DRAWING #10: FIELDS IN ELECTROSTATICS

RECALL OUR DEFINITION OF THE ELECTRIC FIELD $\mathbb{E}$ (FROM ELECTROSTATICS):

$$\mathbb{E}(x,y,z) = \frac{\mathbb{F}(x,y,z)}{q} \qquad (q>0)$$

--- THAT IS: $\mathbb{E}$ DESCRIBES THE FORCE $\mathbb{F}$ FELT PER UNIT CHARGE, IF A CHARGE (e.g. q) WERE TO BE PLACED AT A POINT IN SPACE (x,y,z).

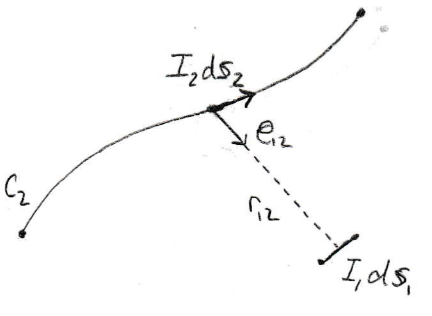
WE SAW A SIMILAR THING RELATING THE ELECTRIC POTENTIAL ϕ TO THE ELECTRIC POTENTIAL ENERGY $\mathcal{U}$ (ALSO FROM ELECTROSTATICS):

$$\phi(x,y,z) = \frac{\mathcal{U}(x,y,z)}{q}$$

--- THAT IS: ϕ DESCRIBES THE AMOUNT OF $\mathcal{U}$ THAT A CHARGE (e.g., q) WOULD HAVE IF PLACED AT ANY POINT IN SPACE (x,y,z).

BOTH $\mathbb{E}$ AND ϕ ARE EXAMPLES OF FIELDS: A MATHEMATICAL FUNCTION ASSIGNING A VALUE TO EVERY POINT IN SPACE (IN THESE CASES, A VECTOR $\mathbb{E}$ AND SCALAR ϕ, RESPECTIVELY).

CONSIDER NOW AGAIN THE EXPRESSION THAT PROVIDES THE AMOUNT OF FORCE ON A TEST WIRE $(I_1 ds_1)$ FROM A CURRENT CARRYING WIRE C_2:



$$d\mathbb{F} = (I_1 ds_1) \times \left[\frac{\mu_0}{4\pi} \int_{C_2} \frac{(I_2 ds) \times e_{12}}{r_{12}^2} \right]$$

IN ANALOGY WITH OUR DEFINITIONS OF THE ELECTRIC FIELD AND ELECTRIC POTENTIAL, THE ABOVE EQUATION ALLOWS US TO DEFINE A NEW FIELD, THAT ALLOWS US TO DESCRIBE THE FORCE THAT A TEST WIRE $(I_1 ds_1)$ WOULD FEEL IF PLACED AT ANY POINT IN SPACE AROUND A CURRENT-CARRYING WIRE:

$$d\mathbb{F} = (I_1 ds_1) \times \left[\frac{\mu_0}{4\pi} \int_{C_2} \frac{(I_2 ds_2) \times e_{12}}{r_{12}^2} \right]$$

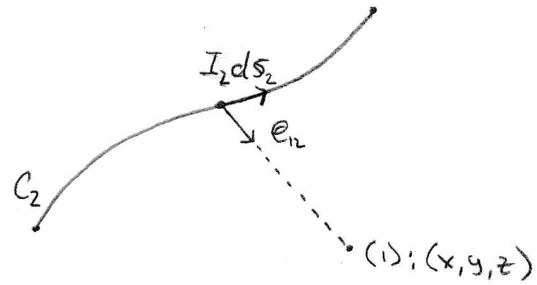
$$= (I_1 ds_1) \times \mathbb{B}$$

WHERE $\mathbb{B}$ IS THE MAGNETIC FIELD (PROVIDED EXPLICITLY ON THE NEXT PAGE).

