DRAWING #1: CAPACITORS

CAPACITORS

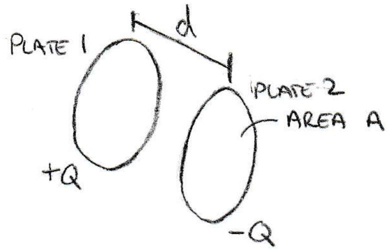
CAPACITORS ARE IMPORTANT ELECTRICAL COMPONENTS BECAUSE OF THEIR ABILITY TO STORE ENERGY (IN AN ELECTRIC FIELD).

(THIS ENERGY CAN BE USED IN A NUMBER OF WAYS --- AS WE'LL SEE)

SYMBOL IN CIRCUIT DIAGRAMS:

DRAWING #2: PARALLEL-PLATE CAPACITOR

CONSIDER TWO PARALLEL CONDUCTING PLATES SEPARATED BY A DISTANCE d:



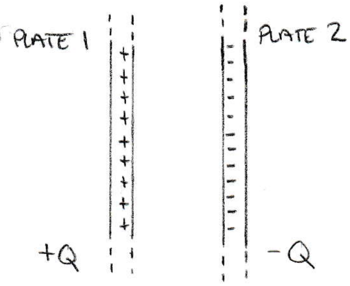
SUPPOSE NOW THAT N ELECTRONS ARE TRANSFERRED FROM ONE PLATE TO THE OTHER; THIS GIVES ONE PLATE A CHARGE OF $-Q = N(-e)$, AND THE OTHER A CHARGE OF $+Q$, WITH A NET CHARGE OF $Q + (-Q) = 0$.

THIS ARRANGEMENT OF TWO PARALLEL CONDUCTING PLATES CHARGED EQUALLY BUT OPPOSITELY IS CALLED A PARALLEL-PLATE CAPACITOR. (OR CAPACITOR)

(AND THE PROCESS OF TRANSFERRING CHARGE BETWEEN THE PLATES IS CALLED CHARGING THE CAPACITOR.)

DRAWING #3: THE PLATES OF A CAPACITOR AS INFINITE PLANES

CONSIDER A SIDE VIEW OF THE PARALLEL-PLATE CAPACITOR:



BECAUSE OPPOSITE CHARGES ATTRACT, THE (EXCESS) CHARGES ON EACH PLATE WILL RESIDE ON THE INNER SURFACES OF THE CAPACITOR (SHOWN ABOVE).

ALSO, BECAUSE LIKE CHARGES REPEL, OVER EACH SURFACE, THE CHARGES WILL WANT TO UNIFORMLY DISTRIBUTE THEMSELVES.

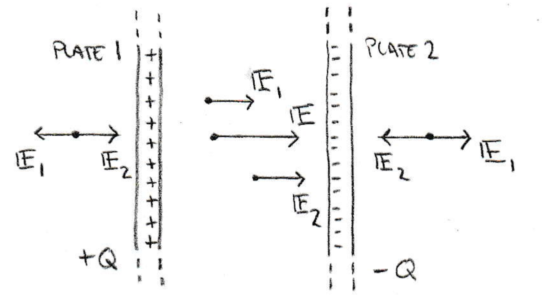
NOTE: THE TWO STATEMENTS ABOVE IGNORE EFFECTS AT THE OUTER EDGES OF THE PLATES.

THE TWO STATEMENTS ABOVE MEAN THAT WE CAN MODEL THE TWO PLATES AS TWO PLANES OF CHARGE, WITH EQUAL AND OPPOSITE SUFACE CHARGE DENSITIES (σ; NOT TO BE CONFUSED WITH CONDUCTIVITY)...

