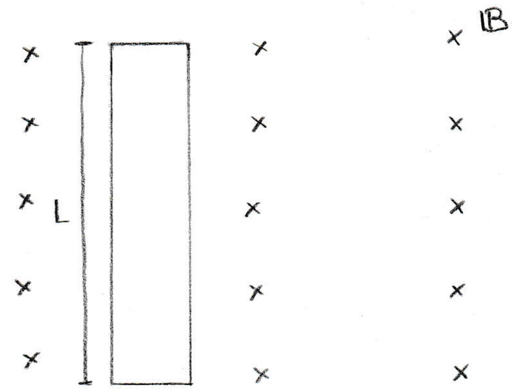
ELECTROMAGNETIC INDUCTION

SOON AFTER THE DISCOVERY THAT CURRENTS CREATE MAGNETIC FIELDS, PEOPLE STARTED TO WONDER WHETHER THE CONVERSE IS TRUE (USING A MAGNETIC FIELD TO MAKE A CURRENT).

THIS IDEA WAS CONFIRMED IN 1831 BY MICHAEL FARADAY AND JOSEPH HENRY, THE PROCESS OF ELECTROMAGNETIC INDUCTION.

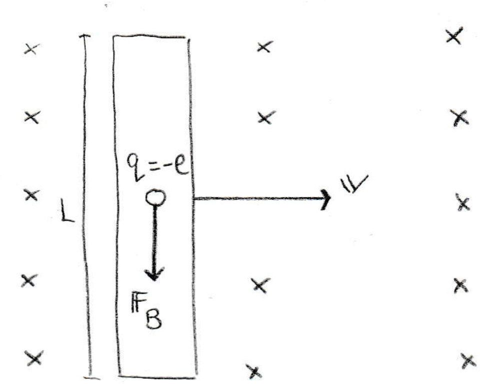
DRAWING #2: ELECTROMOTIVE FORCE I

CONSIDER A CONDUCTOR OF LENGTH L IN A PERPENDICULAR UNIFORM MAGNETIC FIELD $\mathbb{B}$:



SINCE THE WIRE (CONDUCTOR) IS AT REST, IT EXPERIENCES NO FORCE.

NOW CONSIDER THAT THE WIRE MOVES AT A VELOCITY $\mathbb{V}$ THROUGH THE MAGNETIC FIELD:



NOTE: THE DIRECTION OF THE FORCE CAN BE FOUND USING THE <u>RIGHT-HAND RULE</u>.

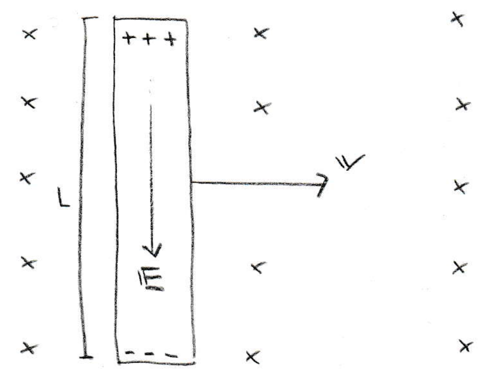
THE RELATIVELY FREE ELECTRONS IN THE CONDUCTOR WILL EXPERIENCE A MAGNETIC FORCE:

$$\mathbb{F} = q\, \mathbb{V} \times \mathbb{B}$$

NOTE: REMEMBER, IT IS ELECTRONS THAT ARE FREE TO MOVE (NOT THE POSITIVELY-CHARGED NUCLEI).

