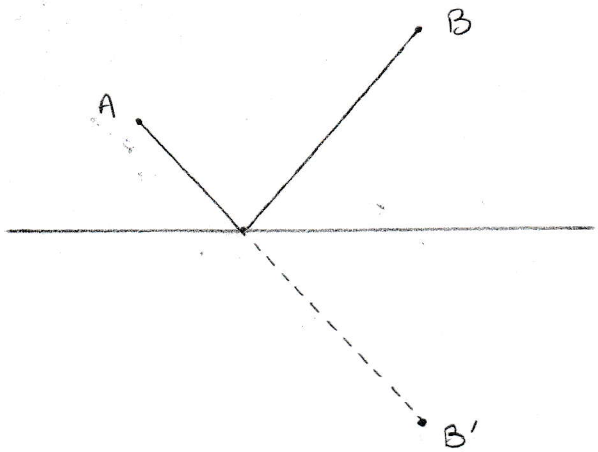
$$AP + PB$$

... WHICH CAN BE WRITTEN IN TERMS OF THE POINT B′ AS:

$$AP + PB'$$

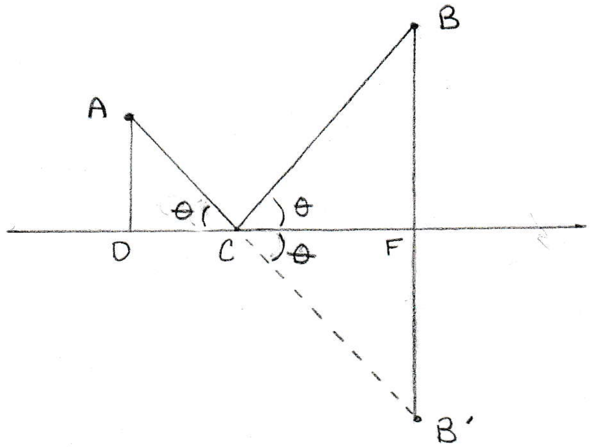
WE MUST THEREFORE DETERMINE POINT P SUCH THAT THE DISTANCE BETWEEN POINTS A AND B′ IS MINIMIZED...

... JUST AS IN THE FIRST EXAMPLE, THIS WILL OCCUR WHEN P IS CHOSEN SUCH THAT THE PATH BETWEEN A AND B′ IS A STRAIGHT LINE:



DRAWING #9: REFLECTION 3

BASED ON THE GEOMETRY OF THE PROBLEM, WE CAN MAKE SOME ADDITIONAL REMARKS:



SINCE:

$$CB = CB'$$

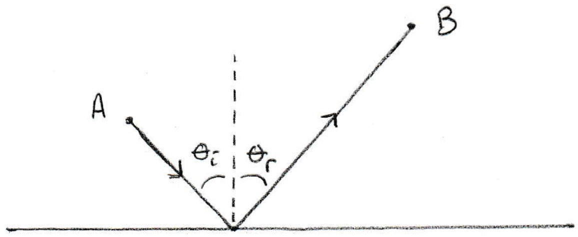
AND

$$FB = FB'$$

THE ANGLE BCF IS EQUAL TO B'CF (DENOTED ABOVE BY θ).

IN ADDITION, SINCE ACB' IS A STRAIGHT LINE, THE ANGLE ACD IS EQUAL TO B'CF (AND HENCE BCF).

THEREFORE, IF WE DRAW LIGHT INCIDENT FROM POINT A, REFLECTING UP TO POINT B:



NOTE: THE DOTTED LINE IS NORMAL TO THE MIRROR

θ_i: ANGLE OF INCIDENCE

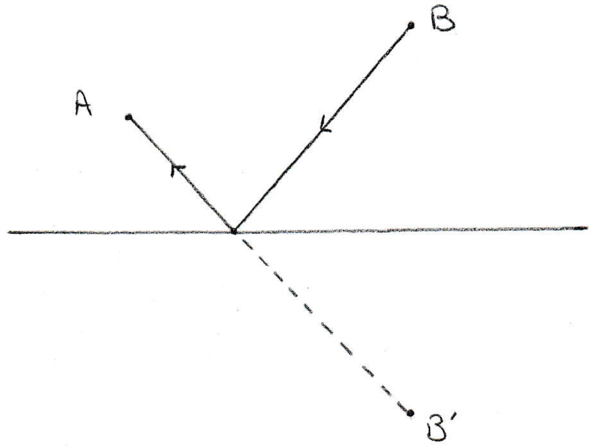
θ_r: ANGLE OF REFLECTION

... WE KNOW GEOMETRICALLY THAT:

$$\boxed{\theta_i = \theta_r}$$

... WHICH IS A CONSEQUENCE OF FERMAT'S PRINCIPLE.

DRAWING #10: REFLECTION 4



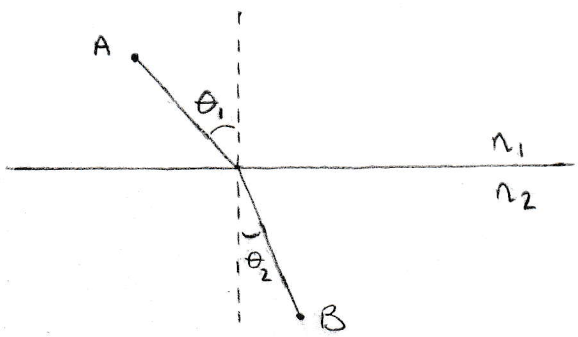
SUPPOSE THAT AN OBSERVER IS AT POINT A, AND A SOURCE OF LIGHT AT POINT B SENDS LIGHT TOWARDS THE MIRROR.

THE LIGHT WHICH ARRIVES AT POINT A DOES SO IN THE EXACT SAME MANNER AS IT WOULD HAVE DONE IF THERE WERE A SOURCE OF LIGHT AT POINT B′ AND NO MIRROR.

THE ILLUSION THAT THAT THERE IS AN OBJECT "BEHIND" THE MIRROR IS DUE PRECISELY TO THIS... THE LIGHT THAT WE OBSERVE IS EXACTLY THE SAME HAD THERE ACTUALLY BEEN AN OBJECT BEHIND IT.

REFRACTION

AS A SECOND (NON-TRIVIAL) EXAMPLE OF FERMAT'S PRINCIPLE, WE'LL SEE HOW IT GIVES RISE TO **SNELL'S LAW OF REFRACTION**:



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

NOTE: THIS FORMULA DESCRIBES THE RELATIONSHIP BETWEEN ANGLES OF RAYS 1 AND 2, WHEN LIGHT PASSES THROUGH A BOUNDARY BETWEEN TWO MATERIALS.

NOTE: THIS LAW DOES NOT EXPLICITELY MENTION WHICH ARE THE INCIDENT AND REFRACTED ANGLES.

NOTE: THIS EXAMPLE WILL SHOW WHY THIS IS THE PRINCIPLE OF LEAST TIME, AND NOT THE PRINCIPLE OF LEAST DISTANCE.

DRAWING #12: INDEX OF REFRACTIONS

LIGHT TRAVELS AT THE SPEED OF LIGHT c IN A VACUUM, BUT AT A SLOWER SPEED AS IT PASSES THROUGH (TRANSPARENT) MATERIALS. (SPACE IN WHICH THERE IS NO MATTER)

NOTE: THIS IS DUE TO THE INTERACTION BETWEEN THE ELECTROMAGNETIC FIELD OF THE LIGHT AND ELECTRONS IN THE MATERIAL.

