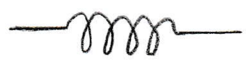
DRAWING #1: INDUCTORS

INDUCTORS

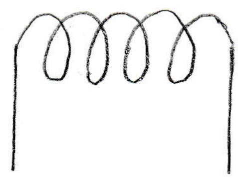
JUST AS CAPACITORS ARE USEFUL CIRCUIT ELEMENTS BECAUSE THEY CAN STORE (POTENTIAL) ENERGY IN AN ELECTRIC FIELD, INDUCTORS ARE USEFUL CIRCUIT ELEMENTS FOR STORING ENERGY IN A MAGNETIC FIELD.

SYMBOL IN CIRCUIT DIAGRAMS:

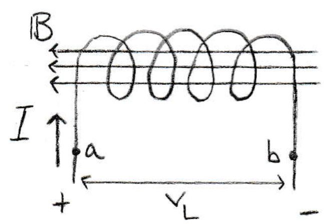


DRAWING #2: A SOLENOID

CONSIDER A WIRE THAT HAS BEEN WOUND MANY TURNS IN THE FORM OF A ~~WIRE~~ COIL, WITH THE ENDS EXTENDING OUT SOME DISTANCE FROM THE COIL:



IF THE ENDS OF THE WIRE ARE THEN CONNECTED TO A VOLTAGE SOURCE, CURRENT WILL FLOW THROUGH THE WIRE, ESTABLISHING A MAGNETIC FIELD:



(E.g., V_L MIGHT BE PROVIDED BY A BATTERY)

A CYLINDRICAL COIL OF WIRE ACTING AS A MAGNET WHEN CARRYING ELECTRIC CURRENT IS CALLED A <u>SOLENOID</u>.

DRAWING #3: A SOLENOID; FARADAY'S LAW I

LET'S CONSIDER THE APPLICATION OF FARADAY'S LAW TO THE SOLENOID:

$$\oint_C \mathbb{E}\cdot ds = -\int_S da\,\frac{d\mathbb{B}}{dt}\cdot\hat{m}$$

$$= -\frac{d\bar{\Phi}_m}{dt} \qquad \left(\bar{\Phi}_m = \int da\,\mathbb{B}\cdot\hat{m}\right)$$

(RECALL THAT THIS SAYS THAT THE INTEGRAL TANGENTIAL COMPONENT OF THE ELECTRIC FIELD AROUND A CLOSED PATH IS EQUAL TO THE (NEGATIVE) RATE CHANGE OF MAGNETIC FLUX THROUGH THAT PATH)

CONSIDERING FIRST THE LEFT-HAND SIDE:

$$\oint_C \mathbb{E}\cdot ds$$

THE CLOSED PATH THAT WE WILL TAKE WILL BE ALONG THE WIRE; IN THIS CASE, THIS INTEGRAL CAN BE BROKEN UP INTO THE SUM OF TWO PARTS:

$$\oint_C \mathbb{E}\cdot ds = \underbrace{\int_a^b \mathbb{E}\cdot ds}_{\text{VIA COIL}} + \underbrace{\int_b^a \mathbb{E}\cdot ds}_{\text{OUTSIDE}}$$

EACH OF THESE INTEGRAL EXPRESSIONS CAN BE WRITTEN IN TERMS OF THE DIFFERENCE IN ELECTRIC POTENTIAL. SINCE THERE IS NO POTENTIAL DIFFERENCE BETWEEN TWO POINTS IN AN IDEAL WIRE:

$$\underbrace{\int_a^b \mathbb{E}\cdot ds}_{\text{VIA COIL}} = 0$$

THE OTHER INTEGRAL EXPRESSION CAN BE WRITTEN:

$$\int_b^a \mathbb{E}\cdot ds = -\left(\phi(a) - \phi(b)\right)$$

$$= \phi(b) - \phi(a)$$

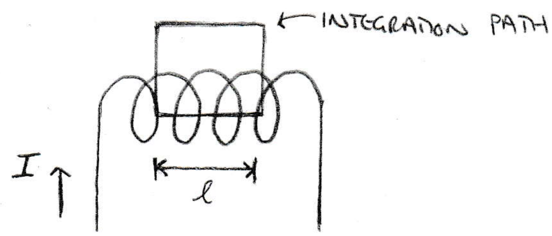
$$= V_L$$

DRAWING #4: A SOLENOID; FARADAY'S LAW 2; AMPÈRE'S LAW

FOR THE RIGHT-HAND SIDE OF FARADAY'S LAW, THE MAJOR COMPONENT OF THE MAGNETIC FIELD WILL BE THAT PASSING THROUGH THE TURNS OF WIRE (WE'LL IGNORE ANY EFFECTS OUTSIDE OF THE COIL).

