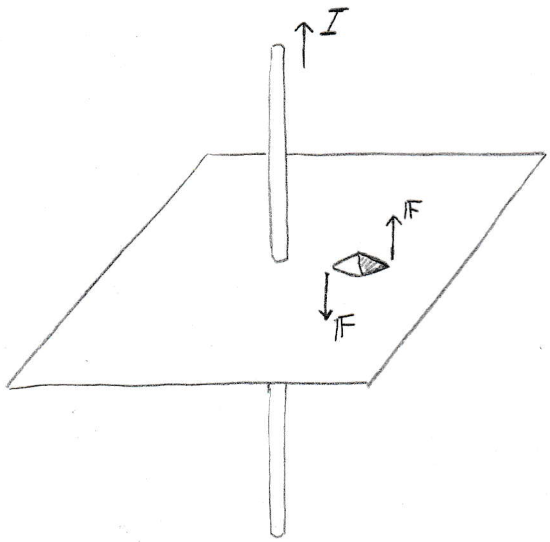
DRAWING #1: THE CONNECTION BETWEEN ELECTRICITY AND MAGNETISM

THE MAGNETIC FIELD

RECALL THE DISCOVERY BY HANS CHRISTIAN ØRSTED SHOWING THAT ELECTRICITY AND MAGNETISM ARE RELATED:



(NOTE THE DEFLECTION OF THE COMPASS NEEDLE IN THE PRESENCE OF A CURRENT)

THIS MEANS THAT:

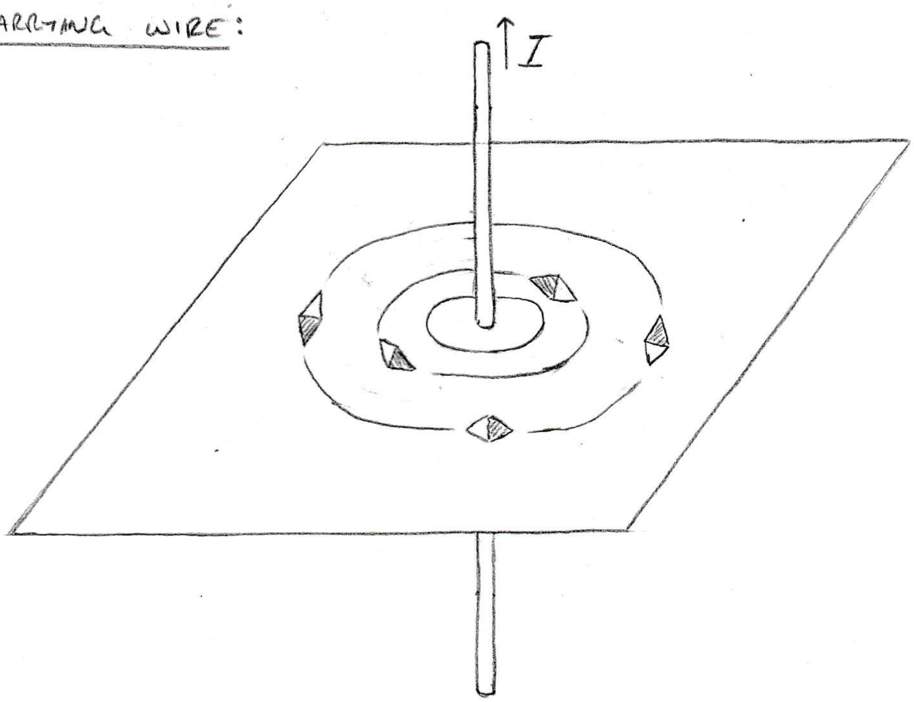
<u>MAGNETISM IS CAUSED BY AN ELECTRIC CURRENT I</u>

DRAWING #2: COMPASS AROUND CURRENT CARRYING WIRES

FOLLOWING ØRSTED'S DISCOVERY, HE CARRIED OUT A SERIES OF FURTHER EXPERIMENTS. (QUESTION TO SELF: DID AMPÈRE DO THESE TOO?)

