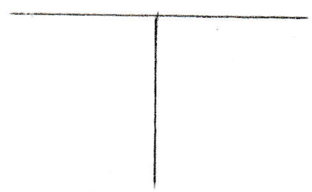
DRAWING #1: WIRES

WIRES

THE BASIC COMPONENT OF AN ELECTRICAL NETWORK IS A WIRE, USED TO CONTROL WHERE CHARGE FLOWS.

SYMBOL(S) IN CIRCUIT DIAGRAMS:

WIRE

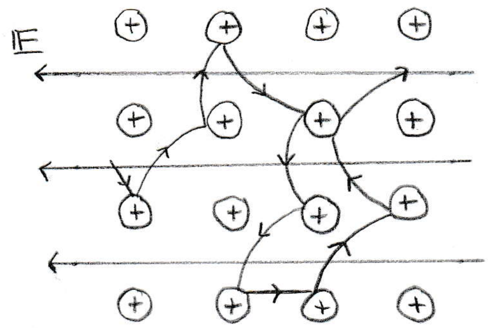


JUNCTION

SWITCH

DRAWING #2: DRUDE MODEL OF ELECTRON CONDUCTION

RECALL OUR MODEL FOR ELECTRON CONDUCTION (THE <u>DRUDE MODEL</u>):



- ELECTRONS ARE TREATED AS AN IDEAL GAS, FREE TO MOVE BETWEEN THE POSITIVELY-CHARGED IONIC CORES

  NON-INTERACTING

- ELECTRONS ACHIEVE THERMAL EQUILIBRIUM WITH THEIR SURROUNDINGS THROUGH COLLISIONS WITH (AND ONLY WITH) THE NUCLEI

- AN ELECTRON UNDERGOES A COLLISION PER UNIT TIME OF τ^{-1} (THE SCATTERING RATE)

THE ABOVE MODEL GIVES NO NET DISPLACEMENT OF THE ELECTRONS

- UNDER THE APPLICATION OF AN ELECTRIC FIELD: RANDOM THERMAL MOTION WILL STILL OCCUR, BUT THE ELECTRICAL FORCE WILL CAUSE A NET DISPLACEMENT OF THE ELECTRONS IN THE DIRECTION OPPOSITE TO THE FIELD

DRAWING #3: ELECTRIC CURRENT DENSITY

EARLIER WE DERIVED THE ELECTRON'S AVERAGE VELOCITY DUE TO THE ELECTRIC FIELD (THE DRIFT VELOCITY $\mathbb{V}_d$):

$$\mathbb{V}_d = \frac{-e\tau}{m_e}\mathbb{E}$$

m_e: MASS OF AN ELECTRON

AND TO DESCRIBE THE DYNAMICS OF MANY ELECTRONS, WE DEFINED THE ELECTRIC CURRENT DENSITY $\mathbb{J}$ (THE AMOUNT OF CHARGE PASSING PER UNIT AREA AND PER UNIT TIME, AT A RIGHT ANGLE TO THE FLOW):

$$\mathbb{J} = \rho \mathbb{V}_d$$

$$= \rho \frac{-e\tau}{m_e}\mathbb{E}$$

CONSIDERING THAT ρ CONSISTS OF INDIVIDUAL CHARGES, IT IS USEFUL TO REWRITE $\mathbb{J}$:

$$\rho = n_e(-e)$$

n_e: THE NUMBER DENSITY OF ELECTRONS (NUMBER OF ELECTRONS PER UNIT VOLUME)

$$\mathbb{J} = n_e(-e)\frac{-e\tau}{m_e}\mathbb{E}$$

$$= \frac{n_e e^2 \tau}{m_e}\mathbb{E}$$

BEFORE WE TREATED ALL MATERIALS AS THE SAME, AND IGNORED THEIR MICROSCOPIC DETAILS...

... WE NOW WISH TO CONSIDER THESE ("REAL" MATERIALS)

DRAWING #4: CONDUCTIVITY

FROM OUR EQUATION FOR $\mathbb{J}$:

$$\mathbb{J} = \frac{n_e e^2 \tau}{m_e} \mathbb{E}$$

THE TERM:

$$\frac{n_e e^2 \tau}{m_e}$$

(IN PARTICULAR n_e AND τ) DEPENDS ONLY ON THE CONDUCTING MATERIAL; AND:

- MATERIALS WITH A HIGHER n_e WILL LEAD TO A HIGHER CURRENT DENSITY (OTHER PARAMETERS BEING FIXED)
- MATERIALS WITH A HIGHER τ WILL LEAD TO A HIGHER CURRENT DENSITY

