DRAWING #1: RC CIRCUITS

ELECTRICAL COMPONENTS CAN BE COMBINED IN ELECTRICAL NETWORKS, FOR APPLICATIONS NOT POSSIBLE WITH THE INDIVIDUAL COMPONENTS ALONE.

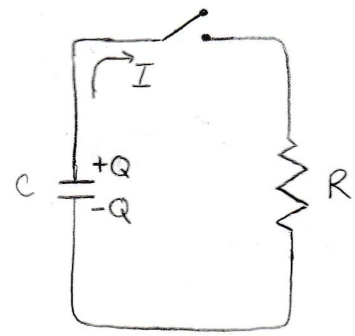
FOR EXAMPLE, THERE ARE MANY APPLICATIONS IN WHICH THE TIME DEPENDANCE OF THE CURRENT IS AN IMPORTANT ASPECT --- SO FAR THOUGH, WE HAVE NOT SEEN ANY CIRCUITS CAPABLE OF PROVIDING THIS.

ONE TYPE OF CIRCUIT THAT DISPLAYS A TIME-DEPENDENT CURRENT IS AN <u>RC CIRCUIT</u>: AN ELECTRICAL CIRCUIT (OR NETWORK) COMPRISED OF RESISTORS AND CAPACITORS.

THE CHARGING AND DISCHARGING OF A CAPACITOR, FOR EXAMPLE, EXHIBITS A TIME DEPEND CURRENT; SUCH CIRCUITS ARE AN ESSENTIAL PART OF TIME-KEEPING CIRCUITS IN DIGITAL ELECTRONICS.

DRAWING #2: DISCHARGING A CAPACITOR 1

CONSIDER A CHARGED CAPACITOR CONNECTED BY IDEAL WIRES TO A RESISTOR, ALONG WITH A SWITCH TO CONTROL WHETHER CURRENT FLOWS:



NOTE: I IS THE DIRECTION CURRENT WILL FLOW WHEN THE SWITCH IS CLOSED, AND IF THE CAPACITOR IS CHARGED

AT TIME $t = t_0$:

- THE CAPACITOR HAS A TOTAL AMOUNT OF CHARGE OF Q_0
- ... AND SO THERE IS A POTENTIAL DIFFERENCE ACROSS THE CAPACITOR OF $V_{C,0} = Q_0/C$
- THERE IS NO CURRENT IN THE CIRCUIT, $I = 0$
- THE POTENTIAL DIFFERENCE ACROSS THE RESISTOR, V_R, IS ZERO $(V_{R,0} = IR)$

NOW SUPPOSE THAT THE SWITCH IS CLOSED (AT $t = t_0$). CURRENT WILL BEGIN TO FLOW THROUGH THE CIRCUIT, AS THE CAPACITOR IS DISCHARGED.

FROM <u>KIRCHHOFF'S LOOP LAW</u>, WE CAN WRITE AT ANY TIME t:

$$V_{\text{CLOSED}} = \sum_{\substack{i \\ \text{CLOSED}}} V_i = 0$$

$$V_C + V_R = 0$$

$$\frac{Q}{C} - IR = 0$$

NOTE: Q AND I ARE THE INSTANTANEOUS VALUES AT TIME t

*NOTE: THE MINUS SIGN COMES IN BECAUSE THERE IS A POTENTIAL INCREASE OVER THE CAPACITOR, AND DECREASE OVER THE RESISTOR.

DRAWING #3: DISCHARGING A CAPACITOR 2

THE CURRENT IS LIMITED BY THE RESISTOR (JUST AS IN THE RESISTOR CIRCUITS THAT WE CONSIDERED BEFORE)...

... THE CURRENT I IS THEREFORE THE RATE THAT CHARGE FLOWS THROUGH THE RESISTOR:

$$I = \frac{dq}{dt}$$

dq: INFITESIMAL AMOUNT OF CHARGE FLOWING THROUGH THE RESISTOR

... WHICH BY THE CONSERVATION OF CHARGE MUST BE EQUAL TO THE RATE THAT IT WAS REMOVED FROM THE CAPACITOR:

$$I = -\frac{dQ}{dt}$$

INSERTING THIS EXPRESSION INTO THE PRIOR EQUATION, AND REARRANGING:

$$\frac{Q}{C} - IR = 0$$

$$\frac{Q}{C} + \frac{dQ}{dt}R = 0 \qquad \text{(INSERT } I\text{)}$$

$$\frac{Q}{RC} + \frac{dQ}{dt} = 0 \qquad \text{(DIVIDE BY } R\text{)}$$

$$\frac{dQ}{dt} = -\frac{Q}{RC} \qquad \text{(MOVE TO OTHER SIDE)}$$

$$\frac{1}{Q}\frac{dQ}{dt} = -\frac{1}{RC} \qquad \text{(DIVIDE BY } Q\text{)}$$

$$\frac{dQ}{Q} = -\frac{1}{RC}\,dt \qquad \text{(MULTIPLY BY } dt\text{)}$$

DRAWING #4: DISCHARGING A CAPACITOR 3

KNOWING THAT THE CHARGE ON THE CAPACITOR WAS Q_0 AT t_0, WE CAN FIND THE CHARGE AT A LATER TIME t BY INTEGRATING THE PRIOR EXPRESSION:

$$\int_{Q_0}^{Q} dQ \frac{1}{Q} = \int_{t_0}^{t} dt \left(-\frac{1}{RC}\right)$$

$$= -\frac{1}{RC}\int_{t_0}^{t} dt$$

$$\ln Q \Big|_{Q_0}^{Q} = -\frac{1}{RC} t \Big|_{t_0}^{t} \qquad \left(\int dx \frac{1}{x} = \ln|x|\right)$$

