DRAWING #1: RESISTORS

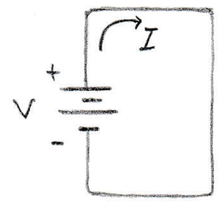
RESISTORS

RESISTORS ARE USEFUL ELECTRICAL COMPONENTS THAT CAN BE USED TO LIMIT THE CURRENT IN AN ELECTRICAL CIRCUIT.

SYMBOL IN CIRCUIT DIAGRAMS:

DRAWING #2: RESISTANCE OF IDEAL WIRES

CONSIDER AN ELECTRICAL CIRCUIT COMPRISED OF A BATTERY AND A WIRE (NON-IDEAL, BUT ALMOST IDEAL):



WE FOUND BEFORE FROM OHM'S LAW THE CURRENT IN THIS CIRCUIT:

$$I = \frac{V}{R}$$

FOR A FIXED V, IT IS EASY TO SEE THAT:

$$I \to \infty \quad \text{AS} \quad R \to 0 \quad \left(\lim_{R \to 0} \frac{V}{R} \text{ IS UNDEFINED, ACTUALLY} \right)$$

(NOTE: THIS IS ANOTHER WAY OF STATING THAT THERE CAN BE A CURRENT IN A CONDUCTOR WITH $R = 0\,\Omega$ EVEN WITHOUT A VOLTAGE --- A SUPERCONDUCTOR)

THEREFORE, A CIRCUIT MADE OUT OF ONLY NEAR-IDEAL WIRES WOULD GENERATE ENORMOUS CURRENTS. (NOTE: MOST METAL WIRES ARE NEAR-IDEAL)

(THIS WOULD, FOR EXAMPLE, QUICKLY DEPLETE THE POTENTIAL SOURCE --- e.g., THE BATTERY)

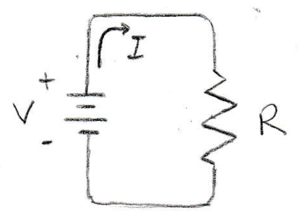
DRAWING #3: RESISTORS

RATHER THAN TRY TO USE VERY NON-IDEAL WIRES TO LIMIT THE CURRENT IN A CIRCUIT (THIS WOULD GENERATE SIGNIFICANT HEAT, NOT ALLOW PRECISE CONTROL OVER THE CURRENT FLOW WITH A JUNCTION, ETC.), IT IS USEFUL TO LIMIT THE CURRENT FLOW IN A CIRCUIT WITH AN OHMIC DEVICE CALLED A RESISTOR.

A RESISTOR IS AN OHMIC DEVICE WHOSE RESISTANCE IS SIGNIFICANTLY HIGHER THAN THE METAL WIRES (e.g., A POORLY CONDUCTING MATERIAL, SUCH AS CARBON):

$$R_{RESIST} >> R_{WIRE}$$

CONSIDER A RESISTOR CONNECTED TO A BATTERY WITH CURRENT-CARRYING WIRES:



EVEN THOUGH CURRENT FLOWS IN AN IDEAL WIRE, THERE IS ZERO POTENTIAL DIFFERENCE BETWEEN ITS TWO ENDS; THIS CAN BE SEEN IN OHM'S LAW (REARRANGED)

$$V = IR_{WIRE} = 0$$

THEREFORE, THERE MUST BE A POTENTIAL DIFFERENCE ACROSS THE RESISTOR EQUAL TO THAT PROVIDED BY THE BATTERY... (OR VOLTAGE DROP)

... FURTHERMORE, SINCE THE SAME AMOUNT OF CURRENT PASSES THROUGH BOTH SEGMENTS OF WIRE AS DOES THE RESISTOR, WE CAN FIND THE CURRENT IN THE WIRE AGAIN VIA OHM'S LAW:

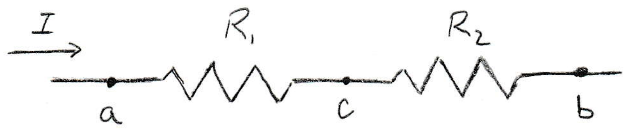
$$I = \frac{V_{DROP}}{R_{RESIST}} = \frac{V}{R_{RESIST}}$$

(i.e., A RESISTOR LIMITS THE CURRENT FLOW IN A CIRCUIT)

DRAWING #4: SERIES RESISTORS

MULTIPLE RESISTORS CAN BE COMBINED IN A SINGLE CIRCUIT.

