DRAWING #7: REFERENCE FRAME(S) FOR THE MAGNETIC FORCE

NOTING THAT VELOCITY IS THE TIME DERIVATIVE OF A POSITION WITH RESPECT TO A FRAME OF REFERENCE LEADS TO THE QUESTION(S):

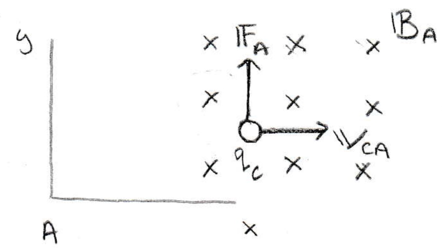
WHAT VELOCITY ENTERS INTO THE EQUATION FOR THE MAGNETIC FORCE?

WITH RESPECT TO WHICH REFERENCE FRAME?

IT TURNS OUT THAT ANY INERTIAL FRAME WILL DO.

DRAWING #8: THE RELATIVITY OF $\mathbf{E}$ AND $\mathbf{B}$

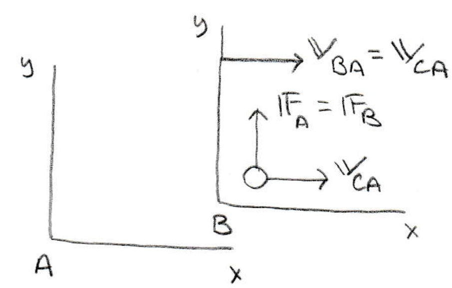
SUPPOSE THAT WITH RESPECT TO REFERENCE FRAME A AN OBSERVER ESTABLISHES A REGION OF MAGNETIC FIELD $\mathbf{B}_A$, BUT NO ELECTRIC FIELD ($\mathbf{E}_A = 0$); FURTHER, A CHARGE q_c IS TRAVELLING WITH VELOCITY $\mathbf{v}_{CA}$ (WITH RESPECT TO REFERENCE FRAME A) THROUGH $\mathbf{B}_A$:



FROM THE LORENTZ FORCE LAW, THIS OBSERVER MEASURES A FORCE ON q_c:

$$\mathbf{F}_A = q_c\left(\mathbf{E}_A + \mathbf{v}_{CA} \times \mathbf{B}_A\right)$$
$$= q_c \mathbf{v}_{CA} \times \mathbf{B}_A$$

NOW SUPPOSE THAT REFERENCE FRAME B TRAVELS AT THE SAME VELOCITY AS q_c:



BECAUSE BOTH A AND B ARE INERTIAL REFERENCE FRAMES, OBSERVERS MUST AGREE ABOUT THE FORCE ACTING ON q_c:

$$\mathbf{F}_A = \mathbf{F}_B$$

SINCE q_c IS STATIONARY IN REFERENCE FRAME B, AN OBSERVER MUST CONCLUDE FROM THE LORENTZ FORCE LAW THAT:

$$\mathbf{F}_B = q_c\left(\mathbf{E}_B + \mathbf{v}_{CB} \times \mathbf{B}_B\right)$$
$$= q_c \mathbf{E}_B$$

ESTABLISHING A REGION OF ELECTRIC FIELD $\mathbf{E}_B$, BUT NO MAGNETIC FIELD ($\mathbf{B}_B = 0$).

DRAWING #9: THE RELATIVITY OF $\mathbb{E}$ AND $\mathbb{B}$ 2

IF WE CHOOSE YET ANOTHER REFERENCE FRAME WE WOULD FIND A MIXTURE OF $\mathbb{E}$ AND $\mathbb{B}$ FIELDS.

THEREFORE, WHETHER A FIELD IS SEEN AS "ELECTRIC" OR "MAGNETIC" DEPENDS ON THE CHOSEN REFERENCE FRAME (e.g., ITS MOTION RELATIVE TO THE SOURCES OF THE FIELD...

DRAWING #10: THE RELATIVITY OF $\mathbb{E}$ AND $\mathbb{B}$ 3

IN GENERAL, SUPPOSE THAT IN REFERENCE FRAME A AN ELECTRIC ($\mathbb{E}_A$) AND MAGNETIC ($\mathbb{B}_A$) FIELD ARE ESTABLISHED. A CHARGE q MOVING WITH VELOCITY $\mathbb{V}_{CA}$ WILL EXPERIENCE A FORCE:

$$\mathbb{F}_A = q\left(\mathbb{E}_A + \mathbb{V}_{CA} \times \mathbb{B}_A\right)$$

NOW SUPPOSE THAT REFERENCE FRAME B MOVES WITH VELOCITY $\mathbb{V}_{BA} = \mathbb{V}_{CA}$. THE FORCE IN THIS FRAME MUST BE:

$$\mathbb{F}_B = q\left(\mathbb{E}_B + \mathbb{V}_{CB} \times \mathbb{B}_B\right)$$

$$= q\,\mathbb{E}_B \qquad (\text{SINCE } \mathbb{V}_{CB} = 0)$$

SINCE OBSERVERS IN THE TWO INERTIAL REFERENCE FRAMES MUST AGREE ON THE FORCES:

$$\mathbb{F}_B = \mathbb{F}_A$$

$$q\,\mathbb{E}_B = q\left(\mathbb{E}_A + \mathbb{V}_{CA} \times \mathbb{B}_A\right)$$

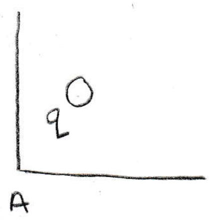
$$\boxed{\mathbb{E}_B = \mathbb{E}_A + \mathbb{V}_{BA} \times \mathbb{B}_A} \qquad (\text{USING } \mathbb{V}_{BA} = \mathbb{V}_{CA})$$

THIS RESULT TRANSFORMS THE ELECTRIC AND MAGNETIC FIELDS MEASURED IN REFERENCE FRAME A INTO AN ELECTRIC FIELD MEASURED MEASURED IN REFERENCE FRAME B THAT MOVES AT A CONSTANT VELOCITY RELATIVE TO B.