HIS FIRST CONSISTED OF USING A SMALL COMPASS TO "MAP" OUT THE FORCE (OR TORQUE) THAT IT EXPERIENCES IN A THE PRESENCE OF A CURRENT-CARRYING WIRE...

...HE FOUND THAT A (COMPASS) NEEDLE MAPS OUT A SERIES OF CONCENTRIC LOOPS IN THE PLANE PERPENDICULAR TO A CURRENT CARRYING WIRE:



NOTE: THE DIRECTION OF CIRCULATION IS BY CONVENTION TAKEN TO BE THE DIRECTION IN WHICH THE NORTH POLE OF A COMPASS NEEDLE POINTS;

THIS CAN BE FOUND USING THE RIGHT-HAND (SCREW) RULE (THUMB IN THE DIRECTION OF THE CURRENT).

USED TO REPRESENT THE ROTATION OF SOMETHING

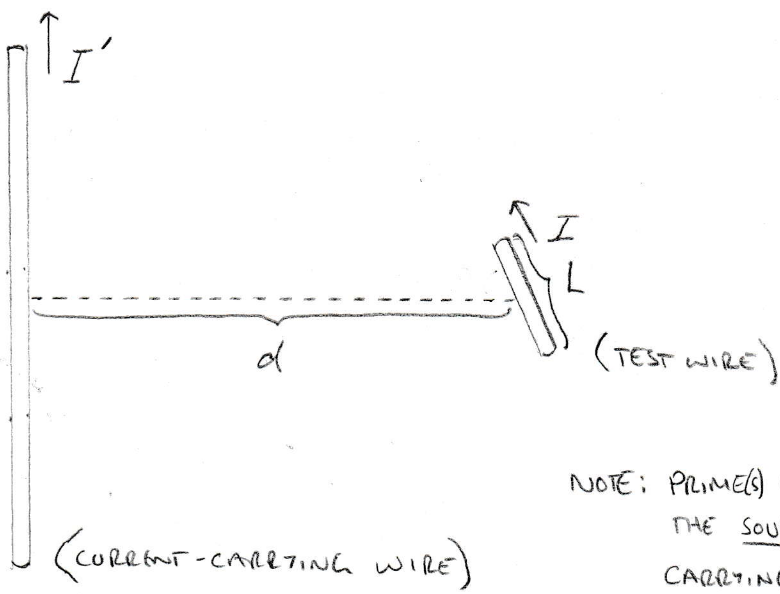
ØRSTED ALSO FOUND:

- THE DIRECTION OF THE CIRCULATION IS DEPENDENT ON THE DIRECTION OF ELECTRICAL CURRENT
- THE STRENGTH OF THIS EFFECT DIMINISHED WITH DISTANCE FROM THE WIRE

DRAWING #3: THE FORCE BETWEEN CURRENT-CARRYING WIRES I

ANDRÉ-MARIE AMPÈRE GOT INVOLVED BY STUDYING THE INTERACTION BETWEEN TWO CURRENT-CARRYING WIRES.

IMAGINE A (LARGE) CURRENT-CARRYING WIRE ORIENTED ALONG SOME AXIS, AND A (SMALL) CURRENT-CARRYING TEST WIRE IS BROUGHT NEAR:



NOTE: PRIME(S) ARE USED ON THE SOURCE (CURRENT-CARRYING) WIRE

AMPÈRE FOUND THAT A FORCE WAS EXERTED ON THE TEST WIRE (AND, OF COURSE, ON THE FIRST WIRE)...

... AND HE MADE THE FOLLOWING OBSERVATIONS:

- THE MAGNITUDE OF THE FORCE EXERTED ON THE TEST WIRE IS DIRECTLY PROPORTIONAL TO ITS LENGTH:

$$F \propto L$$

- THE MAGNITUDE OF THE FORCE IS PROPORTIONAL TO THE PRODUCT OF THE CURRENTS:

$$F \propto I'I$$

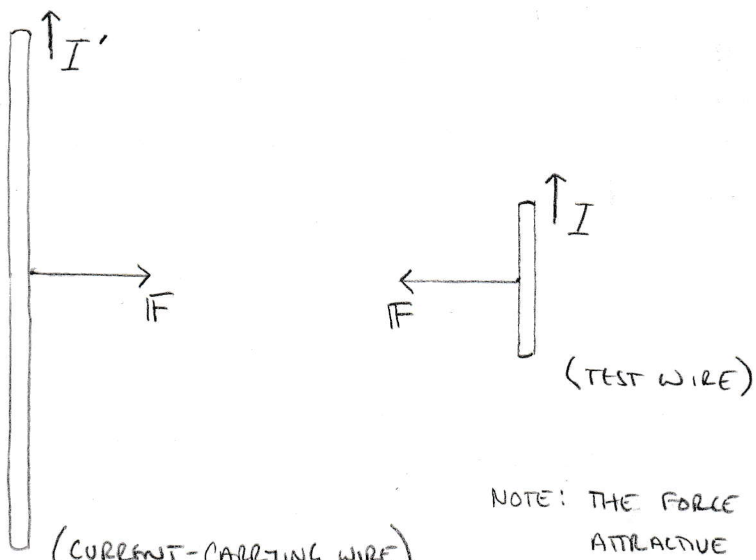
- FOR PARALLEL WIRES, SEPARATED BY d:

$$F \propto 1/d$$

- THE FORCE HAS A VERY STRANGE DIRECTIONAL CHARACTERISTIC...

DRAWING #4: THE FORCE BETWEEN CURRENT-CARRYING WIRES 2

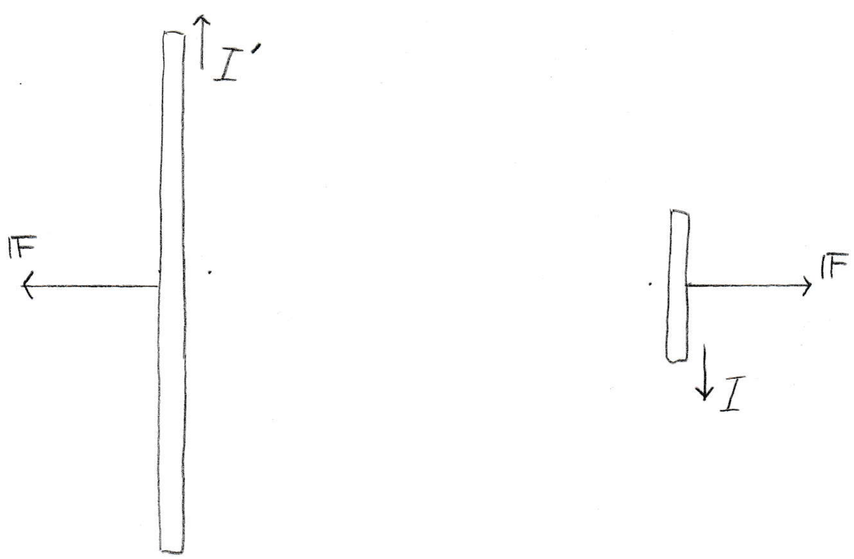
IF THE CURRENT IN THE TEST WIRE FLOWS PARALLEL TO THE CURRENT-CARRYING WIRE, THEN THE FORCE IS ATTRACTIVE:



NOTE: THE FORCE WOULD REMAIN ATTRACTIVE IF BOTH CURRENTS WERE REVERSED...

