ELECTROMAGNETICS

UNTIL NOW, WE HAVE BEEN CONCERNED WITH ELECTROSTATICS : THE ELECTRICAL FORCE(S) GENERATED BY AND ACTING UPON CHARGES AT REST.

NOW, WE WILL CONSIDER THE FORCE(S) GENERATED BY AND ACTING UPON CHARGES IN MOTION...

... WE'LL SEE HOW THIS RESULTS IN A "NEW" FORCE : THE MAGNETIC FORCE.

IN THE END, WE'LL UNIFY THESE TWO FORCES IN THE ELECTROMAGNETIC FORCE ; THE FORCE THAT OCCURS BETWEEN CHARGES.

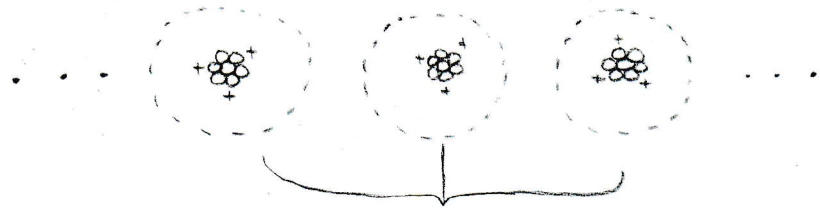
NOTE: THE ELECTROMAGNETIC FORCE IS ONE OF THE FOUR (KNOWN) FUNDAMENTAL INTERACTIONS (ALL OTHER FORCES ARE DERIVED FROM THESE).

NOTE: ELECTROMAGNETISM IS A BRANCH OF PHYSICS THAT INVOLVES THE STUDY OF THE ELECTROMAGNETIC FORCE.

DRAWING #7: TYPES OF MATTER
(FROM LECTURE #2)

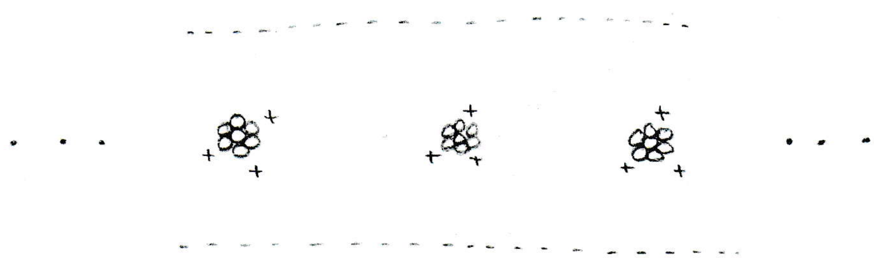
THE INTERPLAY BETWEEN MANY EFFECTS AT SMALL LENGTH SCALES GIVES RISE TO DIFFERENT TYPES OF MATTER.

INSULATORS



THE ELECTRONS ARE VERY TIGHTLY BOUND TO THE NUCLEI

CONDUCTORS



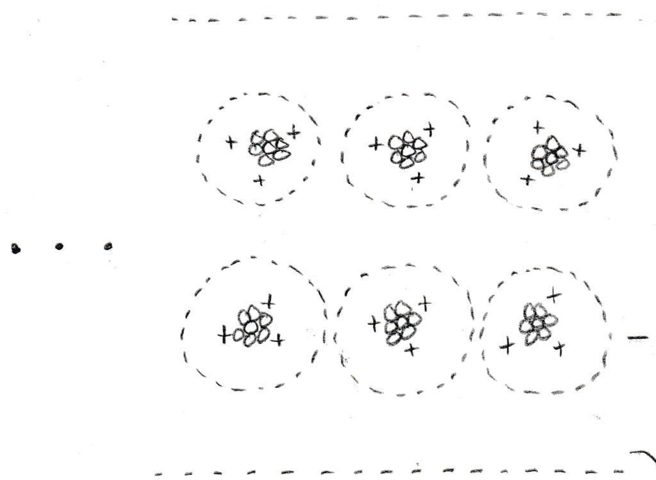
THE ELECTRONS ARE DELOCALIZED

DRAWING #2 : CONDUCTORS AND THE FREE-ELECTRON GAS

WE FIRST NEED A MODEL FOR CHARGES IN MOTION.

RECALL THAT THE INTERPLAY BETWEEN MANY TYPES OF EFFECTS AT SMALL LENGTH SCALES GIVES RISE TO DIFFERENT TYPES OF MATTER.