THE SPEED OF LIGHT IN A MATERIAL IS CHARACTERIZED BY THE MATERIAL'S INDEX OF REFRACTION n:

$$\boxed{n = \frac{\text{SPEED OF LIGHT IN A VACUUM}}{\text{SPEED OF LIGHT IN THE MATERIAL}}}$$

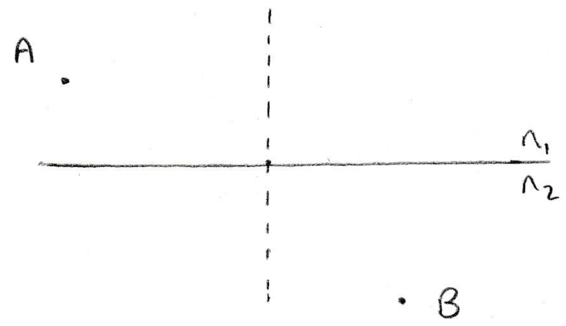
NOTE: n IS ALWAYS GREATER THAN 1.

EXAMPLES:

MATERIAL	n
VACUUM	1
AIR	1.0003
WATER	1.33

DRAWING #13: REFRACTION I

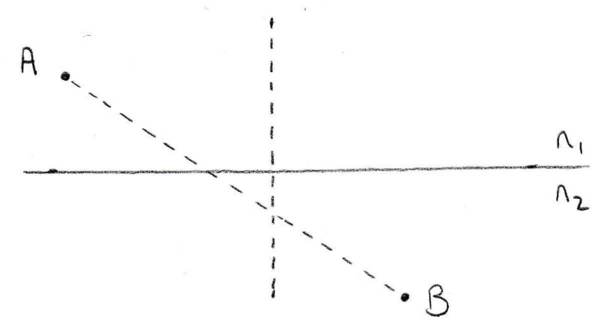
SUPPOSE THAT THERE ARE TWO POINTS A AND B ON EITHER SIDE OF A BOUNDARY BETWEEN TWO MATERIALS:



... AND WE WISH TO FIND THE PATH THAT LIGHT TAKES BETWEEN THEM.

FERMAT'S PRINCIPLE STATES THAT OUT OF ALL POSSIBLE PATHS, LIGHT WILL TAKE THAT BETWEEN POINTS A AND B THAT REQUIRES THE LEAST TIME.

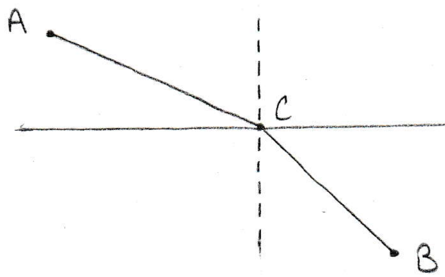
QUALITATIVELY, IT IS EASY TO SEE THAT IF $n_1 \neq n_2$, THE PATH THAT REQUIRES THE LEAST TIME WILL <u>NOT</u> BE A STRAIGHT LINE BETWEEN THEM:



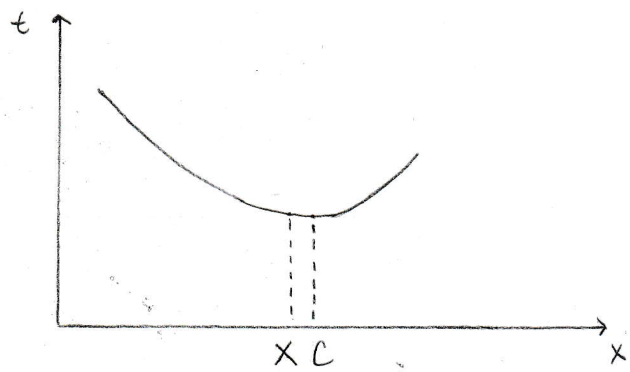
FOR EXAMPLE: IF $n_1 < n_2$, THEN A PATH WITH LESS TIME CAN BE FOUND IF LIGHT WERE TO SPEND MORE TIME IN SIDE 1, AND THUS LESS IN SIDE 2.