... IN THE FOLLOWING, THE CURRENT-CARRYING WIRE REMAINS FIXED

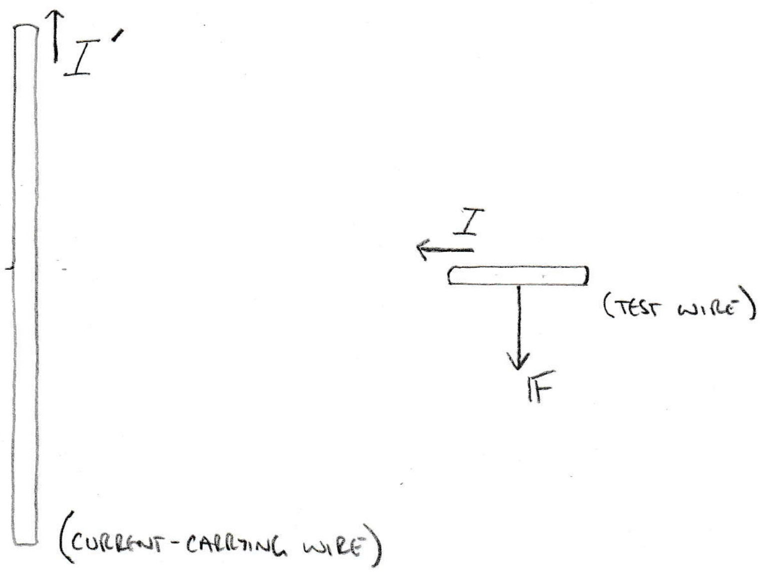
IF THE CURRENT IN THE TEST WIRE IS REVERSED, THE TWO WIRES REPEL:



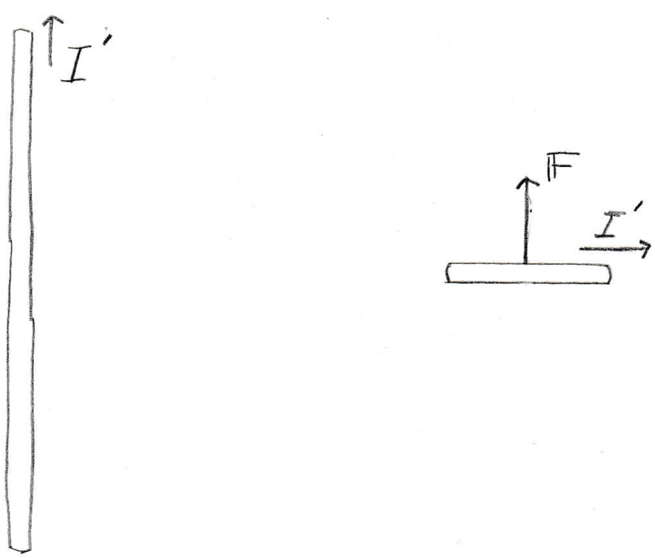
THIS ALL ISN'T TOO STRANGE (PERHAPS), AND WE MIGHT SAY THAT "LIKE" CURRENTS ATTRACT AND "OPPOSITE" CURRENTS REPEL, BUT...

DRAWING #5: THE FORCE BETWEEN CURRENT-CARRYING WIRES 3

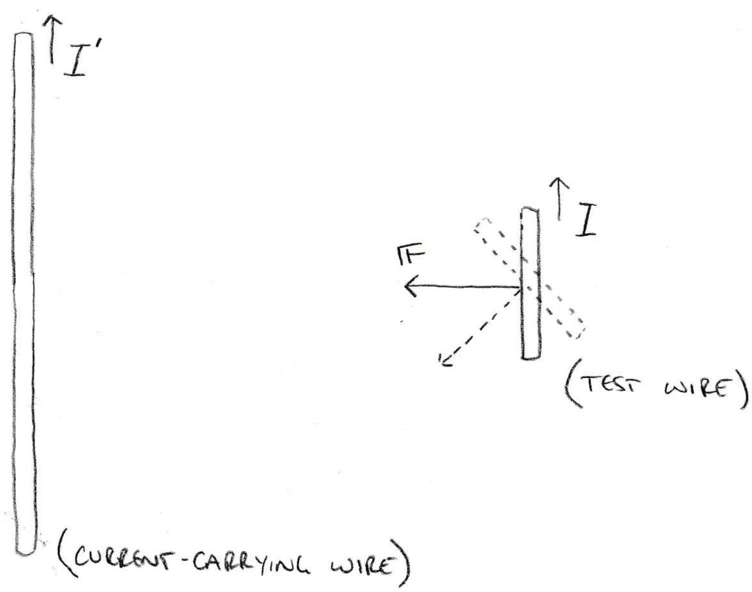
IF THE TEST WIRE CURRENT POINTS RADIALLY INWARD TOWARDS THE CURRENT-CARRYING WIRE, THEN THE TEST WIRE EXPERIENCES A DOWNWARD FORCE:



LIKEWISE, IF THE TEST WIRE CURRENT POINTS RADIALLY OUTWARDS, THE TEST WIRE EXPERIENCES AN UPWARD FORCE:



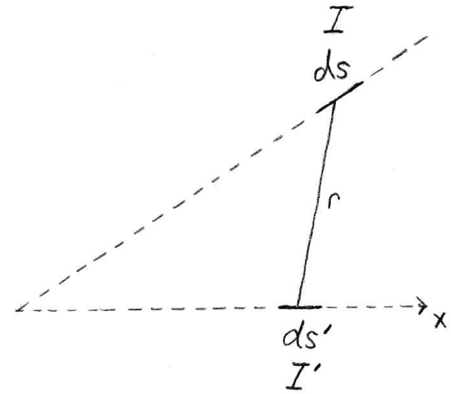
GENERALIZING THESE RESULTS: IF THE TEST CURRENT IS IN THE PLANE OF THE CURRENT-CARRYING WIRE, AND ROTATED, THE FORCE EXERTED ON IT IS ALWAYS OF CONSTANT MAGNITUDE AND AT A RIGHT ANGLE TO THE TEST CURRENT.



THERE ARE SEVERAL EXPRESSIONS FOR THE FORCE CONSISTENT (THOUGH NOT NECESSARILY COMPLETE) CONSISTENT WITH THE ORIGINAL EXPERIMENTS.

EXAMPLE!

THE X-COMPONENT OF THE FORCE BETWEEN TWO LINEAR CURRENTS I AND I WAS GIVEN BY AMPÈRE (IN 1825) AND GAUSS (FROM GAUSS'S LAW; IN 1833):



$$dF_x = K_A I I' ds' \int ds \frac{\cos(x\,ds)\cos(r\,ds') - \cos(rx)\cos(ds\,ds')}{r^2}$$

NOTE: THIS EXPRESSION IS CONSISTENT WITH THE PRIOR OBSERVATIONS:

- $F \propto L \qquad \left(L = \int ds\right)$
- $F \propto I I'$
- STRANGE DIRECTIONAL CHARACTER OF $\mathbb{F}$

... THOUGH IT IS NOT CLEAR THAT $F \propto 1/r$ FOR PARALLEL WIRES (TO COME)

NOTE: K_A IS A PROPORTIONALITY CONSTANT, CALLED THE MAGNETIC FORCE CONSTANT:

$$K_A \stackrel{\text{def}}{=} \frac{\mu_0}{4\pi}$$

(NOTE: $\mu_0/4\pi$ WILL BE USED FROM HERE ON OUT)

WHERE μ_0 IS THE MAGNETIC CONSTANT, ALSO DEFINED:

$$\mu_0 = 4\pi \cdot 10^{-7}\ N/A^2$$

(THESE DEFINITIONS ARISE FROM DECIDING LARGE THE UNIT OF CURRENT (A) SHOULD BE:

THE FORCE PER UNIT LENGTH BETWEEN TWO PARALLEL CONDUCTORS, SPACED BY 1M, EACH CARRYING A CURRENT OF 1A IS:

$$F = 2 \cdot 10^{-7}\ N/M \quad \text{(EXACTLY)}$$
)

DRAWING #8: AMPÈRE'S FORCE LAW 2

THE FORCE LAW DEVELOPED BY AMPÈRE AND GAUSS IS NOT <u>GENERAL</u>:

- IT ONLY DESCRIBES ONE COMPONENT OF THE FORCE (x)
- IT IS NOT IN TERMS OF <u>INFITESIMAL CURRENT ELEMENTS</u> (JUST ds')) (IT ASSUMES LINEAR CURRENT ELEMENTS)

... THAT IS, IT IS NOT A "FUNDAMENTAL" EXPRESSION OF THE FORCE

... NOT TO MENTION THAT IT IS NOT CLEAR INTUITIVELY WHAT IS BEING DESCRIBED

NOTE: THE FIRST POINT IS THE CASE, AS VECTORS HAD ONLY APPEARED AT THE BEGINNING OF THE 19TH CENTURY; AND VECTOR CALCULUS CLOSER TO THE END!