DRAWING #3: ELECTROMOTIVE FORCE 2

THIS SEPARATION OF CHARGE CREATES AN ELECTRIC FIELD INSIDE THE WIRE:



THEREFORE, CHARGES IN THE WIRE WILL (NOW) EXPERIENCE A FORCE GIVEN BY THE LORENTZ FORCE LAW:

$$\mathbb{F} = q\left(\mathbb{E} + \mathbb{v} \times \mathbb{B}\right)$$

NOTING THAT $\mathbb{E}$ AND $\mathbb{v} \times \mathbb{B}$ ARE BOTH ALONG THE AXIS OF THE WIRE, BUT IN OPPOSITE DIRECTIONS:

$$F = q\left(-E + vB\right)$$

THIS SEPARATION OF CHARGES WILL CONTINUE UNTIL THE ELECTRIC FORCE BALANCES THE MAGNETIC FORCE; THIS WILL OCCUR WHEN:

$$-E + vB = 0$$

$$\boxed{E = vB}$$

THIS RESULT MEANS THAT THE MAGNETIC FORCE ON CHARGE CARRIERS IN A MOVING CONDUCTOR CREATES AN ELECTRIC FIELD INSIDE THE CONDUCTOR.

DRAWING #4: ELECTROMOTIVE FORCE 3

AS THE CONDUCTOR IS MOVED THROUGH THE MAGNETIC FIELD, THE MAGNETIC FORCE DOES WORK TO SEPARATE THE CHARGES.

WE CAN EXPRESS THIS BY DEFINING THE ELECTROMOTIVE FORCE ($\mathcal{E}$) (OR emf):

THE WORK PERFORMED PER UNIT CHARGE, TO SEPARATE THE CHARGE.

NOTE: $\mathcal{E}$ IS NOT AN ACTUAL FORCE.

RECALL THE DEFINITION OF WORK:

$$W = \int_a^b \mathbb{F} \cdot ds$$

$$\mathcal{E} = W(\text{UNIT}) = \int_a^b (\mathbb{F}/q) \cdot ds$$

IN THIS CASE $\mathbb{F}$ IS THE MAGNETIC FORCE ($\mathbb{F}_m$), BUT WE JUST FOUND THIS WAS THE NEGATIVE OF THE ELECTRIC FORCE ($\mathbb{F}_E$), SO:

$$\mathcal{E} = W(\text{UNIT}) = \int_a^b (\mathbb{F}_m/q) \cdot ds$$

$$= -\int_a^b (\mathbb{F}_E/q) \cdot ds$$

$$= -\int_a^b \mathbb{E} \cdot ds$$

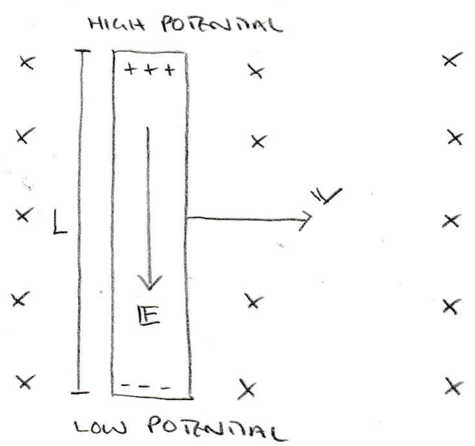
DRAWING # 5: ELECTROMOTIVE FORCE 4

WE'VE ENCOUNTERED THIS EXPRESSION BEFORE, FINDING THAT WE CAN WRITE IT IN TERMS OF THE ELECTRIC POTENTIAL:

$$\mathcal{E} = w(\text{UNIT}) = -\int_a^b \mathbb{E} \cdot ds = \Phi(b) - \Phi(a)$$

THEREFORE, THE MOTION OF A CONDUCTOR THROUGH A MAGNETIC FIELD INDUCES A POTENTIAL DIFFERENCE BETWEEN THE ENDS OF THE CONDUCTOR.