$$\ln Q - \ln Q_0 = -\frac{1}{RC}(t-t_0)$$

$$\ln\left(\frac{Q}{Q_0}\right) = -\frac{(t-t_0)}{RC} \qquad \left(e^{\ln x} = x\right)$$

$$\frac{Q}{Q_0} = e^{-(t-t_0)/RC}$$

$$Q = Q_0 e^{-(t-t_0)/RC}$$

NOTING THAT THE QUANITY IN AN EXPONENTIAL MUST BE DIMENSIONLESS, THE QUANTITY RC MUST HAVE UNITS OF TIME. IT IS USEFUL TO DEFINE THIS AS THE <u>TIME CONSTANT</u> τ OF THE RC CIRCUIT:

$$\boxed{\tau = RC}$$

WITH THIS DEFINITION, WE CAN WRITE:

$$\boxed{Q = Q_0 e^{-(t-t_0)/\tau}}$$

DRAWING #5: DISCHARGING A CAPACITOR 4

RECALLING THAT:

$$V_c = \frac{Q}{C}$$

WE CAN WRITE:

$$V_c = \frac{1}{C} Q_o e^{-(t-t_o)/\tau}$$

$$\boxed{V_c = V_o e^{-(t-t_o)/\tau}}$$

$\left(V_o = Q_o/C\right)$ (NOTE: REMEMBER TOO: $V_R = -V_c$)

SIMILARILY, FOR THE CURRENT THROUGH THE RESISTOR:

$$I = -\frac{dQ}{dt}$$

$$= -\frac{d}{dt}\left(Q_o e^{-(t-t_o)/\tau}\right)$$

$$= Q_o \frac{1}{\tau} e^{-(t-t_o)/\tau}$$

$$= Q_o \frac{1}{RC} e^{-(t-t_o)/\tau}$$

$$= \frac{1}{R} \frac{Q_o}{C} e^{-(t-t_o)/\tau}$$

$$= \frac{1}{R} V_o e^{-(t-t_o)/\tau}$$

$$\boxed{I = I_o e^{-(t-t_o)/\tau}}$$

(NOTE: I_o IS THE INSTANTANEOUS CURRENT AFTER THE SWITCH CLOSES)

DRAWING #6: DISCHARGING A CAPACITOR 5

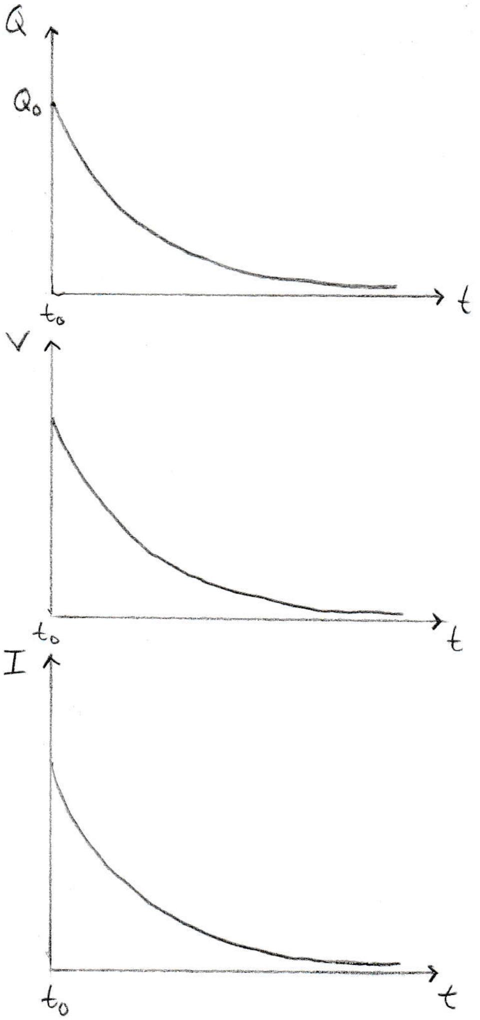
THEREFORE, THE CHARGE ON THE CAPACITOR, THE VOLTAGE ACROSS IT (OR THE RESISTOR), AND THE CURRENT IN THE CIRCUIT ARE ALL EXPONENTIALLY-DECREASING FUNCTIONS OF TIME:

$$Q = Q_0 e^{-(t-t_0)/\tau}$$

$$\begin{aligned} V_C &= -V_R \\ &= V_0 e^{-(t-t_0)/\tau} \end{aligned} \qquad \text{(REPEATED)}$$

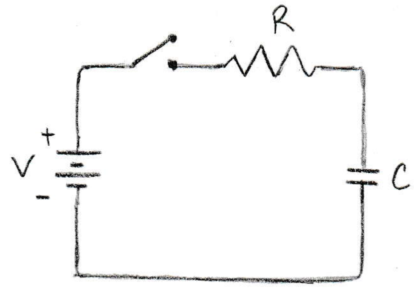
$$I = I_0 e^{-(t-t_0)/\tau}$$

GRAPHICALLY:



DRAWING #7: CHARGING A CAPACITOR

WE CAN USE A SIMILAR TYPE OF ELECTRICAL CIRCUIT TO CHARGE A CAPACITOR:



USING A SIMILAR ANALYSIS AS BEFORE, IT CAN BE SHOWN THAT THE CHARGE AT TIME t IS:

$$\boxed{Q = Q_{MAX}\left(1 - e^{-(t-t_0)/\tau}\right)} \qquad Q_{MAX} = CV$$

GRAPHICALLY: