WAVE OPTICS

IN OUR STUDY OF ELECTRICITY AND MAGNETISM, WE DISCUSSED MAXWELL'S RECONCILIATION OF THESE TWO CONCEPTS INTO ELECTROMAGNETISM. THIS LED TO THE REALIZATION THAT LIGHT IS AN ELECTROMAGNETIC WAVE.

MUCH EARLIER THAN THIS, NEWTON CONCLUDED THAT LIGHT CONSISTS OF CORPUSCLES: VERY SMALL, LIGHT, AND FAST-MOVING PARTICLES.

THE THINKING CHANGED IN 1801 (MUCH EARLIER THAN MAXWELL), HOWEVER, WHEN THOMAS YOUNG PERFORMED AN EXPERIMENT DEMONSTRATING INTERFERENCE BETWEEN LIGHT WAVES.

NOTE: QUANTUM THEORY NOW SHOWS THAT LIGHT BEHAVES AS BOTH PARTICLES AND WAVES --- THOUGH A DISCUSSION OF THIS IS BEYOND THE SCOPE OF THIS COURSE.

DRAWING #1.1: THE WAVE THEORY OF LIGHT

WE DO NOT WISH TO FOCUS ON THE IDEA THAT LIGHT CONSISTS OF CORPUSCLES; THIS IS INCORRECT ANYWAY IN THE FRAMEWORK OF CLASSICAL MECHANICS.

LET US THEREFORE TAKE THE VIEWPOINT THAT LIGHT IS A WAVE, AND ANALYZE THE EXPERIMENTS THAT OCCURED IN THE EARLY 19TH CENTURY WITHIN THIS FRAMEWORK.

USEFULL FOR THIS WILL BE A GEOMETRIC MODEL DEVELOPED BY CHRISTIAN HUYGENS...

DRAWING #2: HUYGENS'S PRINCIPLE 1

CHRISTIAN HUYGENS WAS A CONTEMPORARY OF NEWTON WHO ARGUED THAT LIGHT DID NOT CONSIST OF PARTICLES, BUT THAT LIGHT WAS A WAVE.

THE MATHEMATICAL THEORY OF WAVES NOT BEEN DEVELOPED BY THE TIME OF HUYGENS, AND SO HE DEVELOPED A GEOMETRIC MODEL OF WAVE PROPAGATION, PUBLISHED IN HIS 1678 TREATISE ON THE WAVE THEORY OF LIGHT.

NOTE: FROM HIS GEOMETRIC MODEL, HE WAS ABLE TO DERIVE THE LAWS OF REFLECTION AND REFRACTION . . .

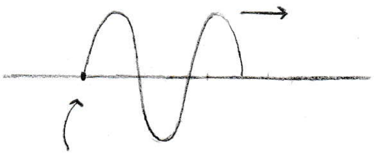
. . . AND THE MATHEMATICAL THEORY OF WAVES (DEVELOPED LATER) CAN BE USED TO JUSIFY IT (ALMOST --- THE MODEL IS DEFICIENT IN THAT IT DOESN'T ACCOUNT FOR THE DIRECTIONALITY OF THE WAVE)

NOTE: WE'RE NOT SO MUCH INTERESTED IN THE MATHEMATICS OF HUYGENS'S PRINCIPLE, BUT ITS USE AS A VISUAL (QUALITATIVE) TOOL.

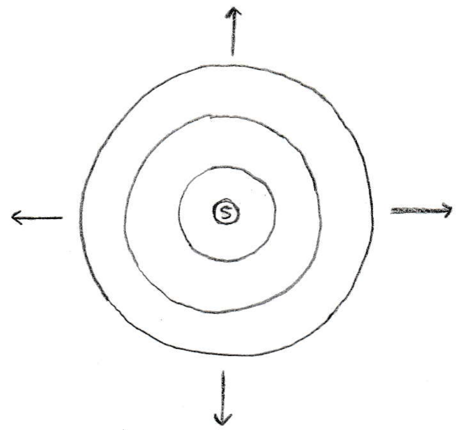
DRAWING #3: HUYGENS'S PRINCIPLE 2

SO FAR, WE HAVE ASSUMED THAT SOURCES OF WAVES ARE <u>POINT SOURCES</u> (SOURCES WITH NO MEASURABLE EXTENT).

