DRAWING #1: THE MAGNETIC FORCE ON A MOVING CHARGE I

THE MAGNETIC FIELD CAN BE DEFINED IN SEVERAL EQUIVALENT WAYS, BASED ON THE EFFECTS IT HAS ON ITS ENVIRONMENT.

EARLIER, WE DEFINED THE MAGNETIC FIELD $\mathbb{B}$ AS THE FIELD NECESSARY TO DESCRIBE THE AMOUNT OF FORCE $d\mathbb{F}$ ON A CURRENT-CARRYING TEST WIRE $(I\,ds)$:

$$d\mathbb{F} = (I\,ds) \times \mathbb{B}$$

NOW WE WISH TO PROVIDE A MORE "FUNDAMENTAL" DEFINITION (IN TERMS OF POINT CHARGES)

RECALL OUR DEFINITION OF THE ELECTRIC CURRENT:

$$I = \frac{dQ}{dt}$$

(THE TOTAL AMOUNT OF CHARGE PASSING THROUGH ANY SURFACE PER UNIT TIME).

WE CAN THEREFORE WRITE:

$$d\mathbb{F} = (I\,ds) \times \mathbb{B}$$

$$= \left(\frac{dQ}{dt}\,ds\right) \times \mathbb{B}$$

$$= \left(dQ\,\frac{ds}{dt}\right) \times \mathbb{B}$$

DRAWING #2: THE MAGNETIC FORCE ON A MOVING CHARGE 2

RECOGNIZING:

$$\frac{d\mathbb{s}}{dt} = \mathbb{v}$$

$\mathbb{v}$: THE VELOCITY OF THE CHARGE MOTION

WE CAN WRITE:

$$d\mathbb{F} = \left(dQ\,\frac{d\mathbb{s}}{dt}\right) \times \mathbb{B}$$

$$= \left(dQ\,\mathbb{v}\right) \times \mathbb{B}$$

RECALLING THAT CHARGE IS QUANTIZED, FOR A SINGLE CHARGE q:

$$dQ = q$$

AND WE CAN THEREFORE WRITE THE FORCE (THE MAGNETIC FORCE) ON A MOVING CHARGE AS:

$$d\mathbb{F} = \left(dQ\,\mathbb{v}\right) \times \mathbb{B}$$

$$\boxed{\mathbb{F} = q\left(\mathbb{v} \times \mathbb{B}\right)}$$

THE MAGNETIC FIELD IS OFTEN DEFINED AS THE VECTOR FIELD NECESSARY TO MAKE THIS FORCE LAW CORRECTLY DESCRIBE THE MOTION OF A CHARGED PARTICLE.

NOTE THAT THE SUPERPOSITION PRINCIPLE APPLIES TO $\mathbb{F}$, AS IT DOES FOR $\mathbb{B}$:

$$\mathbb{F} = q\mathbb{v} \times \mathbb{B}$$

$$= q\mathbb{v} \times \left(\sum_{i=1}^{N} \mathbb{B}_i\right)$$

$$= \sum_{i=1}^{N} q\mathbb{v} \times \mathbb{B}_i$$

$$= \sum_{i=1}^{N} \mathbb{F}_i$$

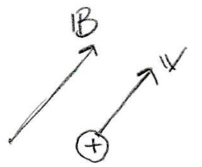
$$\left[\mathbb{a} \times (\mathbb{b} + \mathbb{c}) = (\mathbb{a} \times \mathbb{b}) + (\mathbb{a} \times \mathbb{c})\right]$$

DRAWING #3: THE MAGNETIC FORCE ON A MOVING CHARGE 3

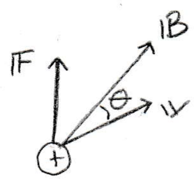
THERE ARE A NUMBER OF IMPORTANT SUBTLETIES IN THE MAGNETIC FORCE EQUATION:

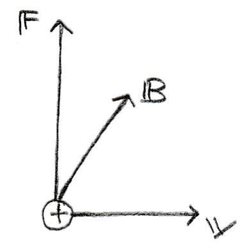
- ONLY A MOVING CHARGE EXPERIENCES A MAGNETIC FORCE
- THERE IS NO MAGNETIC FORCE ON A CHARGE MOVING PARALLEL OR ANTIPARALLEL TO A MAGNETIC FIELD
- BECAUSE OF THE CROSS PRODUCT, THE FORCE IS PERPENDICULAR TO BOTH $\mathbf{v}$ AND $\mathbf{B}$
- THE FORCE ON A NEGATIVE CHARGE IS IN THE DIRECTION OPPOSITE TO $(\mathbf{v} \times \mathbf{B})$

