DRAWING #7: AMPÈRE'S LAW I

IN ELECTROSTATICS, FINDING THE ELECTRIC FIELD IN ANY CIRCUMSTANCE SIMPLY REQUIRED EVALUATING A COMPLICATED INTEGRAL EXPRRESSION ...

... THE SAME IS TRUE IN MAGNETOSTATICS FOR FINDING THE MAGNETIC FIELD.

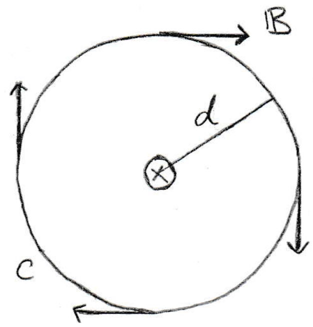
IN ELECTROSTATICS, WE THEREFORE USED CLEVER GUESSWORK TO EVALUATE THE INTEGRAL EXPRESSIONS IN SPECIAL SITUATIONS (e.g., GAUSS'S LAW IN HIGHLY SYMMETRICAL SITUATIONS), AND THIS GAVE US DEEPER INSIGHT INTO THE ELECTRIC FIELD.

... WE CAN DO THE SAME IN MAGNETOSTATICS,

IN THE FOLLOWING, A LAW THAT PLAYS THE SAME ROLE IN MAGNETOSTATICS AS GAUSS'S LAW PLAYED IN ELECTROSTATICS WILL BE DERIVED.

DRAWING #8: AMPÈRE'S LAW 2

SUPPOSE THERE EXISTS A LONG WIRE CARRYING CURRENT INTO THE PAGE, AND WE WISH TO CALCULATE THE MAGNETIC FIELD AT A DISTANCE d AWAY (NOTE: WE WORKED OUT THIS PROBLEM ONCE BEFORE):



IN ANALOGY WITH GAUSS'S LAW, CONSIDER THE LINE INTEGRAL OF $\mathbb{B}$ AROUND THE CIRCLE:

$$\oint_C \mathbb{B} \cdot d\mathbf{s}$$

SINCE $\mathbb{B}$ IS EVERYWHERE TANGENT TO THE CIRCLE:

$$\oint_C \mathbb{B} \cdot d\mathbf{s} = \oint_C ds\, B$$

B: THE MAGNITUDE OF $\mathbb{B}$

NOTE: YOU CAN KEEP THE DIRECTION OF $\mathbb{B}$ IN MIND USING THE RIGHT-HAND SCREW RULE.

ADDITIONALLY, SINCE $\mathbb{B}$ HAS A CONSTANT MAGNITUDE AT ALL POINTS ON THE CIRCLE, WE CAN WRITE:

$$\oint_C ds\, B = B \oint_C ds$$

$$= B(2\pi d)$$

PREVIOUSLY, WE FOUND THE MAGNETIC FIELD AT A DISTANCE d FROM AN INFINITE WIRE IS:

$$B = \frac{\mu_0 I}{2\pi d}$$

INSERTING THIS INTO OUR PRIOR EXPRESSION:

$$\oint_C \mathbf{B} \cdot d\mathbf{s} = B(2\pi d)$$

$$= \frac{\mu_0 I}{2\pi d}(2\pi d)$$

$$\boxed{\oint_C \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{THROUGH}}$$

THIS SAYS THAT THE CIRCULATION OF $\mathbf{B}$ AROUND A CLOSED CURVE IS EQUAL TO THE CURRENT PASSING THROUGH THE CURVE.

THIS RESULT IS KNOWN AS AMPÈRE'S LAW.

NOTE: THOUGH WE HAVE DERIVED AMPÈRE'S LAW USING A SPECIAL CIRCUMSTANCE, IT CAN BE DERIVED IN A GENERAL WAY STARTING FROM THE BIOT--SAVART LAW, USING A FAIRLY HIGH LEVEL OF VECTOR CALCULUS...

... THIS MEANS THAT AMPÈRE'S LAW:

- IS VALID FOR ANY CLOSED CURVE (INDEPENDENT OF THE SHAPE)
- IS INDEPENDENT OF WHERE THE CURRENT PASSES THROUGH THE CURVE
- DEPENDS ONLY ON THE TOTAL AMOUNT OF CURRENT THROUGH THE CURVE

NOTE: AMPÈRE'S LAW IS A FORMAL STATEMENT THAT CURRENTS CREATE MAGNETIC FIELDS.