DRAWING #14: REFRACTION 2

WE WISH TO FIND A POINT C ON THE "X-AXIS" SUCH THAT THE PATH FROM A TO B CORRESPONDS TO THE SHORTEST OF ALL POSSIBLE TIMES:



PLOTTING TIME VS POSITION X MAY LOOK LIKE:



...WHERE C CORRESPONDS TO THE SHORTEST OF ALL POSSIBLE TIMES (THE MINIMUM OF THE GRAPH)

* NOTE: TO A FIRST APPROXIMATION, THERE IS ESSENTIALLY NO CHANGE IN TIME FOR THE POINT(S) X NEAR C (BECAUSE THE SLOPE IS ZERO AT THE BOTTOM OF THIS CURVE)

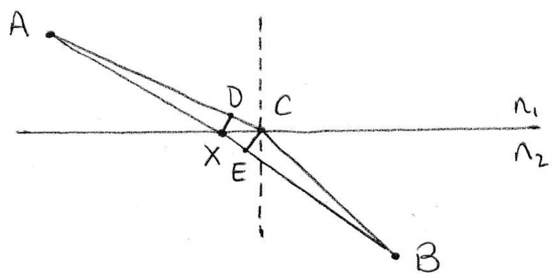
(NOTE: THERE IS A CHANGE TO SECOND ORDER)

WE WILL USE THIS APPROXIMATION TO FIND THE POINT C.

(NOTE: THE USE OF THIS APPROXIMATION IS THE ESSENCE OF THE GENERAL, MODERN FORM OF FERMAT'S PRINCIPLE)

DRAWING #15: REFRACTION 3

WITH THE FORMER APPROXIMATION IN MIND:



... AND WE WANT TO FIND THE POINT C FOR WHICH THE TIME TAKEN FROM ACB IS THE SAME AS AXB (WHERE XC IS OBVIOUSLY SMALL).

FOR THE PATH THROUGH X, RELATIVE TO C:

- THE PATH AC IS SHORTENED BY AN AMOUNT DC ...
- ... WHILE THE PATH CB IS INCREASED BY AN AMOUNT XE

IN TERMS OF TIME:

- THE AMOUNT DC SAVES:

$$\frac{DC}{c/n_1}$$

C: SPEED OF LIGHT IN A VACUUM

- ... WHILE THE PATH XE TAKES:

$$\frac{XE}{c/n_2}$$

SINCE THERE IS NO CHANGE IN TIME FOR POINTS X NEAR C, THE ABOVE TIMES MUST BE EQUAL:

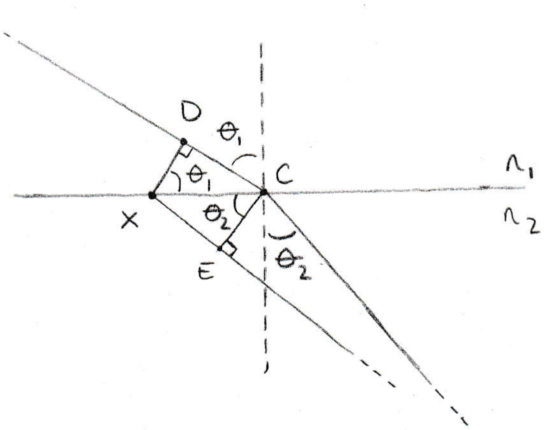
$$\frac{DC}{c/n_1} = \frac{XE}{c/n_2}$$

$$\frac{DC}{1/n_1} = \frac{XE}{1/n_2}$$

$$n_1 DC = n_2 XE$$

DRAWING #16: REFRACTION 4

ZOOMING IN ON THE XC REGION:



NOTE: ANGLES AND THEIR RELATIONSHIPS HAVE BEEN DRAWN

USING TRIGONOMETRY:

$$\sin\theta_1 = \frac{DC}{XC} \longrightarrow DC = \sin\theta_1 \, XC$$

$$\sin\theta_2 = \frac{XE}{XC} \longrightarrow XE = \sin\theta_2 \, XC$$

INSERTING THESE EXPRESSIONS INTO OUR PRIOR EQUATION:

$$n_1 DC = n_2 XE$$

$$n_1 \sin\theta_1 \, XC = n_2 \sin\theta_2 \, XC$$

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

NOTE: THIS EXPRESSION IS KNOWN AS SNELL'S LAW OF REFRACTION

... WHICH WE SEE FOLLOWS FERMAT'S PRINCIPLE OF LEAST TIME.