<u>EXAMPLES:</u>



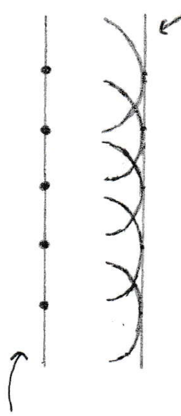
POINT SOURCE



S: POINT SOURCE

CONSIDER NOW AN <u>EXTENDED SOURCE</u>.

<u>EXAMPLE</u>: A LINE OF POINT SOURCES:



DUE TO SUPERPOSITION, THE WAVE FRONT AT A LATER TIME IS TANGENT TO ALL WAVELETS,

EACH OF THESE POINT SOURCES IS THE SOURCE OF A SPHERICAL <u>WAVELET</u> (A SMALL WAVE)

DRAWING #4: HUYGENS'S PRINCIPLE 3

HUYGENS EXTENDED THIS CONCEPT TO DESCRIBE WAVE PROPAGATION.

HUYGENS'S PRINCIPLE CAN BE STATED (IN WEAK FORM*) AS:

- EACH POINT ON A WAVE FRONT IS THE SOURCE OF A SPHERICAL WAVELET THAT SPREADS OUT AT THE WAVE SPEED.
- AT A LATER TIME, THE SHAPE OF THE WAVE FRONT IS THE LINE TANGENT TO ALL OF THE WAVELETS.

*NOTE: THERE IS A STRONGER FORM OF HUYGENS'S PRINCIPLE (FOR ODD NUMBERS OF DIMENSIONS GREATER THAN 1). FOR OUR PURPOSES (AS A VISUAL TOOL) THIS IS NOT NEEDED.

EXAMPLE: PLANE WAVE

NOTE: THIS IS THE SAME DRAWING AS BEFORE, BUT IS NOW DEPICTING SOMETHING DIFFERENT

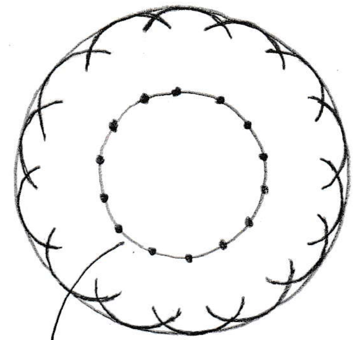
EACH POINT ON THE NEW WAVE FRONT AGAIN ACTS AS A SOURE OF SPHERICAL WAVELETS

WAVE FRONT AT A LATER TIME; THE SHAPE OF THE WAVE FRONT IS THE LINE TANGENT TO ALL OF THE WAVELETS.

INITIAL WAVE FRONT; EACH POINT ON IT ACTS AS A SOURE OF SPHERICAL WAVELETS THAT SPREAD OUT AT THE WAVE SPEED

DRAWING #5: HUYGENS' PRINCIPLE 4

EXAMPLE: SPHERICAL WAVE



THE WAVE FRONT AT A LATER TIME IS TANGENT TO ALL OF THE WAVELETS

INITIAL WAVE FRONT; EACH POINT IS THE SOURCE OF A SPHERICAL WAVELET,

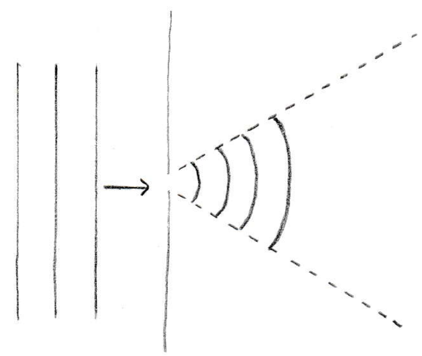
DRAWING #6: LIGHT AS A WAVE EXPERIMENTS

LET'S NOW CONSIDER A FEW EXPERIMENTS THAT SHOW THAT LIGHT IS A WAVE:

- DIFFRACTION
- DOUBLE-SLIT INTERFERENCE
- THE DIFFRACTION GRATING

DRAWING #7: DIFFRACTION BY A SINGLE NARROW SLIT I

CONSIDER A PLANE WAVE INCIDENT ON A SINGLE NARROW SLIT:



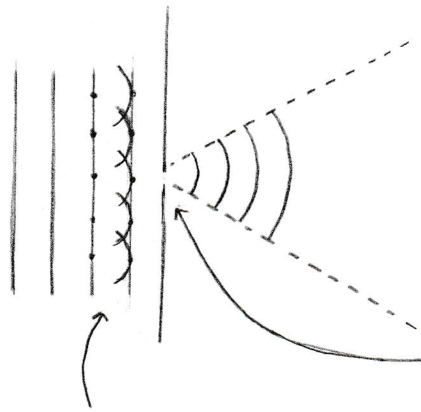
AFTER PASSING THROUGH THE SLIT, THE LIGHT TENDS TO SPREAD OUT.

THIS PHENOMENON IS A TYPE OF <u>DIFFRACTION</u> (WHICH REFERS TO VARIOUS PHENOMENA THAT OCCUR WHEN A <u>WAVE</u> ENCOUNTERS AN OBSTACLE OR A SLIT): THE BENDING OF A WAVE AROUND THE CORNERS OF AN OBSTACLE OR APERTURE INTO THE REGION OF THE GEOMETRIC SHADOW OF THE OBSTACLE.

NOTE: DIFFRACTION IS A DEFINITIVE CHARACTERISTIC OF A WAVE.

DRAWING #8: DIFFRACTION BY A SINGLE NARROW SLIT 2

WE CAN UNDERSTAND DIFFRACTION BY HUYGENS'S PRINCIPLE:



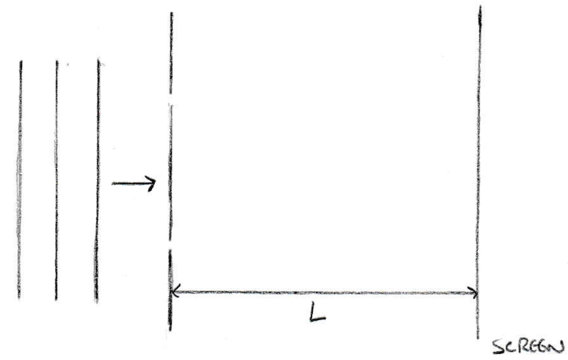
EACH POINT ON THE WAVE FRONT IS THE SOURCE OF A SPHERICAL WAVELET THAT SPREADS OUT AT THE WAVE SPEED.

WHEN THE WAVE FRONT (A LINE OF SOURCES OF SPHERICAL WAVELETS) ENCOUNTERS A SINGLE NARROW SLIT, THIS FILTERS OUT ALL OF THESE SOURCES, EXCEPT THE ONE AT THE SLIT...

... THIS SLIT THEN APPEARS TO BE A SOURCE OF A SINGLE SPHERICAL WAVE.