IN THE PRESENT CASE:



$$\mathbb{E} = -vB$$

(NOTE: THE DIRECTION OF $\mathbb{E}$ SHOULD BE CONSIDERED RELATIVE TO THE MOTION OF A POSITIVE (TEST) CHARGE)

$$\mathcal{E} = -\int_a^b \mathbb{E} \cdot ds$$

$$= -\int_0^L ds\,(-vB)$$

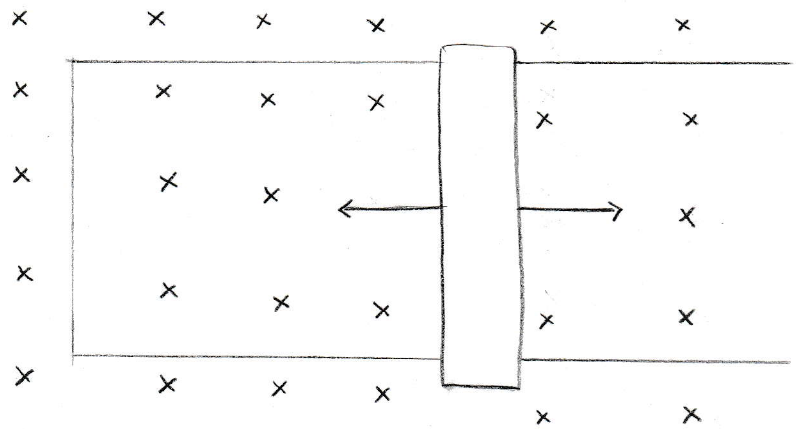
$$= vBL$$

BECAUSE THIS emf IS DUE TO THE MOTION OF THE CONDUCTOR, IT IS SOMETIMES CALLED <u>MOTIONAL emf</u>.

DRAWING #6: ELECTROMOTIVE FORCE 4

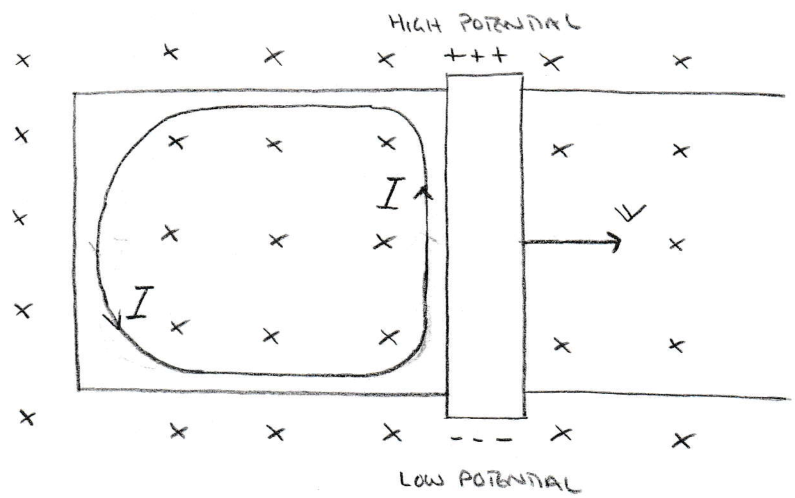
IN THE PRIOR EXAMPLE, THE CHARGES HAD NOWHERE TO GO.

NOW SUPPOSE THE CONDUCTOR IS CONNECTED TO A U-SHAPED WIRE, AND THAT THE CONDUCTOR CAN SLIDE BACK AND FORTH:



AS THE CONDUCTOR IS SLID DOWN THE U-SHAPED WIRE, THE ELECTRONS WILL AGAIN EXPERIENCE A FORCE. IN THIS CASE, HOWEVER, THEY HAVE SOMEWHERE TO GO (AROUND THE U-SHAPED WIRE), AND A CURRENT IS <u>INDUCED</u>:

NOTE: REMEMBER THAT, BY CONVENTION, CURRENT IS IN THE DIRECTION OPPOSITE TO WHICH THE ELECTRONS FLOW



NOTE: THE CURRENT WILL FLOW FROM A REGION OF HIGH POTENTIAL TO LOW POTENTIAL.