NOTE: THE UNITS OF $\mathbb{B}$ MUST BE $N/A{\cdot}M$ CALLED A tesla T.

DRAWING #12: THE BIOT--SAVART LAW

AT A POSITION $(1):(x,y,z)$, THE MAGNETIC FIELD $\mathbb{B}(1)$ GENERATED BY A STEADY CURRENT I_2 OVER THE PATH C_2 IS:



$$\boxed{\mathbb{B}(1) = \frac{\mu_0}{4\pi}\int_{C_2} \frac{(I_2 ds_2)\times e_{12}}{r_{12}^2}}$$

THIS EQUATION IS CALLED THE BIOT--SAVART LAW (AFTER JEAN-BAPTISE BIOT AND FELIX SAVART, WHO DISCOVERED IT IN 1820).

NOTE: THIS EQUATION IS NOT OUR FUNDAMENTAL DEFINITION OF THE MAGNETIC FIELD (WE WILL SEE THAT LATER); IT DOES PROVIDE A WAY TO DIRECTLY OBTAIN THE MAGNETIC FIELD FROM A (STEADY) CURRENT-CARRYING WIRE.

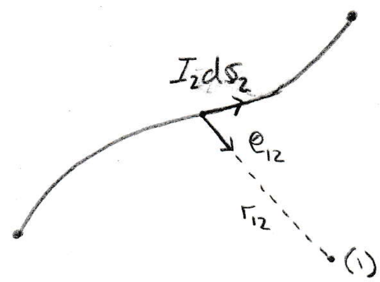
THE AMOUNT OF MAGNETIC FIELD GENERATED BY A SMALL SECTION OF WIRE $(I_2 ds_2)$ IS:

$$d\mathbb{B}(1) = \frac{\mu_0}{4\pi}\frac{(I_2 ds_2)\times e_{12}}{r_{12}^2} \qquad \mathbb{B}(1) = \int_{C_2} d\mathbb{B}(1)$$

NOTE: THE DIRECTION OF $d\mathbb{B}(1)$ CAN BE FOUND USING THE RIGHT-HAND RULE (NOTE THE CROSS PRODUCT)--- FOREFINGER IN THE DIRECTION OF $(I_2 ds_2)$, MIDDLE FINGER IN THE DIRECTION OF e_{12}, AND THE THUMB POINTS IN THE DIRECTION OF $d\mathbb{B}(1)$.

DRAWING #1: THE PRINCIPLE OF SUPERPOSITION

FROM THE BIOT--SAVART LAW :



$$\mathbb{B}(1) = \frac{\mu_0}{4\pi} \int_{C_2} \frac{(I_2 ds_2) \times e_{12}}{r_{12}^2}$$

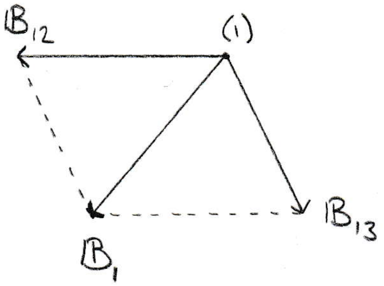
FROM WHICH WE CAN SEE THAT THE AMOUNT OF MAGNETIC FIELD GENERATED BY A SMALL SECTION OF THE WIRE IS:

$$d\mathbb{B}(1) = \frac{\mu_0}{4\pi} \frac{(I_2 ds_2) \times e_{12}}{r_{12}^2} \qquad \mathbb{B}(1) = \int_{C_2} d\mathbb{B}(1)$$

FROM THE ABOVE, WE SEE THAT MAGNETIC FIELDS OBEY THE PRINCIPLE OF SUPERPOSITION!