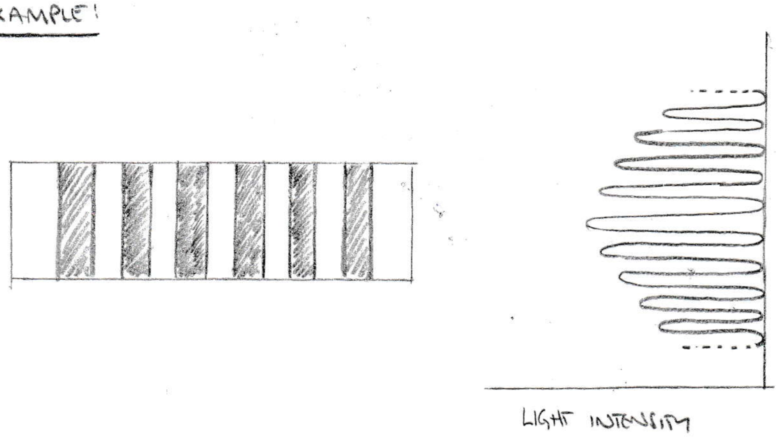
DRAWING #9: DIFFRACTION BY A DOUBLE SLIT I

NOW INSTEAD OF A SINGLE NARROW SLIT, WE CONSIDER A PLANE WAVE INCIDENT ON TWO NARROW SLITS:



THOMAS YOUNG FOUND IN 1801 THAT IF A SCREEN WAS PLACED AT A DISTANCE L AWAY, AN INTERFERENCE PATTERN WOULD BE OBSERVED.

EXAMPLE:



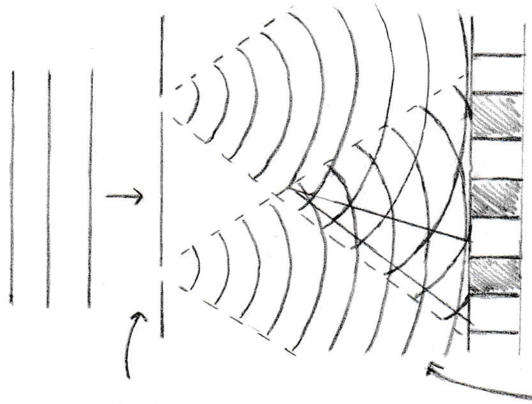
NOTE: THE ALTERNATING BRIGHT AND DARK FRINGES ARE DUE TO CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE.

NOTE: THOMAS YOUNG DID THIS EXPERIMENT USING SUNLIGHT.

AS INTERFERENCE IS A DEFINING PROPERTY OF WAVES, THIS EXPERIMENT LED THOMAS YOUNG TO CONCLUDE THAT LIGHT IS A WAVE.

DRAWING #10: DIFFRACTION BY A DOUBLE SLIT 2

LET'S TRY TO UNDERSTAND THIS BY HUYGENS'S PRINCIPLE:



← NOTE: SHOULD BE MIDWAY BETWEEN THE TWO SLITS

EACH SLIT ACTS AS A SOURCE OF A SINGLE SPHERICAL WAVE

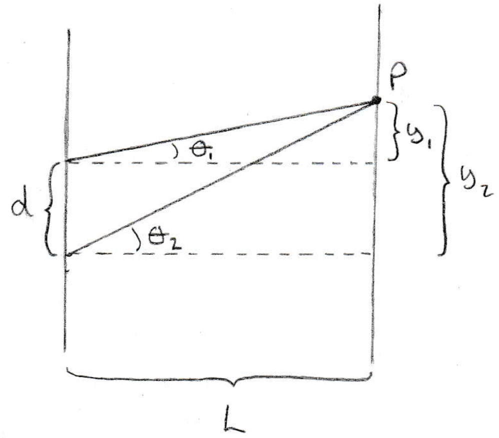
THE TWO WAVES INTERFERE IN THIS REGION, LEADING TO AN INTERFERENCE PATTERN ON THE SCREEN

NOTE: AN IMPORTANT POINT IS THAT THE INHERENT PHASE OF THE TWO WAVES IS THE SAME (BECAUSE THEY BOTH ORIGINATE FROM THE SAME PHASE OF THE INCIDENT PLANE WAVE).

DRAWING #11: DIFFRACTION BY A DOUBLE SLIT 3

LET'S NOW ANALYZE THE SITUATION MATHEMATICALLY.