THEREFORE SOME MATERIALS ARE "BETTER" CONDUCTORS THAN OTHERS (HIGHER CURRENT DENSITY). WE CAN DESCRIBE HOW "GOOD" OF A CONDUCTOR A MATERIAL IS BY DEFINING THE CONDUCTIVITY σ:

$$\boxed{\sigma = \frac{n_e e^2 \tau}{m_e}}$$

(NOTE: n_e AND σ ARE OFTEN TABULATED)

(UNITS: $A \cdot C / N \cdot m^2 = \frac{1}{\Omega \cdot m}$

$\Omega = \frac{N \cdot m}{A \cdot C}$)

OHM

NOTE: WE WILL SEE LATER WHERE AN "OHM" COMES FROM

FROM WHICH WE CAN WRITE:

$$\boxed{\mathbb{J} = \sigma \mathbb{E}}$$

THIS EQUATION TELLS US:

- CURRENT IS CAUSED BY AN ELECTRIC FIELD
- CURRENT DEPENDS LINEARLY ON THE STRENGTH OF THE ELECTRIC FIELD
- CURRENT DEPENDS ON THE CONDUCTIVITY OF A MATERIAL

DRAWING #5: RESISTIVITY

FOR MANY PRACTICAL APPLICATIONS, IT WILL BE USEFUL TO DEFINE THE RESISTIVITY ρ OF A MATERIAL:

$$\boxed{\begin{aligned}\rho &= 1/\sigma \\ &= \frac{m_e}{n_e e^2 \tau}\end{aligned}}$$

(NOTE: RESISTIVITY VALUES ARE ALSO OFTEN TABULATED)

(UNITS: $\Omega \cdot m$)

THE RESISTIVITY DESCRIBES HOW RELUCTANT THE ELECTRONS ARE TO MOVE IN AN ELECTRIC FIELD.

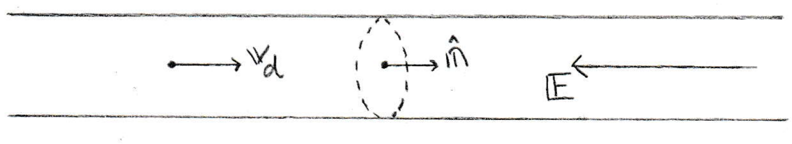
DRAWING #6: ELECTRIC CURRENT

WHEN DESCRIBING A SPECIFIC OBJECT (SHAPE), WE'RE NOT SO MUCH INTERESTED IN THE ELECTRIC CURRENT DENSITY, BUT THE ELECTRIC CURRENT I:

$$I = \int_S da\, \mathbb{J} \cdot \hat{n} \left(= \frac{dQ}{dt} \right)$$

EXAMPLE:

CONSIDER A WIRE (CYLINDRICAL) WITH RADIUS r, THROUGH WHICH THERE IS A (UNIFORM) ELECTRIC FIELD:



THE CURRENT IN THIS WIRE IS:

$$\begin{aligned} I &= \int_S da\, \mathbb{J} \cdot \hat{n} \\ &= \int_S da\, \sigma\, \mathbb{E} \cdot \hat{n} \\ &= -\sigma E \int_S da = -\sigma E (\pi r^2) \end{aligned}$$

E: THE MAGNITUDE OF $\mathbb{E}$

* NOTE: THE CURRENT IS SEEN TO FLOW IN THE DIRECTION OPPOSITE TO WHICH THE ELECTRONS MOVE. THIS IS BY CONVENTION (ARISING FROM OUR DEFINITION OF I).

HOWEVER, IN THE ANALYSIS OF CIRCUITS IT ACTUALLY DOESN'T MATTER (AND IS IN FACT USEFUL) IF WE CONSIDER CURRENT AS THE FLOW OF (EFFECTIVE) POSITIVE CHARGE...

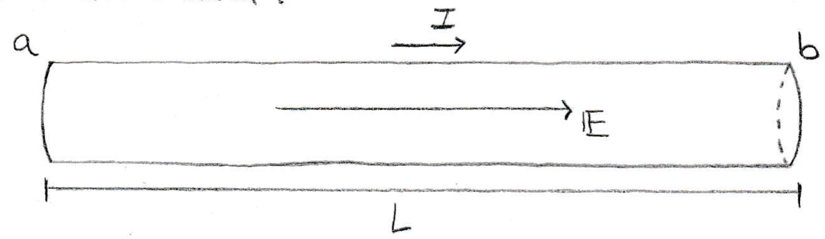
... AT THE MICROSCOPIC LEVEL THOUGH, IT IS ELECTRONS THAT MOVE.

DRAWING #8: ELECTRIC FIELD, POTENTIAL, AND CURRENT I

SO FAR WE'VE CONSIDERED THAT AN ELECTRIC FIELD EXIST INSIDE A CONDUCTOR, AND IT IS THIS FIELD WHICH CAUSES A CURRENT...

... BUT WE HAVEN'T SPECIFIED WHERE THIS ELECTRIC FIELD COMES FROM.