EXAMPLES:



$\mathbf{F} = 0$





NOTE: RECALL OUR GEOMETRIC INTERPRETATION OF THE CROSS PRODUCT

DRAWING #4: THE BIOT--SAVART LAW FOR A POINT CHARGE

RECALL: OUR EXPRESSION FROM THE BIOT--SAVART LAW FOR THE AMOUNT OF MAGNETIC FIELD $d\mathbb{B}$ GENERATED AT A POINT (1) GENERATED BY A SEGMENT OF A CURRENT-CARRYING WIRE AT POINT (2) $(I_2 ds_2)$:

$$d\mathbb{B}(1) = \frac{\mu_0}{4\pi} \frac{(I_2 ds_2) \times e_{12}}{r_{12}^2}$$

WE COULD CARRY OUT THE SAME ARGUMENTS AS WHEN FINDING THE MAGNETIC FORCE ON A MOVING CHARGE TO FIND THE MAGNETIC FIELD GENERATED BY A MOVING CHARGE FROM THIS EXPRESSION:

$$\mathbb{B}(1) = \frac{\mu_0}{4\pi} \frac{q \mathbb{V}_2 \times e_{12}}{r_{12}^2}$$

THIS EQUATION IS KNOWN AS THE BIOT--SAVART LAW FOR A POINT CHARGE (MOVING AT A CONSTANT VELOCITY) (THOUGH IT WAS DERIVED BY OLIVER HEAVISIDE).

NOTE: THIS EXPRESSION IS ONLY VALID WHEN $v^2 \ll c^2$ (c: THE SPEED OF LIGHT)

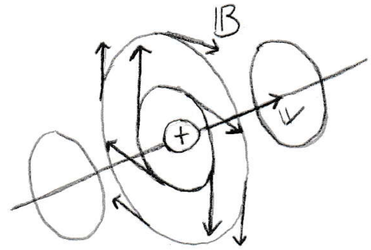
WE CAN THEREFORE SAY THAT MOVING CHARGES ARE THE SOURCE OF THE MAGNETIC FIELD.

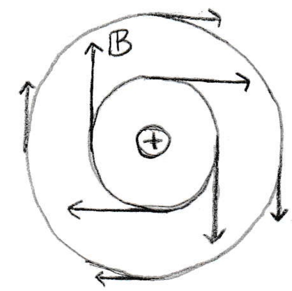
(IN SOME SENSE, THIS IS REPHRASING WHAT WE FOUND BEFORE)

NOTE: WHILE ALL CHARGES CREATE ELECTRIC FIELDS, ONLY MOVING CHARGES CREATE MAGNETIC FIELDS.

DRAWING #5: THE MAGNETIC FIELD OF A MOVING POINT CHARGE

THE MAGNETIC FIELD OF A MOVING POINT CHARGE LOOKS LIKE THE FOLLOWING:





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DRAWING #6: THE LORENTZ FORCE LAW

IN ELECTROSTATICS, WE FOUND THAT A CHARGED PARTICLE q IN AN ELECTRIC FIELD $\mathbb{E}$ WOULD EXPERIENCE A FORCE $\mathbb{F}$ (THE ELECTRIC FORCE):

$$\mathbb{F} = q\,\mathbb{E}$$

NOW WE HAVE FOUND THAT A MOVING CHARGE IN THE PRESENCE OF A MAGNETIC FIELD $\mathbb{B}$ WILL EXPERIENCE A FORCE (THE MAGNETIC FORCE):

$$\mathbb{F} = q\,\mathbb{v} \times \mathbb{B}$$

BECAUSE THE SUPERPOSITION PRINCIPLE HOLDS FOR BOTH OF THESE FORCES, THE TOTAL FORCE ON A POINT CHARGE MOVING IN THE PRESENCE OF ELECTRIC AND MAGNETIC FIELDS IS (THE ELECTROMAGNETIC FORCE):

$$\boxed{\mathbb{F} = q\left(\mathbb{E} + \mathbb{v} \times \mathbb{B}\right)}$$

THIS EQUATION IS CALLED THE LORENTZ FORCE LAW (THOUGH THE FIRST DERIVATION IS COMMONLY ATTRIBUTED TO OLIVER HEAVISIDE, A FEW YEARS BEFORE THE DERIVATION BY HENDRIK LORENTZ).