... ADDITIONALLY, FOR PLATES WITH LARGE AREAS RELATIVE TO THEIR SEPARATION (WHICH WE'LL ASSUME), THEY CAN BE MODELED AS INFINITE PLANES:

DRAWING #4: THE ELECTRIC FIELD INSIDE A CAPACITOR I

FOR EACH PLATE (PLANE) THE (EXCESS) ELECTRIC CHARGE WILL CREATE AN ELECTRIC FIELD:



IN ELECTROSTATICS, WE FOUND THAT THE MAGNITUDE OF THE ELECTRIC FIELD (AT A DISTANCE AWAY) FROM AN INFINITE PLANE WITH SURFACE CHARGE DENSITY IS:

$$E = \frac{\sigma}{2\varepsilon_0}$$

(i.e., INDEPENDENT OF DISTANCE AWAY).

IN ADDITION, THIS FIELD IS PERPENDICULAR TO THE PLANE, AND POINTS AWAY FROM A POSITIVELY-CHARGED PLANE, AND TOWARDS A NEGATIVE ONE.

THEREFORE, INSIDE A PARALLEL-PLATE CAPACITOR, EACH PLANE CREATES AN ELECTRIC FIELD THAT IS EVERYWHERE OF CONSTANT MAGNITUDE:

$$E_1 = \frac{\sigma}{2\varepsilon_0}$$

$$E_2 = \frac{\sigma}{2\varepsilon_0}$$

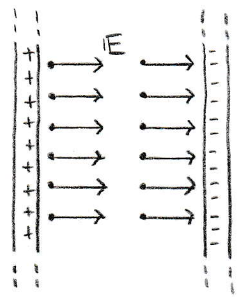
AND POINTS IN THE SAME DIRECTION (FROM THE POSITIVE TO NEGATIVE PLATE).

THEREFORE, THERE IS A UNIFORM ELECTRIC FIELD THAT POINTS FROM THE POSITIVE TO NEGATIVE PLATE WITH MAGNITUDE:

$$\begin{aligned} E &= E_1 + E_2 \\ &= \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} \\ &= \frac{\sigma}{\varepsilon_0} \\ &= \frac{Q}{\varepsilon_0 A} \qquad (\sigma = Q/A) \end{aligned}$$

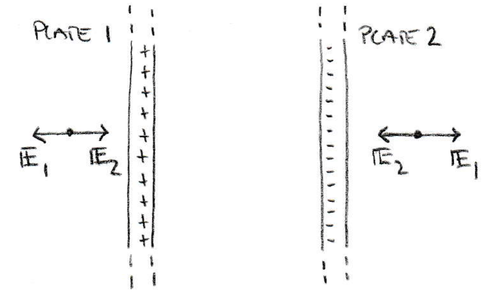
DRAWING #5: THE ELECTRIC FIELD INSIDE A CAPACITOR 2

A SIDE VIEW OF THIS LOOKS LIKE:



DRAWING #6: THE ELECTRIC FIELD OUTSIDE OF A CAPACITOR

OUTSIDE OF A PARALLEL PLATE CAPACITOR, THE ELECTRIC FIELDS STILL HAVE EQUAL MAGNITUDES (INDEPENDENT OF THE DISTANCE AWAY), BUT POINT IN OPPOSITE DIRECTIONS:



THEREFORE, THE NET FIELD OUTSIDE OF THE CAPACITOR IS ZERO.

NOTE: KEEP IN MIND THAT ALL OF THIS NEGLECTS EDGE/BOUNDARY EFFECTS OF REAL, FINITE-AREA PARALLEL PLATES.