THIS TRANSFORMATION IS KNOWN AS A <u>GALILEAN FIELD TRANSFORMATION</u>.

NOTE: REMEMBER THAT THE APPROXIMATION $v^2 \ll c^2$ MUST BE MADE.

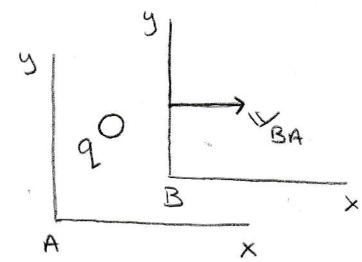
SUPPOSE THAT A CHARGE q IS AT REST IN REFERENCE FRAME A. AN OBSERVER WILL MEASURE AN ELECTRIC FIELD $\mathbb{E}_A$ AND NO MAGNETIC FIELD $\mathbb{B}_A$:



$$\mathbb{E}_A(\mathbb{r}) = \frac{1}{4\pi\epsilon_0}\,\frac{q}{r^2}\,\hat{\mathbb{r}}$$

$$\mathbb{B}_A = 0$$

NOW SUPPOSE THAT REFERENCE FRAME B MOVES WITH VELOCITY $\mathbb{V}_{BA}$:



FROM OUR FIELD TRANSFORMATION EQUATION:

$$\mathbb{E}_B = \mathbb{E}_A + \mathbb{V}_{BA} \times \mathbb{B}_A$$

$$\mathbb{E}_B = \mathbb{E}_A$$

MEANING THAT COULOMB'S LAW IS STILL VALID.

DRAWING #12: THE RELATIVITY OF $\mathbb{E}$ AND $\mathbb{B}$ S

SINCE THE CHARGE NOW ALSO APPEARS TO BE MOVING WITH RESPECT TO B ($\mathbb{V}_{CB} = -\mathbb{V}_{BA}$), THERE WILL BE A MAGNETIC FIELD; FROM THE BIOT--SAVART LAW:

$$\mathbb{B}_B(\mathbb{r}) = \frac{\mu_0}{4\pi} \frac{q(\mathbb{V}_{CB} \times \hat{\mathbb{r}})}{r^2}$$

(NOTE: THIS EXPRESSION HAS BEEN WRITTEN AS A FUNCTION OF RADIAL DISTANCE FROM THE CHARGE)

$$= -\frac{\mu_0}{4\pi} \frac{q}{r^2} (\mathbb{V}_{BA} \times \hat{\mathbb{r}})$$

$$= -\frac{\mu_0 \varepsilon_0}{4\pi \varepsilon_0} \frac{q}{r^2} (\mathbb{V}_{BA} \times \hat{\mathbb{r}})$$

$$= -\varepsilon_0 \mu_0 \mathbb{V}_{BA} \times \left(\frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{\mathbb{r}} \right)$$

$$= -\varepsilon_0 \mu_0 \mathbb{V}_{BA} \times \mathbb{E}_A$$

THESE REARRANGEMENTS SHOW THAT THE <u>BIOT--SAVART LAW FOR THE MAGNETIC FIELD OF A MOVING POINT CHARGE IS SIMPLY COULOMB'S LAW FOR THE ELECTRIC FIELD OF A STATIONARY POINT CHARGE TRANSFORMED INTO A MOVING FRAME OF REFERENCE.</u>

IN GENERAL, THE GALILEAN FIELD TRANSFORMATION FOR $\mathbb{B}$ IS:

$$\boxed{\mathbb{B}_B = \mathbb{B}_A - \varepsilon_0 \mu_0 \mathbb{V}_{BA} \times \mathbb{E}_A}$$

NOTE: TO DERIVE THIS RESULT, ONE MUST USE SPECIAL RELATIVITY, AND MAKE THE APPROXIMATION THAT $v^2 \ll c^2$.

THEREFORE, EVEN THOUGH IN THE STATIC CASE OUR EQUATIONS SEPARATE OUT INTO TWO DISTINCT PAIRS --- ONE FOR THE ELECTRIC FIELD (ELECTROSTATICS) AND ANOTHER FOR THE MAGNETIC FIELD (MAGNETOSTATICS) --- THEY ARE REALLY NOT INDEPENDENT THINGS. THIS SEPARATION DEPENDS ON THE FRAME OF REFERENCE...

... <u>THE ELECTRIC AND MAGNETIC FIELDS SHOULD ALWAYS BE TAKEN **TOGETHER** AS **ONE** COMPLETE ELECTROMAGNETIC FIELD.</u>

NOTE: THIS IS WHY IT IS BEST TO THINK OF THE $\mathbb{E}$ AND $\mathbb{B}$ FIELDS AS ABSTRACT MATHEMATICAL OBJECTS (i.e., WE MUST NOT ATTACH TOO MUCH REALITY TO THEM).