RESISTORS THAT ARE ALIGNED END-TO-END (WITH NO JUNCTIONS BETWEEN THEM) ARE CALLED <u>SERIES RESISTORS</u> ("IN SERIES")



FROM OHM'S LAW, THE POTENTIAL DIFFERENCE (VOLTAGE DROP) ACROSS EACH RESISTOR IS:

$$V_1 = IR_1$$

$$V_2 = IR_2$$

ASSUMING IDEAL WIRES, WE CAN THEREFORE WRITE THE POTENTIAL DIFFERENCE BETWEEN POINTS a AND b AS:

$$V_{ab} = V_1 + V_2 = IR_1 + IR_2$$

$$\left(\begin{array}{l} V_1 = \phi(a) - \phi(c) \\ V_2 = \phi(c) - \phi(b) \\ V_{ab} = V_1 + V_2 = \phi(a) - \phi(b) \end{array} \right)$$

SINCE THE CURRENT MUST BE THE SAME ACROSS BOTH RESISTORS:

$$V_{ab} = IR_1 + IR_2 = I(R_1 + R_2)$$

THE TWO RESISTORS THEREFORE ACT AS A SINGLE RESISTOR WITH A VALUE OF $(R_1 + R_2)$.

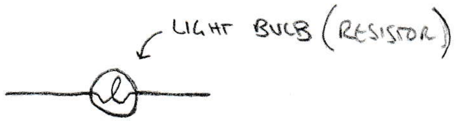
IF WE HAVE N RESISTORS IN SERIES, THEIR <u>EQUIVALENT RESISTANCE</u> IS:

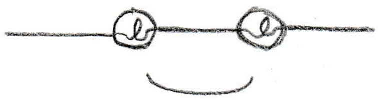
$$\boxed{R_{eq} = \sum_{i=1}^{N} R_i \quad \text{(SERIES RESISTORS)}}$$

DRAWING #4.1; SERIES RESISTORS 2

NOTE: THE EQUIVALENT RESISTANCE OF RESISTORS IN SERIAL IS ALWAYS MORE THAN ANY SINGLE RESISTOR IN THE GROUP --- THIS MEANS THAT CURRENT FLOW THROUGH THE CIRCUIT (FOR THE SAME BATTERY) WILL BE LESS.

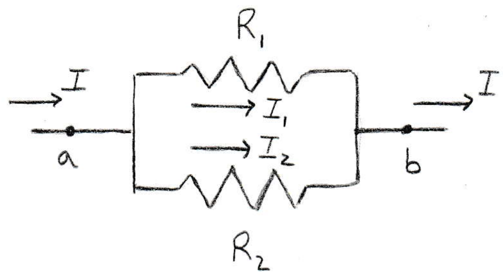
EXAMPLE: LIGHT BULBS IN SERIES:





THESE TWO WILL HAVE A HIGHER EQUIVALENT RESISTANCE THAN THE SINGLE BULB; LESS CURRENT WILL THEREFORE FLOW THROUGH THIS CIRCUIT AND THEY WILL BE DIMMER

RESISTORS CONNECTED AT BOTH ENDS ARE CALLED PARALLEL RESISTORS ("IN PARALLEL"):