DRAWING #7: THE POTENTIAL DIFFERENCE ACROSS A CAPACITOR

RECALL (FROM ELECTROSTATICS) THAT THE ELECTRIC FIELD ($\mathbb{E}$) IS RELATED TO THE ELECTRIC POTENTIAL (ϕ) VIA:

$$\mathbb{E} = -\nabla\phi$$

SINCE THE ELECTRIC FIELD IS EVERYWHERE CONSTANT INSIDE A PARALLEL-PLATE CAPACITOR, SO IS $(-\nabla\phi)$. THEREFORE:

$$-\nabla\phi = -\frac{\phi(2)-\phi(1)}{d}$$

$$= \frac{\phi(1)-\phi(2)}{d}$$

$$= \frac{V_c}{d} \qquad V_c = \phi(1)-\phi(2)$$

(NOTE: THE SUBSCRIPIT C HAS BEEN ADDED TO DENOTE A CAPACITOR)

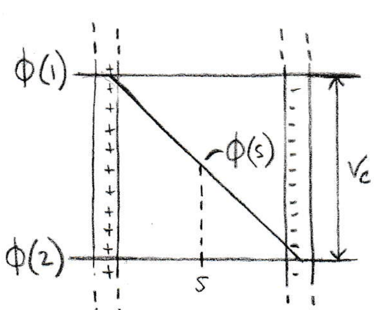
SO:

$$E = \frac{V_c}{d}$$

(NOTE: IT CAN BE SEEN THAT E POINTS FROM PLATE 1 TO 2 IF $\phi(1)>\phi(2)$, AS IN OUR DRAWINGS FROM BEFORE)

SINCE THE ELECTRIC FIELD IS CONSTANT, WE CAN USE THE PRIOR ANALYSIS TO WRITE:

$$V = Es \qquad \text{WHERE } V = \phi(1) - \phi(s)$$



DRAWING #8: CAPACITANCE

USING OUR PRIOR RESULT FOR THE ELECTRIC FIELD INSIDE A CAPACITOR:

$$E = \frac{Q}{\varepsilon_0 A}$$

WE CAN WRITE:

$$E = \frac{V_c}{d}$$

$$\frac{Q}{\varepsilon_0 A} = \frac{V_c}{d}$$

$$Q = \frac{\varepsilon_0 A}{d} V_c$$

THEREFORE, THE CHARGE ON A CAPACITOR IS DIRECTLY PROPORTIONAL TO THE GEOMETRY OF THE CAPACITOR (A AND d) AND THE POTENTIAL DIFFERENCE (VOLTAGE) BETWEEN THE PLATES.

FROM THE PRIOR EXPRESSION, IT IS USEFUL TO DEFINE THE CAPACITANCE (THE CAPACITY OF A CAPACITOR --- HENCE THE NAME(S) --- TO STORE ELECTRICAL CHARGE):

$$\boxed{C \equiv \frac{\varepsilon_0 A}{d}}$$

UNITS: $F = C/V$

FARAD (IN HONOR OF FARADAY)

NOTE: CAPACITANCE IS A PURELY GEOMETRIC PROPERTY (IT ONLY DEPENDS ON A AND d)

IT CAN BE SEEN THAT CAPACITANCE REPRESENTS THE CAPACITY TO STORE ELECTRICAL CHARGE BY WRITING:

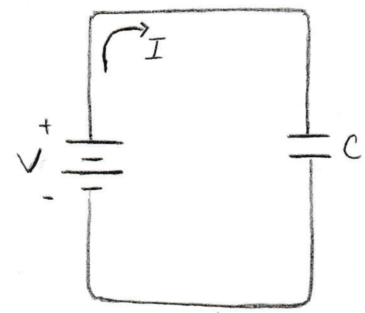
$$\boxed{Q = CV_c}$$

(Q IS THE CHARGE ON A CAPACITOR, WITH CAPACITANCE C AND POTENTIAL DIFFERENCE ACROSS THE PLATES OF V_c)

DRAWING #9: CHARGING A CAPACITOR

A COMMON WAY OF CHARGING A CAPACITOR IS BY CONNECTING IT TO A BATTERY.

CONSIDER A PARALLEL PLATE CAPACITOR CONNECTED TO A BATTERY BY IDEAL WIRES:



REMEMBER THAT EVEN THOUGH CURRENT FLOWS IN AN IDEAL WIRE, THERE IS ZERO POTENTIAL DIFFERENCE BETWEEN ITS TWO ENDS...