EXAMPLE: CONDUCTORS



THE FREE-ELECTRON GAS

IN CONDUCTORS, TWO REASONABLE APPROXIMATIONS ARE:

- THE VALENCE ELECTRONS ARE COMPLETELY DETACHED FROM THEIR NUCLEI
- ELECTRON -- ELECTRON INTERACTIONS ARE IGNORED (AS IN AN IDEAL GAS)

UNDER THESE ASSUMPTIONS, THE VALENCE ELECTRONS FORM A FREE-ELECTRON GAS

NOTE: THE FREE-ELECTRON GAS DOES NOT EXPLICITLY CONSIDER THE NUCLEI. (CRYSTAL LATTICE)

NOTE: WE HAVE SEEN IN ELECTROSTATICS, FOR EXAMPLE, THAT CHARGE IS ABLE TO MOVE FREELY IN CONDUCTORS, DUE TO THE DELOCALIZATION OF THE VALENCE ELECTRONS.

DRAWING #3: DRUDE MODEL OF ELECTRON CONDUCTION I

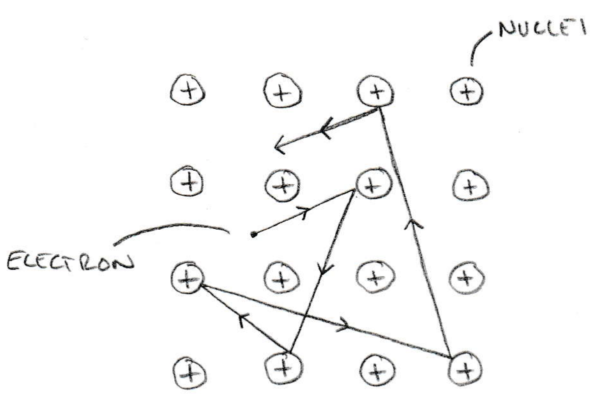
PAUL DRUDE PRESENTED THE FIRST (CLASSICAL) THEORY TO DESCRIBE THE TRASPORT PROPERTIES OF ELECTRONS IN MATERIALS: <u>THE DRUDE MODEL OF ELECTRON CONDUCTION.</u>

THE STARTING POINT OF THE <u>DRUDE MODEL</u> IS THAT OF AN IDEAL GAS OF ELECTRONS, FREE TO MOVE BETWEEN POSITIVELY-CHARGED IONIC CORES.

THE ASSUMPTIONS OF THE MODEL ARE THEN:

- A COLLISION INDICATES THE SCATTERING OF AN ELECTRON BY (AND ONLY BY) A NUCLEI
- ELECTRON -- ELECTRON INTERACTIONS ARE IGNORED
- COLLISIONS ARE INSTANTANEOUS, AND RESULT IN A CHANGE IN VELOCITY
- AN ELECTRON SUFFERS A COLLISION PER UNIT TIME τ^{-1} (THE <u>SCATTERING RATE</u>)
- ELECTRONS ACHIEVE THERMAL EQUILIBRIUM WITH THEIR SURROUNDINGS ONLY THROUGH COLLISIONS.

WHAT THIS LOOKS LIKE:



NOTE: THERE IS NO NET DISPLACEMENT OF THE ELECTRONS

THE ELECTRON DYNAMICS BETWEEN TWO TIMES $t = t_0$ AND $t = t_0 + dt$ CAN BE DESCRIBED AS FOLLOWS:

ON AVERAGE, A FRACTION $(1 - dt/\tau)$ OF THE ELECTRONS WILL NOT HAVE EXPERIENCED A COLLISION; AND THE ONES THAT HAVE WILL TRANSFER THEIR MOMENTUM TO THE NUCLEI, AND CONTRIBUTE TO THE TOTAL (ELECTRON) MOMENTUM TO ONLY A NEGLIGIBLE ORDER.