FOR EACH RESISTOR, WE CAN USE OHM'S LAW TO WRITE:

$$I_1 = \frac{V_1}{R_1}$$

$$I_2 = \frac{V_2}{R_2}$$

ADDING THESE EQUATIONS:

$$I_1 + I_2 = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

FROM THE JUNCTION LAW (OF KIRCHHOFF):

$$I = I_1 + I_2$$

SO:

$$I = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

FURTHER, THE POTENTIAL MUST BE THE SAME FOR BOTH RESISTORS AT POINTS a AND b; FROM WHICH:

$$V_{ab} = V_1 = V_2$$

HENCE:

$$I = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

$$= \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2}$$

$$= V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

DRAWING #6: PARALLEL RESISTORS 2

APPLYING OHM'S LAW AGAIN, WE CAN FIND THE RESISTANCE BETWEEN POINTS a AND b:

$$R_{ab} = \frac{V_{ab}}{I}$$

THEREFORE:

$$R_{ab} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{V_{ab}}{I}$$

FOR N RESISTORS IN PARALLEL, THEIR <u>EQUIVALENT RESISTANCE</u> IS:

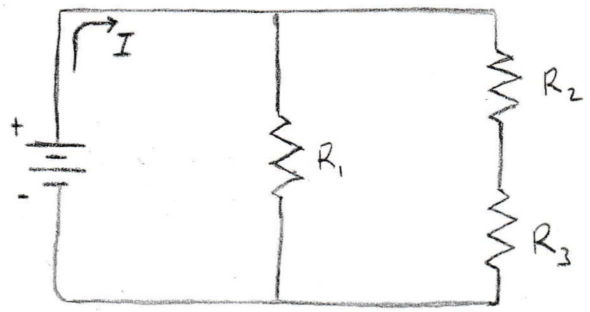
$$R_{eq} = \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)^{-1} \quad \text{(PARALLEL RESISTORS)}$$

NOTE: THE EQUIVALENT RESISTANCE OF RESISTORS IN PARALLEL IS <u>LESS</u> THAN ANY SINGLE RESISTOR IN THE GROUP --- THIS IS BECAUSE WHILE A RESISTOR "RESISTS" THE FLOW OF CHARGE, IN PARALLEL, THERE ARE MORE PATHWAYS FOR CURRENT TO FLOW.

DRAWING #7: COMBINATIONS OF RESISTORS

COMPLEX CIRCUITS (e.g., COMBINATIONS OF RESISTORS) CAN OFTEN BE SIMPLIFIED BY COMBINING SERIES AND PARALLEL RESISTORS INTO AN EQUIVALENT RESISTOR(S).

EXAMPLE:



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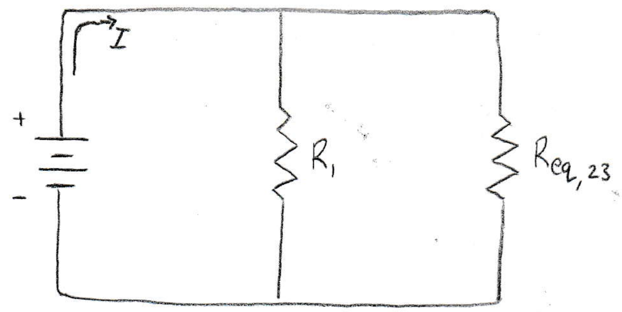
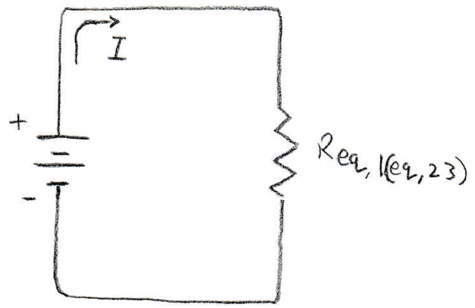


IT IS FIRST USEFUL TO COMBINE THE SERIES RESISTORS R_2 AND R_3 INTO AN EQUIVALENT RESISTOR $R_{eq,23}$...



... THEN, THE PARALLEL RESISTORS R_1 AND $R_{eq,23}$ CAN BE COMBINED INTO (ANOTHER) EQUIVALENT RESISTOR $R_{eq,1(eq,23)}$.

NOTE: THIS CIRCUIT IS MUCH MORE SIMPLE TO ANALYZE THAN WHAT WE STARTED WITH.