$$\mathbb{B}(1) = \sum_{i=2}^{N} \mathbb{B}_{1i}$$

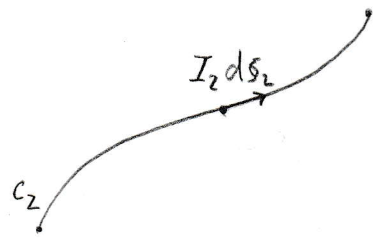
$\mathbb{B}_{1i}$: THE MAGNETIC FIELD AT (1) DUE TO SOURCE i



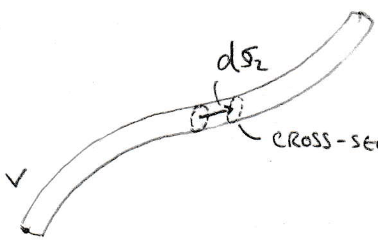
NOTE: THIS MIGHT NOT BE TOO SURPRISING, SINCE WITH THE MAGNETIC FIELD WE ARE STILL TALKING ABOUT FORCES

DRAWING #2: BIOT--SAVART LAW IN MULTIPLE DIMENSIONS

THE BIOT--SAVART LAW AS WE HAVE BEEN DISCUSSING WORKS WELL WHEN THE CURRENT CAN BE APPROXIMATED AS RUNNING THROUGH AN INFINITELY-NARROW WIRE:



IMAGINE NOW THAT OUR CONDUCTOR HAS SOME THICKNESS (i.e., IT IS EXTENDED OVER SOME VOLUME V):



OVER A SMALL DISTANCE ds_2, WE CAN WRITE:

$$
\begin{aligned}
I_2 ds_2 &= (J_2 A) ds_2 \\
&= J_2 (A ds_2) \\
&= J_2 dV_2
\end{aligned}
$$

NOTE: RECALL THAT THE ELECTRIC CURRENT DENSITY IS THE CURRENT THROUGH A SURFACE AT A RIGHT ANGLE TO THE FLOW

NOTE: ON THE LEFT-HAND SIDE, THE DIRECTION OF THE CURRENT WAS GIVEN BY ds_2; ON THE RIGHT THIS IS INDICATED IN J.

WE CAN THEREFORE WRITE THE BIOT--SAVART LAW AS:

$$\mathbb{B}(1) = \frac{\mu_0}{4\pi} \int_{C_2} \frac{(I_2 \, d\mathbb{s}_2) \times \mathbb{e}_{12}}{r_{12}^2} \quad (1D)$$

$$\mathbb{B}(1) = \frac{\mu_0}{4\pi} \iiint_{V_2} \frac{(\mathbb{J} \, dV) \times \mathbb{e}_{12}}{r_{12}^2} \quad (3D)$$

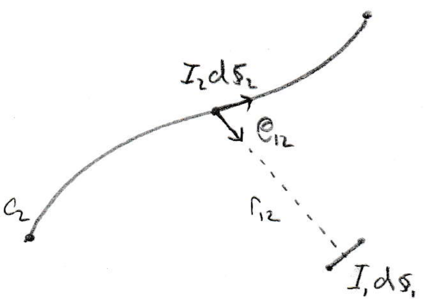
NOTE: THERE IS AN ANALOGOUS EXPRESSION FOR 2D

JUST AS FINDING THE ELECTRIC FIELD IN ELECTROSTATICS REQUIRED EVALUATING A (COMPLICATED) INTEGRAL EXPRESSION, FINDING THE MAGNETIC FIELD IN MAGNETOSTATICS REQUIRES EVALUATING A (COMPLICATED) INTEGRAL EXPRESSION...

... LATER WE'LL FIND WAYS TO EASILY SOLVE THESE INTEGRALS IN SPECIAL CIRCUMSTANCES (JUST AS WE DID WITH GAUSS'S LAW IN ELECTROSTATICS).