THEREFORE:

$$\langle \mathbb{P}(t_0+dt)\rangle = \left(1 - \frac{dt}{\tau}\right)\langle \mathbb{P}(t_0)\rangle$$

THIS EXPRESSION CAN BE REARRANGED:

$$\langle \mathbb{P}(t_0+dt)\rangle = \langle \mathbb{P}(t_0)\rangle - \frac{dt}{\tau}\langle \mathbb{P}(t_0)\rangle$$

$$\langle \mathbb{P}(t_0+dt)\rangle - \langle \mathbb{P}(t_0)\rangle = -\frac{dt}{\tau}\langle \mathbb{P}(t_0)\rangle$$

$$\frac{\langle \mathbb{P}(t_0+dt)\rangle - \langle \mathbb{P}(t_0)\rangle}{dt} = -\frac{\langle \mathbb{P}(t_0)\rangle}{\tau}$$

HENCE:

$$\frac{d}{dt}\langle \mathbb{P}(t)\rangle = -\frac{\langle \mathbb{P}(t)\rangle}{\tau}$$

THE SOLUTION TO THIS EQUATION IS:

$$\langle \mathbb{P}(t)\rangle = \langle \mathbb{P}(t_0)\rangle e^{-t/\tau}$$

(NOTE: THIS EQUATION IS CONSISTENT WITH THE PROBABILITY THAT AN ELECTRON HAS NOT SCATTERED AFTER A TIME t: $P(t) = e^{-t/\tau}$)

THAT IS, GIVEN AN INITIAL MOMENTUM, IT DECREASES EXPONENTIALLY VIA COLLISIONS OF ELECTRONS WITH THEIR SURROUNDINGS...

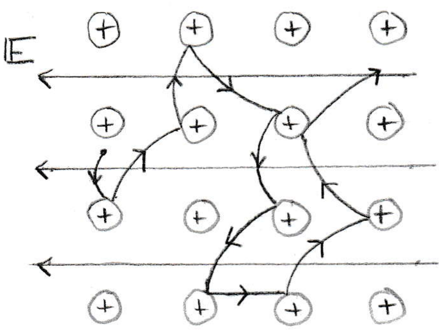
... AND FOR $t \to \infty$, THE ELECTRONS ACHIEVE THERMAL EQUILIBRIUM WITH THEIR SURROUNDINGS.

NOTE: THIS ALL IMPLIES NO NET DISPLACEMENT OF THE ELECTRONS.

DRAWING #5: DRUDE MODEL OF ELECTRON CONDUCTION 3

CONSIDER NOW THE APPLICATION OF AN ELECTRIC FIELD...

... THE COLLISIONS WITH NUCLEI (RANDOM THERMAL MOTION) WILL STILL OCCUR, BUT THE ELECTRICAL FORCE WILL CAUSE A NET DISPLACEMENT (IN THE DIRECTION OPPOSITE TO THE FIELD).



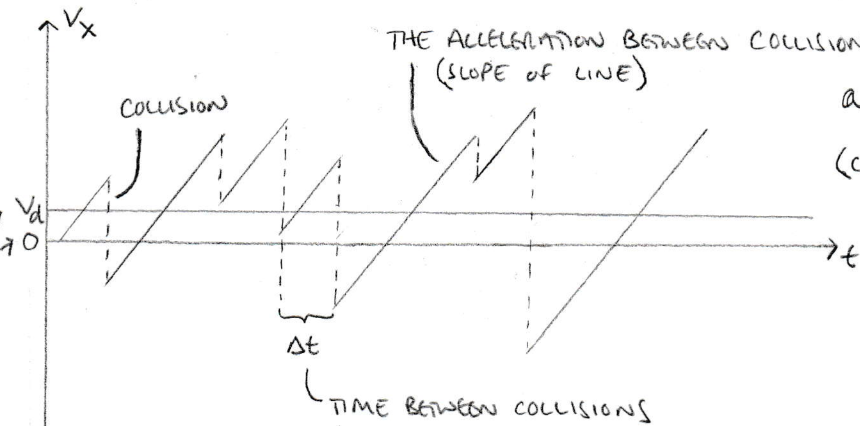
NOTE: THE "PULL" TO THE RIGHT CAUSES PARABOLIC TRAJECTORIES

THE NET DISPLACEMENT CAN BE SEEN BY LOOKING AT THE VELOCITY AS A FUNCTION OF TIME:

THE ACCELERATION BETWEEN COLLISIONS IS:
(SLOPE OF LINE)

$$a = \mathbb{F}/m = \frac{-e\mathbb{E}}{m_e}$$

(CONSTANT)



TIME BETWEEN COLLISIONS (THE MEAN TIME IS τ)

THE AVERAGE REBOUND VELOCITY IS ZERO...