CONSIDER WAVES REACHING A GENERAL POINT P FROM TWO SLITS SEPARATED BY d:



(NOTE: ONLY THE PART OF THE SPHERICAL --- OR CYLINDRICAL --- WAVES THAT REACH POINT P ARE SHOWN)

WE CAN WRITE FOR THE ANGLES θ THAT THE WAVES MUST TRAVEL FROM THE SLITS TO P:

$$\tan\theta_1 = \frac{y_1}{L}$$

$$\tan\theta_2 = \frac{y_2}{L}$$

SUBTRACTING THESE:

$$\tan\theta_2 - \tan\theta_1 = \frac{y_2}{L} - \frac{y_1}{L}$$

$$= \frac{y_2 - y_1}{L}$$

$$= \frac{d}{L}$$

IF $d \ll L$:

$$\tan\theta_2 - \tan\theta_1 \approx 0$$

$$\tan\theta_2 \approx \tan\theta_1$$

$$\theta_2 \approx \theta_1$$

--- i.e., THE ANGLES THAT THE WAVES MAKE WITH RESPECT TO EACH SLIT TO REACH POINT P ARE THE SAME (APPROXIMATELY).

DRAWING #12: DIFFRACTION BY A DOUBLE SLIT 4

TO DETERMINE THE INTERFERENCE BETWEEN THE TWO WAVES, AT POINT P, WE NEED TO FIGURE OUT THE PHASE DIFFERENCE BETWEEN THEM.

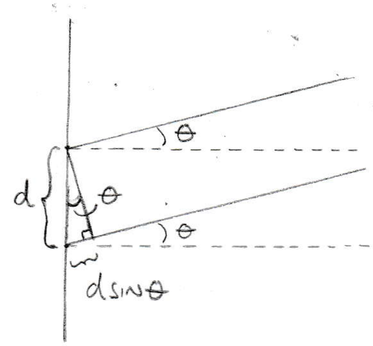
IN OUR EXAMPLE, EACH SLIT ACTS AS A SOURCE OF WAVES WHICH HAVE THE SAME WAVELENGTH, FREQUENCY, AND INHERENT PHASE...

(NOTE: THIS HAS TO DO WITH THE INITIAL SOURCE USED TO GENERATE FIELDS AT THE SLITS.)

... THEREFORE, THE PHASE DIFFERENCE ARISES FROM A PATH-LENGTH DIFFERENCE (i.e., A DIFFERENCE IN DISTANCE EACH WAVE MUST TRAVEL FROM THE SLIT TO P).

THE DIFFERENCE IN PATH LENGTH IS EASY TO FIND BY CONSIDERING THAT EACH WAVE MAKES THE SAME ANGLE (APPROXIMATELY, IF $d \ll L$) WITH RESPECT TO THE HORIZONTAL AXIS OF THE SLIT.

AT THE REGION NEAR THE SLITS:



... THUS THE PATH-LENGTH DIFFERENCE IS $d\sin\theta$

DRAWING #13: DIFFRACTION BY A DOUBLE SLIT 5

RECALL THAT CONSTRUCTIVE INTERFERENCE OCCUR FOR SPHERICAL AND CYLINDRICAL SOURCES WHEN:

$$\Delta\phi = k\Delta r + \Delta\phi' = m2\pi \qquad m = 0, 1, 2, \ldots \qquad \text{(CONSTRUCTIVE)}$$

$$\Delta\phi = k\Delta r + \Delta\phi' = \left(m + \tfrac{1}{2}\right)2\pi \qquad m = 0, 1, 2, \ldots \qquad \text{(DESTRUCTIVE)}$$

NOTE: m IS USED INSTEAD OF n, FROM PRIOR EQUATIONS.