... THEREFORE, THERE MUST BE POTENTIAL DIFFERENCE ACROSS THE CAPACITOR EQUAL TO THAT PROVIDED BY THE BATTERY:

$$V_c = V$$

OF COURSE, THIS PROCESS IS NOT INSTANTANEOUS. THE POTENTIAL DIFFERENCE BETWEEN THE PLATES OF THE CAPACITOR INCREASES FROM 0 TO V_c AS CHARGE IS TRANSFERRED TO THE PLATES (CHARGING THE CAPACITOR).

WE CAN THEREFORE SAY THAT A CAPACITOR ATTACHED TO A BATTERY CHARGES UNTIL $V_c = V$.

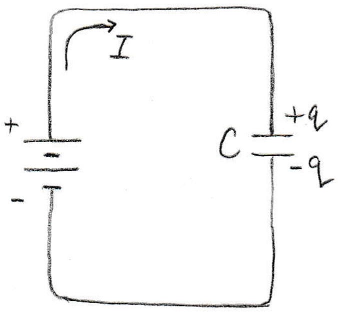
THE CHARGE ON A CAPACITOR (IN AN ELECTRICAL CIRCUIT) IS A PROPERTY OF BOTH THE POTENTIAL DIFFERENCE SUPPLIED BY THE BATTERY AND CAPACITANCE (GEOMETRY) OF THE CAPACITOR.

NOTE: ONCE CHARGED, AN IDEAL CAPACITOR IN VACUUM WILL STAY CHARGED (FOREVER).

DRAWING #10: THE ENERGY STORED IN A CAPACITOR 1

CAPACITORS ARE USEFUL CIRCUIT COMPONENTS BECAUSE OF THEIR ABILITY TO STORE ENERGY.

CONSIDER A CAPACITOR THAT HAS JUST BEEN CONNECTED TO A BATTERY AND IS IN THE PROCESS OF BEING CHARGED:



(NOTE: WITH IDEAL WIRES AND AN IDEAL BATTERY, THIS PROCESS WOULD (MATHEMATICALLY) BE INSTANTANEOUS)

AT ANY INSTANT THE CHARGES ON THE PLATES OF THE CAPACITOR ARE +/- q, WHICH ESTABLISHES A POTENTIAL DIFFERENCE ACROSS THE CAPACITOR OF:

$$V = q/C$$

NOW CONSIDER AN AMOUNT OF CHARGE(S) dq TRANSFERRED TO THE PLATES OF THE CAPACITOR.

RECALLING THAT THE ELECTRIC POTENTIAL ENERGY IS RELATED TO THE ELECTRIC POTENTIAL VIA:

$$U = q\phi$$

WE CAN WRITE THAT THE TRANSFER OF dq INCREASES THE POTENTIAL ENERGY STORED IN THE CAPACITOR BY:

$$dU = dq\, V \qquad \left(\text{NOTE: } V = \frac{(q+dq)}{C} \approx \frac{q}{C}\right)$$

DOES NOT CHANGE

(NOTE: ENERGY MUST BE CONSERVED, AND THIS INCREASE IN POTENTIAL ENERGY IS PROVIDED BY THE BATTERY.)

DRAWING #11: THE ENERGY STORED IN A CAPACITOR 2

THE TOTAL CHARGE TRANSFERRED TO A CAPACITOR CAN BE FOUND BY INTEGRATING THE PRIOR EXPRESSION (FROM THE START OF THE CHARGING, $q = 0$, TO THE END, $q = Q$):

$$\begin{aligned} U_c &= \int dU \\ &= \int_0^Q dq\, V \\ &= \int_0^Q dq\, q \frac{1}{C} \\ &= \frac{1}{C} \int_0^Q dq\, q \\ &= \frac{1}{C} \left. \frac{q^2}{2} \right|_0^Q \\ &= \frac{1}{C} \frac{Q^2}{2} \end{aligned}$$