... WHILE THE AVERAGE VELOCITY IS NONZERO

IN THE PRESENCE OF AN EXTERNAL ELECTRIC FIELD (ELECTRICAL FORCE) THE ELECTRON DYNAMICS CAN BE DESCRIBED BY:

$$\langle \mathbb{P}(t_0+dt)\rangle = \left(1-\frac{dt}{\tau}\right)\left(\langle \mathbb{P}(t_0)\rangle - e\mathbb{E}\,dt\right) \qquad \left(\begin{matrix}\mathbb{F}=q\mathbb{E}\\ d\mathbb{P}=\mathbb{F}dt\end{matrix}\right)$$

REARRANGING THIS EXPRESSION, AND DROPPING THE TERMS OF ORDER dt^2:

$$\frac{d}{dt}\langle \mathbb{P}(t)\rangle = -\frac{\langle \mathbb{P}(t)\rangle}{\tau} - e\mathbb{E}$$

THIS EQUATION MAY BE SOLVED TO GIVE:

$$\langle \mathbb{P}(t)\rangle = \langle \mathbb{P}(t_0)\rangle e^{-t/\tau} - e\tau\mathbb{E}\left(1-e^{-t/\tau}\right)$$

OF PARTICULAR INTEREST IS THE STEADY-STATE SOLUTION TO THE PRIOR EQUATION(S) $\left(\frac{d}{dt}\langle \mathbb{P}(t)\rangle = 0 \text{, OR } t\to\infty\right)$:

$$\langle \mathbb{P}\rangle = -e\tau\mathbb{E}$$

NOTE: THE AVERAGE MOMENTUM IS IN THE DIRECTION <u>OPPOSITE</u> THE ELECTRIC FIELD.

FROM WHICH WE CAN FIND THE ELECTRON'S AVERAGE VELOCITY DUE TO THE ELECTRIC FIELD, <u>THE DRIFT VELOCITY $\mathbb{V}_d$</u>:

$$\langle \mathbb{P}\rangle = m_e\langle \mathbb{V}_d\rangle = -e\tau\mathbb{E}$$

$$\langle \mathbb{V}_d\rangle = \frac{-e\tau}{m_e}\mathbb{E}$$

... WHICH HAS A MAGNITUDE CALLED THE <u>DRIFT SPEED V_d</u>:

$$V_d = \|\langle \mathbb{V}_d\rangle\| = \frac{e\tau}{m}E$$

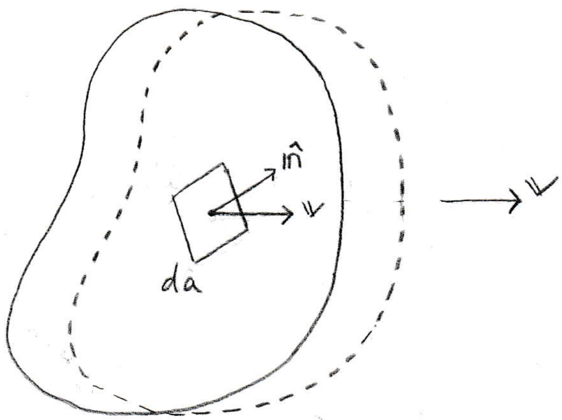
E: THE MAGNITUDE OF $\mathbb{E}$

DRAWING #7: CURRENT DENSITY

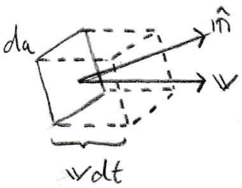
THE DRUDE MODEL DESCRIBES THE DYNAMICS OF INDIVIDUAL ELECTRONS...

... WE NOW WISH TO DESCRIBE THE DYNAMICS OF MANY ELECTRONS.

CONSIDER A COLLECTION OF ELECTRONS, DESCRIBED AS A CHARGE DISTRIBUTION WITH CHARGE DENSITY ρ. UNDER THE APPLICATION OF AN EXTERNAL ELECTRIC FIELD, EACH ELECTRON (IN THE DRUDE MODEL) WILL MOVE (DRIFT) WITH AN AVERAGE VELOCITY $\langle \mathbb{v} \rangle$; WE CAN DESCRIBE THEIR COLLECTIVE MOTION (OF ρ) WITH $\mathbb{v}$.