DUE TO THE SYMMETRY OF THE COIL, FINDING THE MAGNETIC FIELD IS WELL SUITED FOR AMPÈRE'S LAW:

$$\oint \mathbb{B} \cdot d\mathbf{s} = \mu_0 I_{ENC}$$



A PARTICULARLY CONVENIENT INTEGRATION PATH IS A BOX OF SIDE LENGTH ℓ, ORIENTED AS SHOWN ABOVE.

IN THIS CASE, THE LEFT-HAND SIDE OF AMPÈRE'S LAW BECOMES:

$$\oint \mathbb{B} \cdot d\mathbf{s} = B\ell \qquad (B\text{: MAGNITUDE OF } \mathbb{B})$$

(SINCE THE MAGNETIC FIELD IS ONLY NON-ZERO ALONG THE BOTTOM)

AND THE RIGHT-HAND SIDE IS:

$$\mu_0 I_{ENC} = \mu_0 NI \qquad (N\text{: THE NUMBER OF TURNS IN THE LENGTH } \ell)$$

HENCE:

$$B\ell = \mu_0 NI$$

$$B = \mu_0 \frac{N}{\ell} I$$

DRAWING #5: A SOLENOID; FARADAY'S LAW 3; INDUCTANCE

SINCE THE MAGNETIC FIELD IS EVERYWHERE NORMAL TO THE COIL OF WIRE, THE MAGNETIC FLUX (THROUGH ONE TURN OF WIRE) CAN BE WRITTEN:

$$\Phi_{m,\text{TURN}} = \int da\, \mathbf{B}\cdot\hat{m}$$

$$= \mu_0 \frac{N}{\ell} I A \qquad (A: \text{AREA OF ONE TURN OF WIRE})$$

THROUGH THE ENTIRE SOLENOID:

$$\Phi_m = N \Phi_{m,\text{TURN}}$$

$$= N \mu_0 \frac{N}{\ell} I A$$

$$= \mu_0 \frac{N^2}{\ell} I A$$

$$= \left(\mu_0 \frac{N^2}{\ell} A\right) I$$

IT IS USEFUL TO DEFINE THE QUANTITY IN PARANTHESIS AS THE (SELF-)INDUCTANCE L OF THE SOLENOID:

$$L_{\text{SOLENOID}} = \mu_0 \frac{N^2}{\ell} A$$

(NOTE: WE'LL SEE SHORTLY WHY IT IS CALLED INDUCTANCE)

WHICH WE CAN INTERPRET/DEFINE AS THE ABILITY TO CREATE A MAGNETIC FLUX WHEN THERE IS A CURRENT:

$$\boxed{L \equiv \frac{\Phi_m}{I}}$$

NOTE: THIS DEFINITION HOLDS, IN GENERAL

UNITS: HENRY (H) $\quad 1\,H \equiv 1\,\frac{Wb}{A} = 1\,\frac{Tm^2}{A}$

NOTE: THIS DEFINITION IS DIRECTLY ANALOGOUS TO CAPACITANCE $(C = Q/V)$...

