RL CIRCUITS

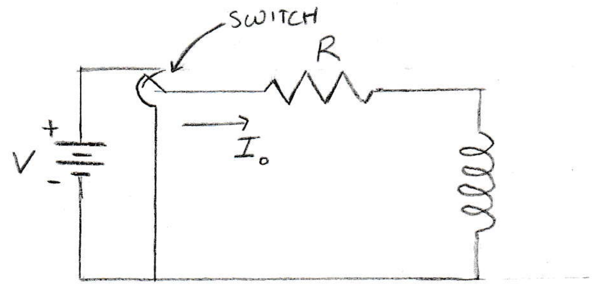
EARLIER WE CONSIDERED A BASIC, YET IMPORTANT, ELECTRICAL CIRCUIT THAT EXHIBITED A TIME-DEPENDENT CURRENT, THE RC CIRCUIT.

GIVEN THE ANALOGIES BETWEEN INDUCTORS AND CAPACITORS, ONE MIGHT SUSPECT THAT WE COULD FORM AN ANALOGOUS CIRCUIT FROM A RESISTOR AND INDUCTOR.

A CIRCUIT THAT CONSISTS OF A RESISTOR, INDUCTOR, AND PERHAPS A BATTERY IS CALLED AN <u>RL CIRCUIT</u>.

DRAWING #2: RL CIRCUITS 2

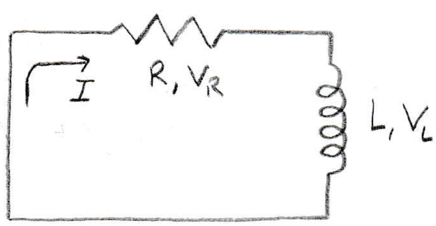
CONSIDER THE FOLLOWING RL CIRCUIT:



UP TO TIME $t = t_0$:

- ASSUME THAT THE SWITCH HAS BEEN IN THE UP POSITION FOR A LONG TIME
- ... THEN I_0 IS STEADY (i.e., $dI/dt = 0$) ...
- ... WHICH MEANS THAT THERE IS NO POTENTIAL DIFFERENCE ACROSS THE INDUCTOR...
- ... AND THEREFORE THE CURRENT IN THE CIRCUIT IS DETERMINED ENTIRELY BY THE BATTERY AND RESISTOR ($I_0 = V/R$).

SUPPOSE THAT AT $t = t_0$ THE SWITCH IS MOVED TO THE DOWN POSITION. THE CIRCUIT THEN LOOKS LIKE:



NOTE: THE CURRENT DOES NOT STOP IMMEDIATELY.

ACCORDING TO THE KIRCHHOFF LOOP LAW:

$$V_R + V_L = 0$$

$$-IR - L\frac{dI}{dt} = 0$$

$\left(V_R = -IR \; , \; V_L = -L\frac{dI}{dt}\right)$

(NOTE: THE MINUS SIGN IN V_R ARISES BECAUSE THERE IS A VOLTAGE DROP ACROSS THE RESISTOR, IN THE DIRECTION OF THE CURRENT)

DRAWING #3: RL CIRCUITS 3

REARRANGING THE PRIOR EXPRESSION:

$$-IR - L\frac{dI}{dt} = 0$$

$$-L\frac{dI}{dt} = IR$$

$$\frac{dI}{dt} = -I\frac{R}{L}$$

$$\frac{1}{I}dI = -\frac{R}{L}dt$$

$$\frac{1}{I}dI = -\frac{1}{(L/R)}dt$$

THE CURRENT AS A FUNCTION OF TIME CAN BE FOUND BY INTEGRATING THE PRIOR EXPRESSION:

$$\int_{I_0}^{I} dI\,\frac{1}{I} = \int_{t_0}^{t} dt\left(-\frac{1}{(L/R)}\right)$$ (NOTE THE LIMITS OF INTEGRATION)

$$\int_{I_0}^{I} dI\,\frac{1}{I} = -\frac{1}{(L/R)}\int_{t_0}^{t} dt$$

$$\ln I\Big|_{I_0}^{I} = -\frac{1}{(L/R)}\,t\Big|_{t_0}^{t}$$

$$\ln I - \ln I_0 = -\frac{1}{(L/R)}(t-t_0)$$

$$\ln\left(\frac{I}{I_0}\right) = -\frac{1}{(L/R)}(t-t_0)$$

$$\frac{I}{I_0} = e^{-(t-t_0)/(L/R)}$$

$$I = I_0\,e^{-(t-t_0)/(L/R)}$$

SINCE THE ARGUMENT OF THE EXPONENT MUST BE DIMENSIONLESS, (L/R) MUST HAVE DIMENSIONS OF TIME. WE CAN DEFINE THIS AS THE <u>TIME CONSTANT τ</u> OF THE RL CIRCUIT:

$$\boxed{\tau = (L/R)}$$

IN TERMS OF τ, WE CAN WRITE THE CURRENT IN THE RL CIRCUIT AS:

$$\boxed{I = I_0 e^{-(t-t_0)/\tau}}$$

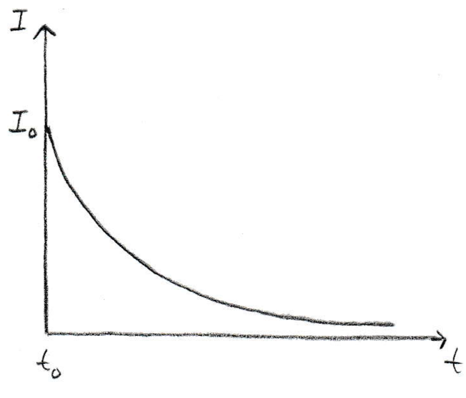
EXPRESSIONS FOR V_R AND V_L CAN BE DERIVED ANALOGOUSLY TO THE WAY V_R WAS DERIVED FOR AN RC CIRCUIT.

→ DRAWING #4,1

IT CAN BE SEEN THAT THE CURRENT AS A FUNCTION OF TIME BEHAVES THE <u>SAME</u> AS IN AN RC CIRCUIT (THE EQUATIONS ARE THE <u>SAME</u>).

NOTE: WE MIGHT HAVE EXPECTED THIS, GIVEN THE SIMILARITIES BETWEEN CAPACITORS AND INDUCTORS.

GRAPHICALLY:



NOTE: IN PRACTICE, CAPACITORS (RC CIRCUITS) ARE USUALLY PREFERRED TO INDUCTORS (RL CIRCUITS), SINCE THEY CAN BE MORE EASILY MANUFACTURED AND ARE GENERALLY PHYSICALLY SMALLER.

DRAWING #4.1: THE TIME-CONSTANT OF AN RC(RL) CIRCUIT

THE TIME CONSTANT OF AN RC(RL) CIRCUIT IS A USEFUL WAY TO CHARACTERIZE THE TIME-DEPENDENCE OF THE CURRENT (OR OTHER QUANTITY):

- AFTER τ, I WILL HAVE DECREASED TO $e^{-1}I_0$
- AFTER 2τ, " " $e^{-2}I_0$
- ... AND SO ON