DRAWING #4: THE "MAGNETIC" FIELD

WE HAVE BEEN CALLING THIS "NEW" FIELD THAT DESCRIBES THE AMOUNT OF FORCE ($d\mathbb{F}$) ON A TEST WIRE ($I_1 d\mathbf{s}_1$) IN THE SPACE AROUND A CURRENT-CARRYING WIRE ($I_2 d\mathbf{s}_2$) BY THE MAGNETIC FIELD $\mathbb{B}$!



$$d\mathbb{F} = (I_1 d\mathbf{s}_1) \times \left[\frac{\mu_0}{4\pi} \int_{C_2} \frac{(I_2 d\mathbf{s}_2) \times \mathbf{e}_{12}}{r_{12}^2} \right]$$

$$= (I_1 d\mathbf{s}_1) \times \mathbb{B}(1)$$

WHERE:

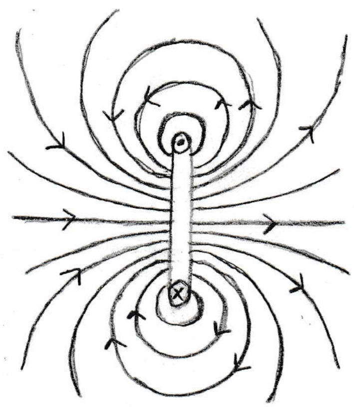
$$\mathbb{B}(1) = \frac{\mu_0}{4\pi} \int_{C_2} \frac{(I_2 d\mathbf{s}_2) \times \mathbf{e}_{12}}{r_{12}^2}$$

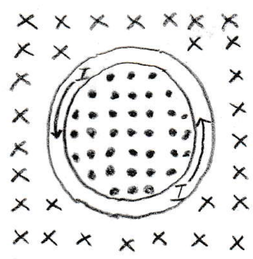
HOWEVER, THE CONNECTION BETWEEN THE MAGNETIC FIELD AND MAGNETISM IS NOT YET CLEAR...

... WE NOW WISH TO MAKE THIS CONNECTION ...

DRAWING #5: CURRENT LOOPS

CONSIDER THE MAGNETIC FIELD AROUND AN IDEAL CURRENT LOOP:





NOTE: DETERMINING THESE MAPPINGS REQUIRES NUMERICAL INTEGRATION

NOTE: RECALL OUR NOTATIONS FOR VECTORS IN 3D

NOTE: UNLIKE ELECTRIC-FIELD LINES WHICH BEGIN AT POSITIVE POINT CHARGES AND END AT NEGATIVE POINT CHARGES, MAGNETIC FIELD LINES DO NOT BEGIN OR END; THEY OFTEN FORM CLOSED LOOPS.

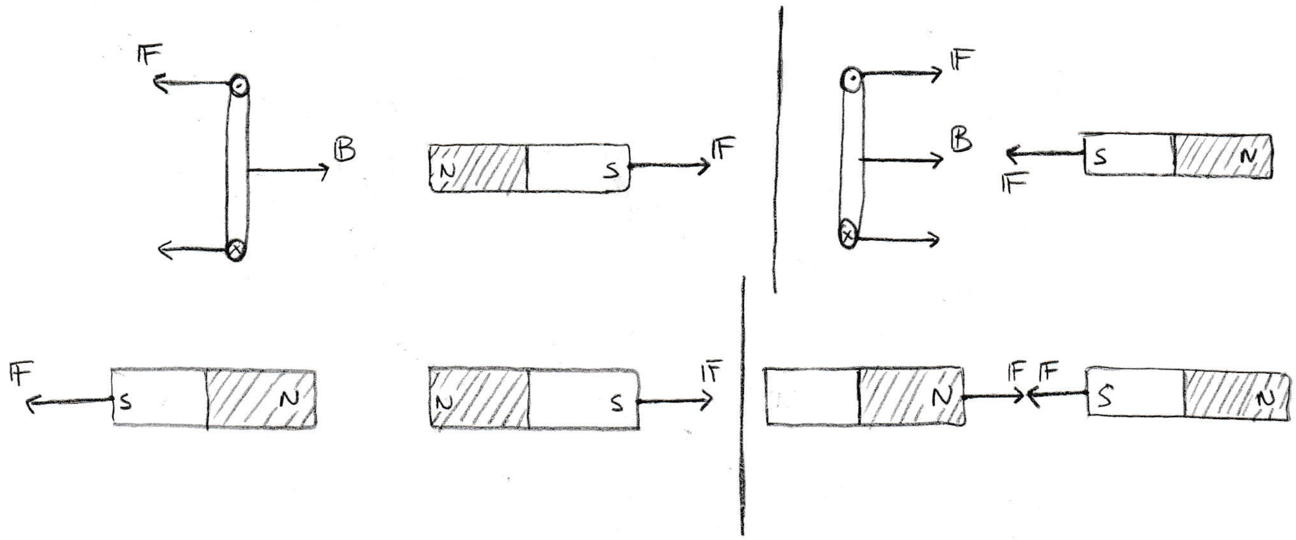
IT APPEARS THAT THE MAGNETIC FIELD LEAVES ONE SIDE OF THE LOOP, FLOWS AROUND THE OUTSIDE, AND RETURNS TO THE LOOP...