... IN FACT, JUST AS CAPACITANCE ONLY DEPENDED ON THE GEOMETRY OF THE CAPACITOR, INDUCTANCE ONLY DEPENDS ON THE GEOMETRY OF THE INDUCTOR (A COMPONENT USED IN A CIRCUIT FOR PROVIDING INDUCTANCE)

DRAWING #6: FARADAY'S LAW 4

THEREFORE, THE RIGHT-HAND SIDE OF FARADAY'S LAW BECOMES:

$$-\frac{d\Phi_M}{dt} = -\frac{d}{dt}(LI)$$
$$= -L\frac{dI}{dt}$$

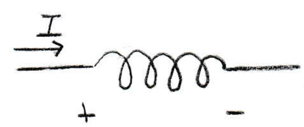
FINALLY, PUTTING FARADAY'S LAW TOGETHER GIVES:

$$\int_C \mathbb{E}\cdot d\mathbf{s} = -\frac{d\Phi_M}{dt}$$

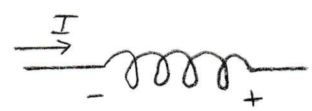
$$\boxed{V_L = -L\frac{dI}{dt}}$$

NOTE: V_L IS THE POTENTIAL DIFFERENCE ACROSS THE INDUCTOR AS MEASURED ALONG THE DIRECTION OF CURRENT; e.g.:

IF $\frac{dI}{dt} > 0$: THE INPUT SIDE OF AN INDUCTOR IS AT A HIGHER POTENTIAL



IF $\frac{dI}{dt} < 0$: THE OPPOSITE



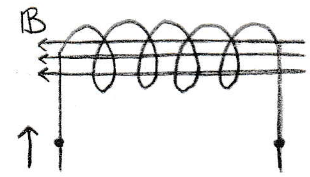
DRAWING #7: VOLTAGE ACROSS AN INDUCTOR

WE'VE ARRIVED MATHEMATICALLY AT A RESULT THAT SHOWS A POTENTIAL DIFFERENCE (CAN) EXISTS ACROSS AN INDUCTOR...

... THIS SEEMS TO BE A PARADOX THOUGH: HOW CAN A POTENTIAL DIFFERENCE EXIST AT TWO POINTS ALONG AN IDEAL WIRE, AND NOTHING IN BETWEEN (IT'S SIMPLY SHAPED INTO A COIL)?

TO ANSWER THIS QUESTION, WE MUST CONSIDER WHAT THE EQUATIONS THAT WE HAVE USED AND DERIVED TELL US.

CONSIDER A STEADY CURRENT:



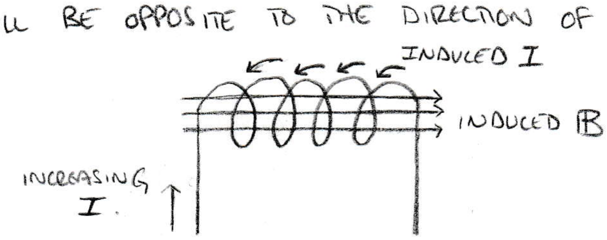
IN THIS CASE, $dI/dt = 0$, SO:

$$V_L = -L\frac{dI}{dt} = 0$$

AS WE EXPECT.

NOW CONSIDER THAT THE CURRENT IS INCREASING:

- THIS WILL CREATE AN INCREASING FLUX TO THE LEFT (AS THE MAGNETIC FIELD GROWS IN STRENGTH)
- THIS INCREASE IN FLUX WILL INDUCE A CURRENT IN THE COIL...
- ... WHICH (BY LENZ'S LAW) WILL BE IN A DIRECTION SUCH THAT IT INDUCES A MAGNETIC FIELD THAT OPPOSES THE CHANGE IN FLUX...
- ... WHICH WILL BE OPPOSITE TO THE DIRECTION OF CURRENT IN THE SOLENOID:



THE INDUCED CURRENT WILL LEAD TO A BUILD-UP OF POSITIVE CHARGE ON THE LEFT (UNTIL A STATIC SITUATION IS ACHIEVED), ESTABLISHING A POTENTIAL DIFFERENCE ACROSS THE SOLENOID.

NOTE: HENCE THE TERMINOLOGY INDUCTOR / INDUCTANCE.

DRAWING #8: ENERGY STORED IN AN INDUCTOR I

RECALL THAT INDUCTORS ARE USEFUL CIRCUIT ELEMENTS BECAUSE THEY CAN STORE ENERGY IN THEIR MAGNETIC FIELD.

CONSIDER THE ELECTRIC POWER PRODUCED BY THE CURRENT AS IT PASSES THROUGH AN INDUCTOR.