WRITING THIS EXPRESSION IN TERMS OF THE CAPACITORS POTENTIAL DIFFERENCE:

$$\begin{aligned} U_c &= \frac{1}{C} \frac{Q^2}{2} \\ &= \frac{1}{2} C V_c^2 \end{aligned} \qquad (V_c = Q/C)$$

NOTE: A CAPACITOR IS ANALOGOUS TO A SPRING: $U = \frac{1}{2} k x^2$

DRAWING #12: THE ENERGY STORED IN A CAPACITOR 3

EARLIER WE FOUND THAT:

$$V_c = Ed$$

THEREFORE:

$$U_c = \frac{1}{2} C V_c^2$$
$$= \frac{1}{2} C (Ed)^2$$

INSERTING THE CAPACITANCE:

$$U_c = \frac{1}{2} \frac{\varepsilon_0 A}{d} (Ed)^2$$
$$= \frac{1}{2} \varepsilon_0 (Ad) E^2$$

NOTE: THE QUANTITY (Ad) IS THE VOLUME INSIDE THE CAPACITOR

NOTE: ALTHOUGH WE SAY "THE ENERGY STORED IN A CAPACITOR", IT IS REALLY THE ENERGY STORED IN THE ELECTRIC FIELD OF THE CAPACITOR.

DRAWING #13: USEFULNESS OF A CAPACITOR

CAPACITORS ARE USEFUL BECAUSE THEY CAN BE CHARGED (e.g., BY A BATTERY), STORING ENERGY IN THE ELECTRIC FIELD; THIS ENERGY CAN THEN BE RELEASED VERY QUICKLY. (NOTE: CHARGING BY A BATTERY IS OFTEN SLOW)

EXAMPLES:

- THE FLASH UNIT ON A CAMERA: THE CAMERA BATTERIES CHARGE A CAPACITOR, THE ENERGY STORED IN WHICH IS QUICKLY DISCHARGED INTO A FLASHLAMP.

- DEFIBRILLATOR: A DEFRIBRILLATOR HAS A LARGE CAPACITOR THAT CAN STORE UP TO 360 J OF ENERGY. THIS ENERGY IS RELEASED IN ABOUT 2 ms THROUGH PADDLES PRESSED AGAINST A PATIENTS CHEST (WHICH COMPLETES THE CIRCUIT).

IN TERMS OF THE ELECTRIC FIELD, CHARGING A CAPACITOR STORES ENERGY IN ITS ELECTRIC FIELD AS IT GROWS IN STRENGTH...

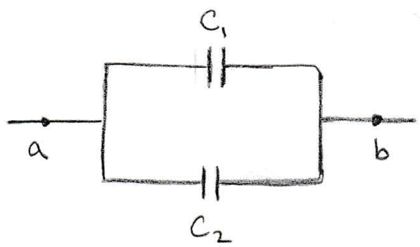
... DISCHARGING A CAPACITOR RELEASES THIS ENERGY AS THE FIELD COLLAPSES.

NOTE: IT IS IMPORTANT TO KEEP IN MIND THE CONSERVATION OF ENERGY.

DRAWING #14: PARALLEL CAPACITORS

MULTIPLE CAPACITORS CAN BE COMBINED IN AN ELECTRICAL NETWORK (JUST LIKE OTHER ELECTRICAL COMPONENTS).