THE ELECTROMOTIVE FORCE THEREFORE INDUCES A CURRENT IN THE CIRCUIT.

(NOTE: THE emf IS THE SAME AS BEFORE $\mathcal{E} = vBL$)

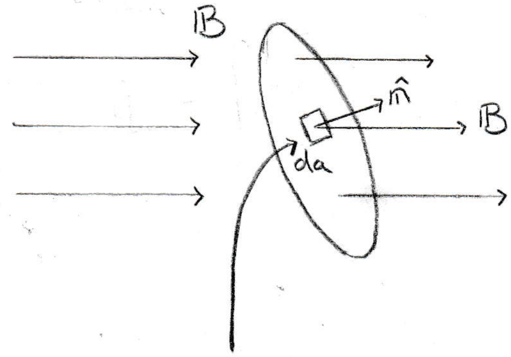
DRAWING #7: MAGNETIC FLUX

ONE OF THE THINGS THAT FARADAY FOUND WAS THAT A CURRENT COULD BE INDUCED IN A LOOP OF WIRE WHEN THE AMOUNT OF MAGNETIC FIELD PASSING THROUGH IT CHANGED . . .

. . . THIS IS PRECISELY WHAT OCCURRED IN THE LAST EXAMPLE.

THE "AMOUNT" OF MAGNETIC FIELD PASSING THROUGH A SURFACE CAN BE QUANTITATIVELY DESCRIBED BY THE MAGNETIC FLUX Φ_m:

(NOTE: WE'VE USED THIS CONCEPT BEFORE TO TALK ABOUT THE ELECTRIC FLUX)



THE AMOUNT OF MAGNETIC FLUX PASSING THROUGH THIS SMALL AREA da IS:

$$d\Phi_m = \mathbb{B}\cdot\hat{n}\, da$$

THE TOTAL AMOUNT OF FLUX THROUGH THE ENTIRE LOOP IS:

$$\Phi_m = \int d\Phi_m$$

$$= \int da(\mathbb{B}\cdot\hat{n})$$

$$= \int da\, B_n$$

B_n: THE NORMAL COMPONENT OF $\mathbb{B}$ ($B_n = \mathbb{B}\cdot\hat{n}$)

DRAWING #8: CHANGING MAGNETIC FLUX

WE CAN DESCRIBE A CHANGING MAGNETIC FLUX BY ITS TIME DERIVATIVE:

$$\frac{d\Phi_M}{dt}$$

THIS MAY OCCUR IN ONE OF THREE WAYS:

- THE MAGNETIC FIELD THROUGH THE LOOP CHANGES
- THE LOOP CHANGES AREA OR ORIENTATION (IN A MAGNETIC FIELD)
- THE LOOP MOVES IN OR OUT OF A MAGNETIC FIELD

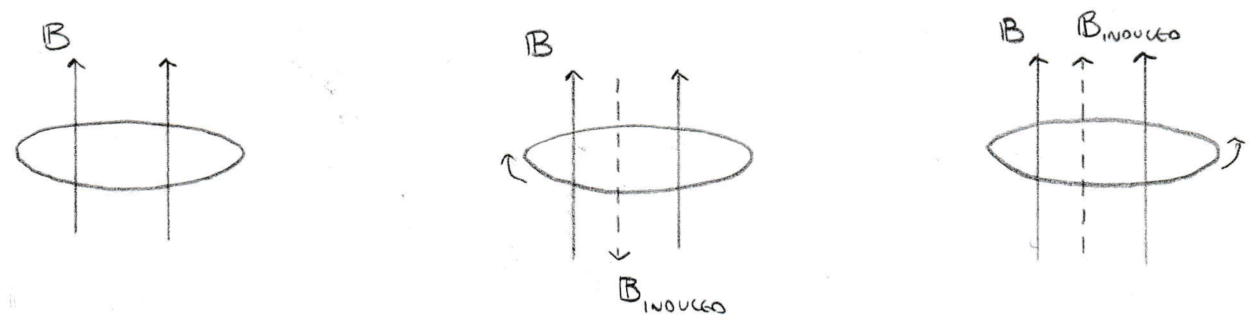
DRAWING #9: LENZ'S LAW

SHORTLY AFTER FARADAY'S DISCOVERY, HEINRICH LENZ DETERMINED A RULE FOR DETERMINING THE DIRECTION OF THE INDUCED CURRENT:

<u>THERE IS AN INDUCED CURRENT IN A CLOSED, CONDUCTING LOOP IF AND ONLY IF THE MAGNETIC FLUX THROUGH THE LOOP IS CHANGING. THE DIRECTION OF THE INDUCED CURRENT IS SUCH THAT THE INDUCED MAGNETIC FIELD OPPOSES THE CHANGE IN FLUX.</u>

THIS RULE IS KNOWN AS <u>LENZ'S LAW</u>.

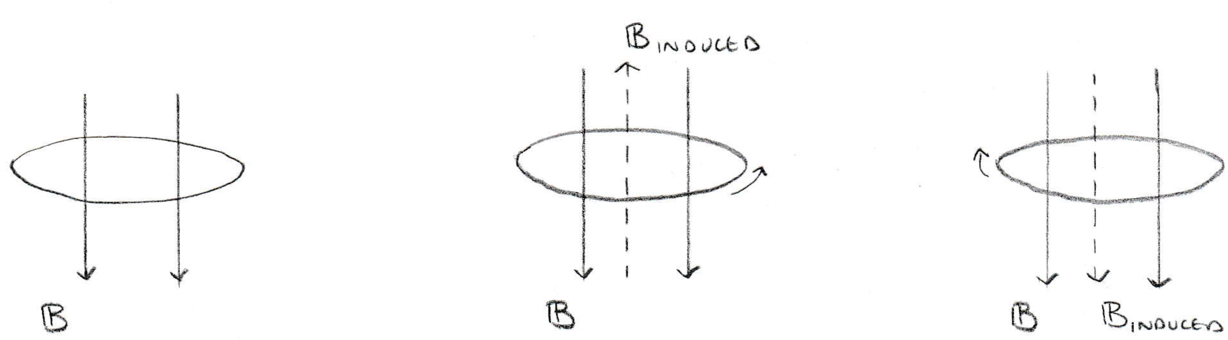
EXAMPLES:



$\mathbb{B}$ STEADY

$\|\mathbb{B}\|$ INCREASING

$\|\mathbb{B}\|$ DECREASING



DRAWING 10: FARADAY'S LAW

FORMALLY, WE CAN WRITE FARADAY'S LAW AS:

AN emf $\mathcal{E}$ IS INDUCED AROUND A CLOSED LOOP IF THE MAGNETIC FLUX THROUGH THE LOOP CHANGES:

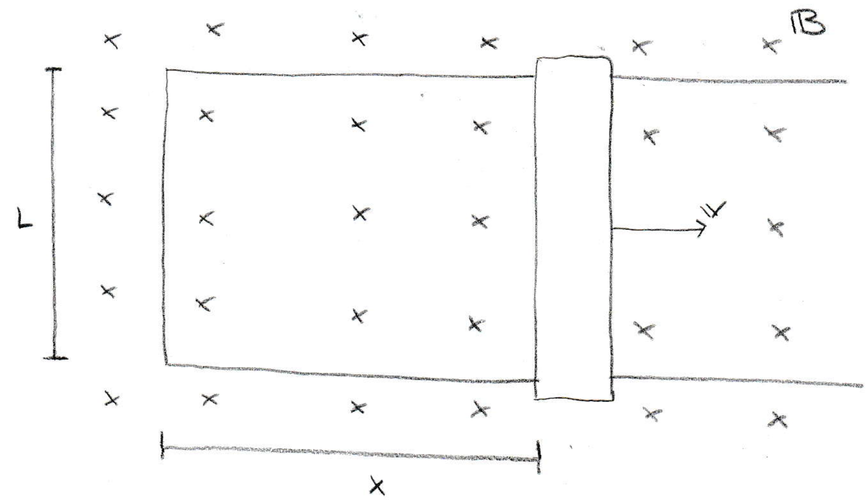
$$\mathcal{E} = -\frac{d\Phi_m}{dt}$$

NOTE: THE MINUS SIGN COMES FROM LENZ'S LAW

NOTE: THE emf IN THIS CASE IS SOMETIMES CALLED AN INDUCED emf.

EXAMPLE:

CONSIDER OUR PREVIOUS EXAMPLE OF CROSSBAR MOTION:



IN THIS CASE:

$$\Phi_m = xL(-B)$$

B: THE MAGNITUDE OF $\mathbb{B}$

NOTE: THE MINUS SIGN ARISES BECAUSE $\mathbb{B}$ POINTS INTO THE PLANE

HENCE:

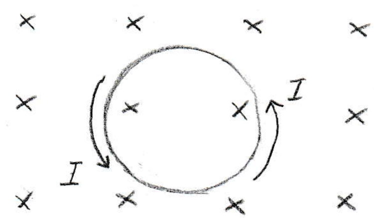
$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_m}{dt} \\ &= -\frac{d}{dt}(-xLB) \\ &= LB\frac{dx}{dt} = LBv\end{aligned}$$

NOTE: THIS AGREES WITH WHAT WE FOUND BEFORE

DRAWING #11: INDUCED FIELDS I

FARADAY'S LAW (AS WRITTEN) TELLS US ABOUT THE emf (AND HENCE CURRENT) INDUCED IN A CONDUCTING LOOP, BUT DOESN'T TELL US ABOUT THE FORCES THAT ACTUALLY MAKE THE CHARGES MOVE.

SUPPOSE A CONDUCTING LOOP IS IN A REGION OF MAGNETIC FIELD:

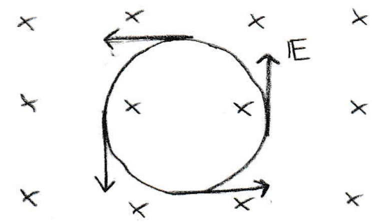


IF THE MAGNETIC FLUX THROUGH THE LOOP CHANGES, A CURRENT WILL BE INDUCED IN THE LOOP. HOWEVER, WHAT IS THE CAUSE?

WE KNOW FROM THE LORENTZ FORCE LAW THAT THE TOTAL ELECTROMAGNETIC FORCE ON A CHARGE IS:

$$\mathbb{F} = q\left(\mathbb{E} + \mathbb{v} \times \mathbb{B}\right)$$

SINCE $\mathbb{v} = 0$ (IN THE REFERENCE FRAME OF THE LOOP), WE MUST CONCLUDE THAT THERE IS AN ELECTRICAL FORCE (FIELD):



IN OTHER WORDS, A CHANGING MAGNETIC FIELD HAS SOMEHOW INDUCED AN ELECTRIC FIELD.

THE emf THAT APPEARS IN FARADAY'S LAW IS THEREFORE THE WORK DONE (PER UNIT CHARGE) BY THE FORCE OF AN INDUCED ELECTRIC FIELD, AS THE CHARGE MOVES AROUND A CLOSED LOOP:

$$\mathcal{E} = \omega(\text{UNIT}) = \oint \mathbb{E} \cdot ds$$

NOTE: CONSIDERING THAT A CHARGE IS BEING FORCED AROUND A CLOSED LOOP BY $\mathbb{E}$ ($\mathbb{E}$ IS ALWAYS IN THE DIRECTION OF ds FOR A CIRCULAR LOOP), IT IS CLEAR THAT THE WORK AROUND THIS PATH IS NON ZERO.