CONSIDER AN (IMAGINARY) AREA ELEMENT da. AFTER A TIME dt:



THE VOLUME OF THIS PARALLELEPIPED IS:

$$dV = (da)(\mathbb{v} \cdot \hat{n}\, dt)$$

THE AMOUNT OF CHARGE PASSING THROUGH da IN A TIME dt IS THEREFORE:

$$\begin{aligned} dq &= \rho\, dV \\ &= \rho (da)(\mathbb{v} \cdot \hat{n}\, dt) \end{aligned}$$

WE CAN DESCRIBE THIS MORE SUCCINCTLY BY DEFINING THE ELECTRIC CURRENT DENSITY $\mathbb{J}$: THE AMOUNT OF CHARGE PASSING PER UNIT AREA AND PER UNIT TIME THROUGH A SURFACE, AT RIGHT ANGLES TO THE FLOW:

(NOTE: AN ELECTRIC CURRENT IS JUST A FLOW OF CHARGE --- WILL BE DEFINED BELOW)

$$\frac{dq}{dt} = \mathbb{J} \cdot \hat{n}\, da$$

COMPARING WITH OUR PRIOR EXPRESSION (REARRANGED):

$$\frac{dq}{dt} = \rho \mathbb{v} \cdot \hat{n}\, da$$

WE CAN SEE:

$$\mathbb{J} = \rho \mathbb{v}$$

NOTE: IF ρ CONSISTS OF INDIVIDUAL CHARGES q (e.g., ELECTRONS; $q = -e$), IT CAN BE USEFUL TO REWRITE ρ AS:

$$\rho = n q$$

WHERE n IS THE NUMBER DENSITY OF THE CHARGES (NUMBER OF CHARGES PER UNIT VOLUME).

NOTE: THE NUMBER DENSITY OF CONDUCTION ELECTRONS n_e IS OFTEN TABULATED...

DRAWING #9: ELECTRIC CURRENT

THE TOTAL AMOUNT OF CHARGE PASSING THROUGH ANY SURFACE S PER UNIT TIME CAN BE FOUND VIA INTEGRATION:

$$\frac{dq}{dt} = \mathbf{J} \cdot \hat{n} \, da$$

$$\frac{dQ}{dt} = \int_S da \, \mathbf{J} \cdot \hat{n}$$

$$\left(dQ = \int da \left(dq/da \right) \right)$$

WE'RE STILL TALKING IN TERMS OF SMALL AMOUNTS

THIS IS CALLED THE ELECTRIC CURRENT I:

$$I = \frac{dQ}{dt} = \int_S da \, \mathbf{J} \cdot \hat{n}$$

(UNITS: $A = C/s$)

"AMPERE"

THE TOTAL CHARGE FLOWING THROUGH A SURFACE BETWEEN TWO TIMES t_1 AND t_2:

$$Q = \int_{t_1}^{t_2} dt \, I$$

DRAWING #10: CONSERVATION OF CHARGE

CONSIDER NOW THE CURRENT OUT OF A CLOSED SURFACE:

$$I = \frac{dQ}{dt}$$
$$= \oint_S da\, \mathbf{J} \cdot \hat{n}$$

SINCE CHARGE IS CONSERVED, THE RATE THAT CHARGE LEAVES THE SURFACE MUST BE EQUAL TO THE RATE THAT THE CHARGE INSIDE DECREASES:

$$-\frac{d}{dt} Q_{INSIDE} = \oint_S da\, \mathbf{J} \cdot \hat{n}$$

CONSIDER THE APPLICATION OF THE PRIOR EXPRESSION TO A SMALL VOLUME ELEMENT dV. IN THIS CASE:

$$Q_{INSIDE} = dq$$
$$= \rho\, dV$$

AND FROM VECTOR CALCULUS:

$$\oint_S da\, \mathbf{J} \cdot \hat{n} = \nabla \cdot \mathbf{J}\, dV \qquad \left(\oint_S da\, \mathbf{A} \cdot \hat{n} = \nabla \cdot \mathbf{A}\, dV \right)$$

HENCE:

$$-\frac{d}{dt} \rho\, dV = \nabla \cdot \mathbf{J}\, dV$$

$$\boxed{-\frac{d\rho}{dt} = \nabla \cdot \mathbf{J}}$$

THIS SAYS THAT THE VOLUME DENSITY OF OUTWARD FLUX (FLOW) OF $\mathbf{J}$ (ELECTRIC CURRENT DENSITY) FROM AN INFITESIMAL VOLUME AROUND A GIVEN POINT IS EQUAL TO THE RATE AT WHICH CHARGE DENSITY IS LEAVING (THE SAME REGION)...

... THIS IS JUST ANOTHER WAY OF STATING THE LAW OF CONSERVATION OF CHARGE.