THE COMMON FORM OF THE FORCE LAW BETWEEN TWO CURRENT-CARRYING WIRES, CALLED <u>AMPÈRE'S FORCE LAW</u>, IS THAT DERIVED BY JAMES CLERK MAXWELL:

NOTE: SLIGHTLY ROTATED OUT OF THE PLANE

$$d^2 F = -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r_{12}^2}\left[(ds_1 \cdot ds_2)e_{12} - (e_{12}\cdot ds_2)ds_1 - (e_{12}\cdot ds_1)ds_2\right]$$

NOTE: $d^2 f$ IS THE <u>SECOND-ORDER DIFFERENTIAL</u> OF f;

$\left(\frac{\partial^2 f}{\partial x \partial y}\right)$ IS THE <u>MIXED PARTIAL DERIVATIVE</u> OF $f(x,y)$.

NOTE: THIS EXPRESSION SHOWS THAT THERE MAY BE COMPONENTS OF THE FORCE ALONG EACH OF THE DIRECTIONS e_{12}, ds_1, AND ds_2, DEPENDING ON THEIR GEOMETRIC RELATIONSHIP.

DRAWING #9: AMPÈRE'S FORCE LAW 3

FROM VECTOR CALCULUS, THE VECTOR TRIPLE PRODUCT IDENTITY:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$$

(MNEMONIC: "BAC - CAB")

MAXWELL'S EXPRESSION ...

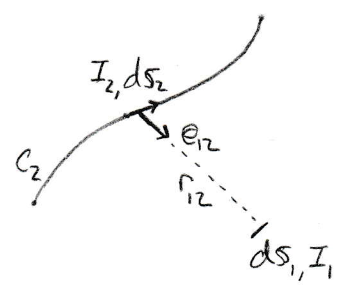
$$d^2\mathbf{F} = -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r_{12}^2}\left[(d\mathbf{s}_1 \cdot d\mathbf{s}_2)\mathbf{e}_{12} - (\mathbf{e}_{12} \cdot d\mathbf{s}_2)d\mathbf{s}_1 - (\mathbf{e}_{12} \cdot d\mathbf{s}_1)d\mathbf{s}_2\right]$$

... THEREFORE CAN BE WRITTEN:

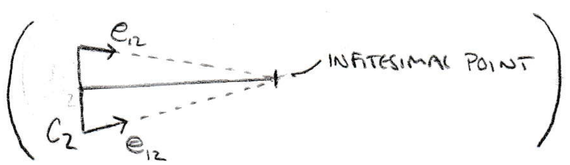
$$d^2\mathbf{F} = \frac{\mu_0}{4\pi}\frac{I_1 I_2}{r_{12}^2}\left[d\mathbf{s}_1 \times (d\mathbf{s}_2 \times \mathbf{e}_{12}) + (\mathbf{e}_{12} \cdot d\mathbf{s}_1)d\mathbf{s}_2\right]$$

WE CAN FIND THE FORCE ON A TEST WIRE $(d\mathbf{s}_1)$ FROM A CURRENT-CARRYING WIRE $(d\mathbf{s}_2)$ BY INTEGRATING THE PRIOR EXPRESSION:

$$\boxed{d\mathbf{F} = (I_1 d\mathbf{s}_1) \times \left[\frac{\mu_0}{4\pi}\int_{C_2}\frac{(I_2 d\mathbf{s}_2) \times \mathbf{e}_{12}}{r_{12}^2}\right]}$$



NOTE: $\int d\mathbf{s}_2 \, (\mathbf{e}_{12} \cdot d\mathbf{s}_1) = 0$



NOTE: THIS EXPRESSION IS VALID FOR ANY CURRENT-CARRYING WIRE WITH CURVE (SHAPE) C_2

NOTE: THIS EXPRESSION HAS BEEN PURPOSEFULLY REARRANGED, IN ANTICIPATION OF RESULTS TO COME.

NOTE: THE CURRENT I_2 MAY BE POSITION DEPENDENT, BUT IT MUST BE STEADY (NOT CHANGING IN TIME) --- IN A WIRE, IT IS HARD TO SEE HOW THAT MAY HAPPEN