THEREFORE, <u>AN INDUCED ELECTRIC FIELD IS NON-CONSERVATIVE</u> ...

... AND <u>WE CANNOT THEREFORE ASSOCIATE AN ELECTRIC POTENTIAL WITH IT.</u>

DRAWING #13: FARADAY'S LAW (AGAIN)

WE CAN WRITE FARADAY'S LAW IN TERMS OF A CHANGING MAGNETIC FIELD AND INDUCED ELECTRIC FIELD AS FOLLOWS:

$$\mathcal{E} = -\frac{d}{dt}\Phi_m$$

NOTING THE LEFT-HAND SIDE IS THE WORK DONE (PER UNIT CHARGE) BY THE FORCE OF AN INDUCED ELECTRIC FIELD, AS THE CHARGE MOVES AROUND A CLOSED LOOP:

$$w(\text{UNIT}) = \oint \mathbb{E}\cdot d\mathbf{s} = -\frac{d}{dt}\Phi_m$$

WRITING OUT EXPLICITLY THE MAGNETIC FLUX:

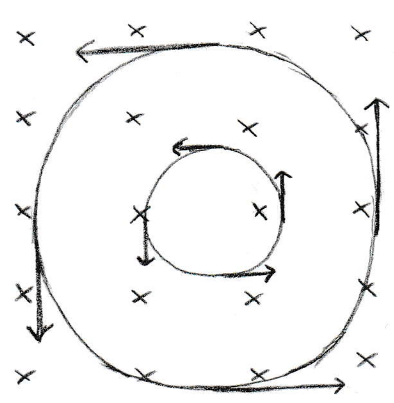
$$\boxed{\oint_C \mathbb{E}\cdot d\mathbf{s} = -\frac{d}{dt}\int_S da\, \mathbb{B}\cdot\hat{\mathbf{n}} = -\int_S da\, \frac{d\mathbb{B}}{dt}\cdot\hat{\mathbf{n}}}$$

FARADAY'S LAW IN INTEGRAL FORM

NOTE: THE CURVE C BOUNDS THE SURFACE S

THIS EXPRESSION SHOWS THAT THE RATE OF CHANGE OF THE NORMAL COMPONENT OF THE MAGNETIC FIELD THROUGH A SURFACE IS EQUAL TO THE LINE INTEGRAL OF ELECTRIC FIELD AROUND THAT SURFACE.

NOTE: THIS APPLIES EVEN IF THERE IS NO CONDUCTOR PRESENT.



NOTE THE CURL OF THE VECTOR FIELD.

FROM VECTOR CALCULUS:

$$\oint_C \mathbb{A} \cdot d\mathbf{s} = \int_S da\, (\nabla \times \mathbb{E}) \cdot \hat{n}$$

WE CAN USE THIS RESULT TO WRITE FARADAY'S LAW IN YET A DIFFERENT FORM:

$$\oint_C \mathbb{E} \cdot d\mathbf{s} = - \int_S da\, \frac{d\mathbb{B}}{dt} \cdot \hat{n}$$

$$\int_S da\, (\nabla \times \mathbb{E}) \cdot \hat{n} = - \int da\, \frac{d\mathbb{B}}{dt} \cdot \hat{n}$$

THEREFORE:

$$\boxed{\nabla \times \mathbb{E} = - \frac{d\mathbb{B}}{dt}}$$

FARADAY'S LAW IN DIFFERENTIAL FORM

THIS SAYS THAT A CHANGING MAGNETIC FIELD CREATES AN ELECTRIC FIELD (WITH A CURL).

NOTE: FOR STATIC FIELDS, $\frac{d\mathbb{B}}{dt} \to 0$, AND THIS EQUATION REDUCES TO OUR SECOND FUNDAMENTAL EQUATION OF ELECTROSTATICS:

$$\nabla \times \mathbb{E} = 0$$