CONSIDER A FINITE CONDUCTOR (LENGTH L) (OF ARBITRARY SHAPE) THROUGH WHICH THERE IS AN ELECTRIC FIELD DRIVING A CURRENT:



RECALL NOW FROM ELECTROSTATICS THAT THE WORK PER UNIT CHARGE CAN BE EXPRESSED AS A DIFFERENCE IN A SCALAR FUNCTION (THE ELECTRIC POTENTIAL ϕ) EVALUATED AT THE END POINTS:

$$w(\text{UNIT}) = \underset{\text{ANY PATH}}{\int_a^b} \mathbb{E}(s)\cdot ds = -\left(\phi(b) - \phi(a)\right)$$

IN A CURRENT-CARRYING CONDUCTOR (OF CONSTANT CROSS-SECTIONAL AREA A), A CONSEQUENCE OF THE CONSERVATION OF CURRENT IS THAT THE ELECTRIC FIELD IS EVERYWHERE CONSTANT AND POINTS TANGENTIAL TO THE DIRECTION OF THE WIRE; THEREFORE:

$$\int_a^b \mathbb{E}(s)\cdot ds = E\int_a^b ds = EL = -\left(\phi(b) - \phi(a)\right)$$
$$= \phi(a) - \phi(b)$$

$$E = \frac{\Delta\phi}{L} \qquad \Delta\phi = \phi(a) - \phi(b)$$

THEREFORE, THE FIELD THAT DRIVES THE CURRENT IN A CONDUCTOR IS THE POTENTIAL DIFFERENCE BETWEEN THE ENDS OF THE CONDUCTOR (DIVIDED BY ITS LENGTH).

NOTE: (EFFECTIVE) POSITIVE CHARGE WILL WANT TO MOVE FROM A REGION OF HIGH-TO-LOW ELECTRIC POTENTIAL ENERGY.

DRAWING #9: ELECTRIC FIELD, POTENTIAL, AND CURRENT 2

WE CAN USE THIS ELECTRIC FIELD TO FIND THE CURRENT IN A CONDUCTOR.

FOR A CONDUCTOR WITH CONSTANT CROSS-SECTIONAL AREA A, WE CAN WRITE;

$$I = -\sigma E A$$

(NOTE: THIS IS A GENERALIZATION OF OUR RESULT FOR A CYLINDRICAL WIRE FROM BEFORE)

$$= -\sigma \frac{V}{L} A$$

(NOTE: REMEMBER THAT $V = \phi(a) - \phi(b)$)

$$= -\frac{A}{\rho L} V$$

NOTE: THIS RESULT CONTAINS DIRECTIONAL INFORMATION, (THE MINUS SIGN), AND IT MAY BE USEFUL TO CONSIDER THE AMOUNT OF CURRENT, AND WRITE;

$$I = \frac{A}{\rho L} V$$

... THOUGH ONE MUST MANUALLY THEN WORK OUT THE DIRECTION OF CURRENT "FLOW".

(IN ANY CASE) IT CAN BE SEEN THAT THE CURRENT IN A CONDUCTOR ARISES FROM A POTENTIAL DIFFERENCE BETWEEN THE END POINTS...

... AND THE DIRECTION IS SUCH THAT CURRENT (POSITIVE CHARGES) "FLOW" FROM A REGION OF HIGHER POTENTIAL TO LOWER POTENTIAL.

DRAWING #10: RESISTANCE AND CONDUCTANCE

FROM THE LAST EQUATION:

$$I = \frac{A}{\rho L} V$$

IT IS USEFUL TO DEFINE THE RESISTANCE R OF A CONDUCTOR:

$$\boxed{R = \frac{\rho L}{A}} \qquad (\text{UNITS}: \Omega)$$

NOTE: RESISTIVITY IS A PROPERTY OF A MATERIAL; RESISTANCE DESCRIBES BOTH THE RESISTIVITY AND GEOMETRY OF A CONDUCTOR.

FROM THIS EQUATION:

- THE RESISTANCE OF A CONDUCTOR INCREASES WITH ITS LENGTH
- THE RESISTANCE OF A CONDUCTOR INCREASES INVERSELY WITH ITS CROSS-SECTIONAL AREA

WE CAN ALSO DEFINE THE CONDUCTANCE G OF A CONDUCTOR:

$$G = \frac{1}{R}$$

$$= \frac{A}{\rho L}$$

$$\boxed{G = \sigma \frac{A}{L}}$$

DRAWING #11: OHM'S LAW

USING OUR DEFINITION OF RESISTANCE, WE CAN WRITE THE CURRENT THROUGH A CONDUCTOR AS:

$$I = \frac{A}{\rho L} V$$

$$\boxed{I = \frac{V}{R}}$$

*NOTE: REMEMBER, WE HAVE DROPPED THE SIGN GIVING US DIRECTIONAL INFORMATION; WE MUST KEEP IN MIND THAT CURRENT FLOWS FROM HIGHER-TO-LOWER POTENTIAL.