... i.e., IT APPEARS THAT THERE ARE TWO DISTINCT SIDES TO THE CURRENT LOOP.

DRAWING #6: CURRENT LOOPS AS PERMANANT MAGNETS

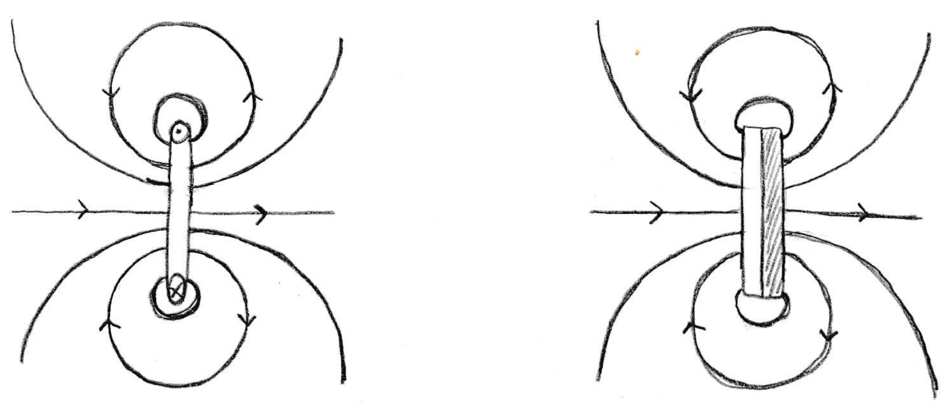
RECALL FROM ØRSTEDS EXPERIMENTS WITH CURRENT-CARRYING WIRES AND COMPASS NEEDLES (BAR MAGNETS) THAT THE DIRECTION OF CIRCULATION OF THE FIELD (THE MAGNETIC FIELD) IS BY CONVENTION THAT IN WHICH THE NORTH POLE OF A COMPASS (MAGNET) POINTS.

WE MIGHT THEREFORE EXPECT IDEAL CURRENT LOOPS TO INTERACT WITH PERMANANT MAGNETS, JUST LIKE OTHER PERMANANT MAGNETS!



IN FACT, CURRENT LOOPS ACT IN EVERY WAY LIKE PERMANANT MAGNETS (THEY ALSO ALIGN WITH THE MAGNETIC FIELD OF THE EARTH, LIKE A COMPASS, ETC.)

THEY THEREFORE GENERATE THE SAME MAGNETIC FIELD:



HENCE THE DESCRIPTION AS THE <u>MAGNETIC FIELD</u>.

NOTE: A MAGNET CREATED BY A CURRENT IN A COIL IS CALLED AN <u>ELECTROMAGNET</u>.