SINCE $\Delta\phi' = 0$:

$$\Delta\phi = k\Delta r = m2\pi \qquad m = 0, 1, 2, \ldots \qquad \text{(CONSTRUCTIVE)}$$

$$\Delta\phi = k\Delta r = \left(m + \tfrac{1}{2}\right)2\pi \qquad m = 0, 1, 2, \ldots \qquad \text{(DESTRUCTIVE)}$$

INSERTING $k = 2\pi/\lambda$:

$$\frac{2\pi}{\lambda}\Delta r = m2\pi$$

$$\Delta r = m\lambda \qquad m = 0, 1, 2, \ldots \qquad \text{(CONSTRUCTIVE)}$$

$$\frac{2\pi}{\lambda}\Delta r = \left(m + \tfrac{1}{2}\right)2\pi$$

$$\Delta r = \left(m + \tfrac{1}{2}\right)\lambda \qquad m = 0, 1, 2, \ldots \qquad \text{(DESTRUCTIVE)}$$

INSERTING $\Delta r = d\sin\theta$:

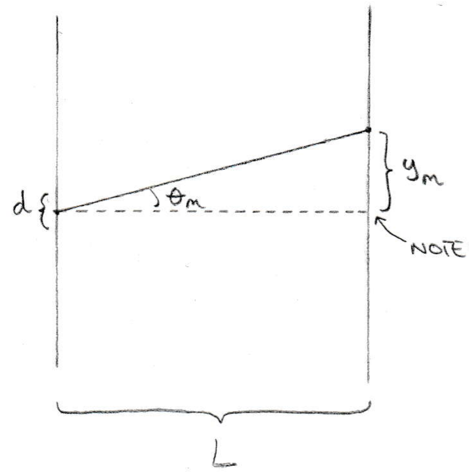
$$d\sin\theta = m\lambda \qquad m = 0, 1, 2, \ldots \qquad \text{(CONSTRUCTIVE)}$$

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \qquad m = 0, 1, 2, \ldots \qquad \text{(DESTRUCTIVE)}$$

THE PRIOR TWO EQUATIONS SHOW THAT THE m^{th} BRIGHT (DARK) FRINGE OCCURS WHEN THE WAVE FROM ONE SLIT TRAVELS m WAVELENGTHS (HALF WAVELENGTHS) FARTHER THAN THE WAVE FROM THE OTHER SLIT.

DRAWING #14: DIFFRACTION BY A DOUBLE SLIT 5

ON A ZOOMED-OUT SCALE:



NOTE: ON OUR ZOOMED-OUT SCALE, THE SLITS ARE INDISTINGUISHABLE

NOTE: THE DISTANCE y_m IS BEING MEASURED FROM THE CENTER OF THE SLITS

WE CAN FIND THE ANGLES (θ_m) AND DISTANCES (y_m) OF THE m^{th} BRIGHT (DARK) FRINGE USING THE PRIOR EQUATIONS:

FOR θ_m:

$$\sin\theta_m = m\frac{\lambda}{d} \qquad m = 0, 1, 2, \ldots \qquad \text{(BRIGHT FRINGES)}$$

$$\sin\theta_m = \left(m+\frac{1}{2}\right)\frac{\lambda}{d} \qquad m = 0, 1, 2, \ldots \qquad \text{(DARK FRINGES)}$$

OUR ASSUMPTION (FROM EARLIER) THAT $d \ll L$ MEANS THAT θ_m (UNDER THIS APPROXIMATION, THE SAME FOR BOTH SLITS) IS SMALL (e.g., $<1°$); WE CAN THEREFORE USE THE <u>SMALL ANGLE APPROXIMATION</u>:

(NOTE: THE VALIDITY OF THIS ALSO DEPENDS ON λ BEING SMALL RELATIVE TO L)

$$\sin x \approx x \qquad \text{FOR SMALL } x \qquad (x \text{ IN RADIANS})$$

... TO WRITE:

$$\boxed{\begin{array}{lll} \theta_m = m\dfrac{\lambda}{d} & m = 0, 1, 2, \ldots & \text{(BRIGHT FRINGES)} \\ \theta_m = \left(m+\dfrac{1}{2}\right)\dfrac{\lambda}{d} & m = 0, 1, 2, \ldots & \text{(DARK FRINGES)} \end{array}}$$

NOTE: θ_m IS SYMMETRIC ABOUT THE HORIZONTAL CENTER LINE

WE CAN ALSO SOLVE FOR THE POSITIONS OF THE FRINGES:

$$y_m = L \tan\theta_m$$

IF THE ANGLE θ_m IS SMALL, WE CAN AGAIN USE THE SMALL-ANGLE APPROXIMATION:

$$\tan x \approx x \qquad \text{IF } x \text{ IS SMALL} \quad (x \text{ IN RADIANS})$$

AND WRITE:

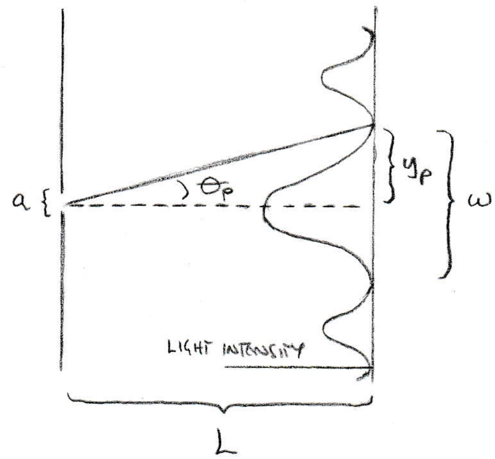
$$y_m \approx L\theta_m$$

INSERTING θ_m FROM BEFORE:

$$\boxed{\begin{array}{lll} y_m = mL\dfrac{\lambda}{d} & m = 0, 1, 2, \ldots & \text{(BRIGHT FRINGES)} \\[2ex] y_m = \left(m + \dfrac{1}{2}\right)L\dfrac{\lambda}{d} & m = 0, 1, 2, \ldots & \text{(DARK FRINGES)} \end{array}}$$

DRAWING #16: SINGLE-SLIT DIFFRACTION I

WHEN LIGHT PASSES THROUGH A SINGLE SLIT WITH A FINITE-WIDTH a, IT WILL CREATE A INTERFERENCE PATTERN ON A SCREEN PLACED BEHIND IT:



THIS PROBLEM IS ANALOGOUS TO DOUBLE-SLIT INTERFERENCE, AND CAN BE ANALYZED BY USING HUYGENS'S PRINCIPLE TO CONSIDER A LINE OF POINT SOURCES AT THE SLIT, AND DETERMINING THEIR CONSTRUCTIVE/DESTRUCTIVE INTERFERENCE ON THE SCREEN PLACED L AWAY.

DRAWING #17: SINGLE-SLIT DIFFRACTION 2

THE ANGLE θ_p OF COMPLETE DESTRUCTIVE INTERFERENCE IS GIVEN BY:

$$a \sin\theta_p = p\lambda \quad p = 1, 2, 3, \ldots$$

... WHICH CAN BE SIMPLIFIED USING THE SMALL-ANGLE APPROXIMATION, IF θ_p IS SMALL:

$$\boxed{\theta_p \approx p\frac{\lambda}{a} \quad p = 1, 2, 3, \ldots}$$

THE POSITION OF THE DARK FRINGES CAN BE FOUND FROM:

$$y_p = L \tan\theta_p$$

... WHICH CAN ALSO BE SIMPLIFIED USING THE SMALL-ANGLE APPROXIMATION, IF θ_p IS SMALL:

$$y_p = L \tan\theta_p$$

$$\approx L\theta_p$$

$$\boxed{y_p \approx p\frac{\lambda L}{a} \quad p = 1, 2, 3, \ldots}$$