THIS RESULT IS KNOWN AS <u>OHM'S LAW</u>, AS IT DESCRIBES THE <u>EXPERIMENTS</u> DONE BY GEORG OHM IN THE YEARS 1825--1826.

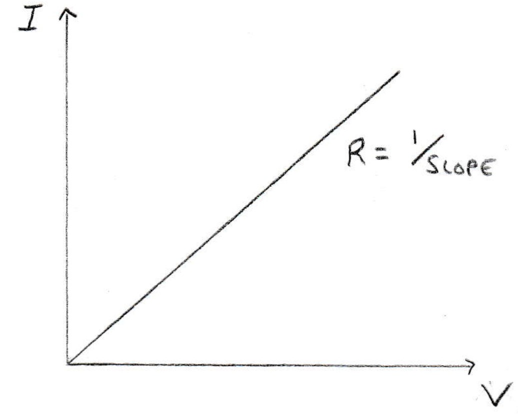
(THIS IS LONG BEFORE MAXWELL'S EQUATIONS AND THE DRUDE MODEL OF ELECTRON CONDUCTION.)

NOTE: OHM'S LAW WAS PROBABLY ONE OF THE MOST IMPORTANT EARLY QUANTITATIVE DESCRIPTIONS OF THE PHYSICS OF ELECTRICITY, BUT WASN'T WIDELY ACCEPTED UNTIL THE 1840s, DUE TO THE PREVAILING SCIENTIFIC PHILOSOPHY IN GERMANY THAT EXPERIMENTS DIDN'T NEED TO BE DONE TO UNDERSTAND NATURE --- THAT THIS COULD BE DONE THROUGH REASONING ALONE.

DRAWING #12: OHMIC MATERIALS

OHM'S LAW IS ACTUALLY NOT A LAW OF NATURE...

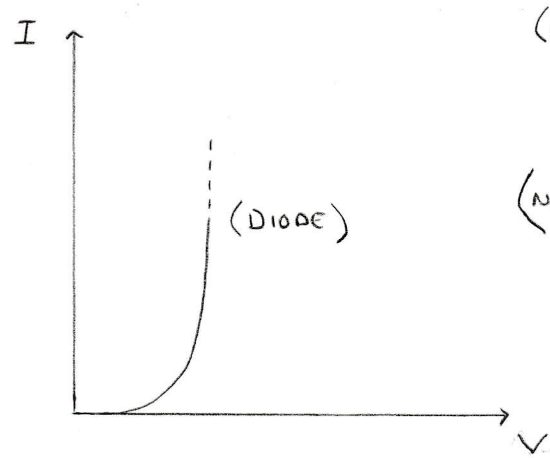
... AND IS LIMITED TO MATERIALS FOR WHICH OHM'S LAW HOLDS:



THE MATERIALS ARE CALLED OHMIC MATERIALS.

EXAMPLE: NON-OHMIC MATERIALS

MATERIALS THAT DO NOT OBEY OHM'S LAW ARE NONOHMIC:



(NOTE: A DIODE IS A CIRCUIT ELEMENT THAT CONDUCTS, IDEALLY, IN ONLY ONE DIRECTION)

(NOTE: DIODES AND TRANSISTORS ARE THE BASIC CIRCUIT ELEMENTS OF DIGITAL ELECTRONICS)

DRAWING #13: OHMIC MATERIALS

THERE ARE THREE IMPORTANT CLASSES OF OHMIC MATERIALS:

- WIRES: CONDUCTORS WITH VERY SMALL RESISTIVITIES AND HENCE RESISTANCE ($R \ll 1\,\Omega$).

  AN IDEAL WIRE HAS A RESISTANCE OF $R = 0\,\Omega$. THIS MEANS THAT A CURRENT CAN BE SUSTAINED, EVEN WITHOUT A POTENTIAL DIFFERENCE BETWEEN THE END POINTS (e.g., A SUPERCONDUCTOR).

  IN DISCUSSING CIRCUITS, WE'LL ASSUME THAT ALL WIRES ARE IDEAL, UNLESS OTHERWISE SPECIFIED.

- RESISTORS: "POOR" CONDUCTORS ($R \sim 10^1 \text{--} 10^6\,\Omega$)

- INSULATORS (NON-CONDUCTORS): $R \gg 10^9\,\Omega$

  AN IDEAL INSULATOR HAS $R \to \infty\,\Omega$. THEREFORE, NO CURRENT WILL FLOW IN IT, EVEN IF THERE IS A POTENTIAL DIFFERENCE ACROSS IT.