CONSIDER TWO CAPACITORS CONNECTED IN PARALLEL (BY IDEAL WIRES):



$$V_{ab} = \Phi(a) - \Phi(b)$$

THE VOLTAGE MUST BE THE SAME AT POINTS a AND b FOR BOTH CAPACITORS; IN OTHER WORDS, TWO (OR MORE) CAPACITORS IN PARALLEL HAVE THE SAME POTENTIAL DIFFERENCE. (VOLTAGE)

THE CHARGES ON THE CAPACITORS ARE:

$$Q_1 = C_1 V_{ab}$$

$$Q_2 = C_2 V_{ab}$$

ADDING THESE EQUATIONS:

$$Q_1 + Q_2 = C_1 V_{ab} + C_2 V_{ab}$$

$$Q = (C_1 + C_2) V_{ab} \qquad Q = Q_1 + Q_2$$

THEREFORE, TWO CAPACITORS IN PARALLEL ACT AS A SINGLE CAPITOR WITH A CHARGE OF $Q = (Q_1 + Q_2)$ AND CAPACITANCE $C = (C_1 + C_2)$.

IF THERE ARE N CAPACITORS IN PARALLEL, THEIR EQUIVALENT CAPACITANCE IS:

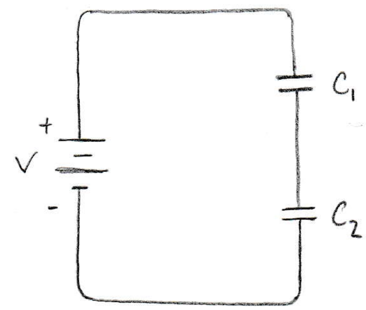
$$\boxed{C_{eq} = \sum_{i=1}^{N} C_i \quad \text{(PARALLEL CAPACITORS)}}$$

NOTE: THIS SUMMATION IS SIMILAR TO SERIES RESISTORS.

NOTE: THE EQUIVALENT CAPACITANCE OF PARALLEL CAPACITORS IS LARGER THAN THE CAPACITANCE OF ANY CAPACITOR IN THE GROUP.

DRAWING #15: SERIES CAPACITORS I

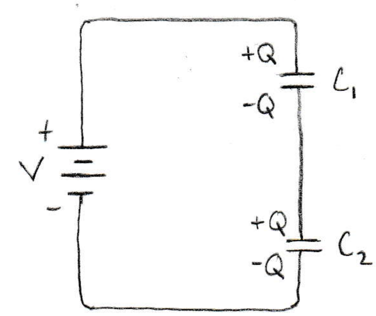
CONSIDER TWO CAPACITORS CONNECTED IN SERIES TO A BATTERY (OR OTHER SOURCE OF CONSTANT POTENTIAL) USING IDEAL WIRES:



THE POTENTIAL DIFFERENCE OF THE BATTERY CAUSES NEGATIVE CHARGE TO TRANSFER FROM THE TOP OF C_1 TO THE BOTTOM OF C_2...

... BETWEEN C_1 AND AND C_2, HOWEVER, THERE IS NO NET FLOW OF CHARGE.

THEREFORE, TWO CAPACITORS IN SERIES MUST HAVE EQUAL MAGNITUDE AND POSSIBLY OPPOSITE CHARGES:



THE POTENTIAL DIFFERENCE ACROSS EACH CAPACITOR IS:

$$V_1 = Q/C_1$$

$$V_2 = Q/C_2$$

ADDING THESE TWO EQUATIONS:

$$V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

DRAWING #16: SERIES CAPACITORS 2

$$V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$V_{12} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

THEREFORE, TWO CAPACITORS IN SERIES ACT AS A SINGLE CAPACITOR WITH CAPACITANCE $C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$.

IF THERE ARE N CAPACITORS IN SERIES, THEIR <u>EQUIVALENT CAPACITANCE</u> IS:

$$\boxed{C_{eq} = \left(\sum_{i=1}^{N} \frac{1}{C_i}\right)^{-1}}$$

NOTE: THIS EXPRESSION IS SIMILAR TO <u>PARALLEL</u> RESISTORS.

NOTE: THE EQUIVALENT CAPACITANCE OF SERIES CAPACITORS IS <u>SMALLER</u> THAN THE CAPACITANCE OF ANY CAPACITOR IN THE GROUP.