NOTE: ELECTRIC POWER IS THE RATE AT WHICH ELECTRICAL ENERGY IS TRANSFERRED BY AN ELECTRICAL CIRCUIT.

IT IS DEFINED AS:

$$P = \text{WORK DONE PER UNIT TIME} = V\frac{Q}{t} = VI$$

(THIS SAYS THAT THE ELECTRIC POWER IS PRODUCED BY AN ELECTRIC CURRENT CONSISTING OF A CHARGE OF Q COULOMBS EVERY t SECONDS PASSING THROUGH AN ELECTRIC POTENTIAL DIFFERENCE V)

IN THE PRESENT CASE:

$$P = V_L I$$

$$= -L\frac{dI}{dt}I$$

NOTE: P IS NEGATIVE BECAUSE THE CURRENT IS LOSING ENERGY, WHICH IS BEING TRANSFERRED TO THE INDUCTOR:

$$\frac{dU_L}{dt} = -P$$

HENCE:

$$\frac{dU_L}{dt} = LI\frac{dI}{dt}$$

$$dU_L = LI\,dI$$

DRAWING #9: ENERGY STORED IN AN INDUCTOR 2

WE CAN FIND THE TOTAL ENERGY STORED IN AN INDUCTOR BY INTEGRATING THE PRIOR EXPRESSION:

$$U_L = \int dU_L$$

$$= \int_0^I dI\, L I$$

(NOTE: LIMITS OF INTEGRATION HAVE BEEN INSERTED)

$$= L \int_0^I dI\, I$$

$$= L \left. \frac{I^2}{2} \right|_0^I$$

$$U_L = \frac{1}{2} L I^2$$

NOTE: THIS EXPRESSION IS ANALOGOUS TO WHAT WE FOUND FOR A CAPACITOR:

$$U_C = \frac{1}{2} C V_C^2$$

DRAWING #10: ENERGY STORED IN AN INDUCTOR 3

EARLIER WE FOUND THAT FOR A SOLENOID:

$$L_{SOLENOID} = \mu_0 \frac{N^2}{\ell} A$$

INSERTING THIS INTO OUR EXPRESSION FOR U_L:

$$U_L = \frac{1}{2} L I^2$$

$$= \frac{1}{2} \mu_0 \frac{N^2}{\ell} A I^2 \quad \text{(INSERTED L)}$$

$$= \frac{1}{2\mu_0} \mu_0^2 \frac{N^2}{\ell} A I^2 \quad \text{(MULTIPLIED AND DIVIDED BY } \mu_0\text{)}$$

$$= \frac{1}{2\mu_0} \mu_0^2 \ell \frac{N^2}{\ell^2} A I^2 \quad \text{(MULTIPLIED AND DIVIDED BY } \ell\text{)}$$

$$= \frac{1}{2\mu_0} A \ell \mu_0^2 \frac{N^2}{\ell^2} I^2 \quad \text{(REARRANGED)}$$

$$= \frac{1}{2\mu_0} (A\ell) \left(\mu_0 \frac{N}{\ell} I\right)^2 \quad \text{(REARRANGED)}$$

NOTING THAT THE MAGNETIC FIELD INSIDE A SOLENOID IS:

$$B = \mu_0 \frac{N}{\ell} I$$

WE CAN WRITE:

$$U_L = \frac{1}{2\mu_0} (A\ell) B^2$$

NOTE: EVEN THOUGH WE HAVE DERIVED THIS RESULT FOR A SOLENOID, IT IS TRUE FOR ANY INDUCTOR (OR, IN FACT, ANYWHERE THAT THERE IS A MAGNETIC FIELD).

NOTE: $(A\ell)$ IS THE VOLUME INSIDE THE INDUCTOR.

NOTE: THIS EXPRESSION IS ANALOGOUS TO WHAT WE FOUND FOR A CAPACITOR:

$$U_C = \frac{1}{2} \varepsilon_0 E^2$$

NOTE: ALTHOUGH WE SAY "THE ENERGY STORED IN AN INDUCTOR", IT IS REALLY THE ENERGY STORED IN THE MAGNETIC FIELD OF THE INDUCTOR.