ELECTRIC POTENTIAL

THE ELECTRIC POTENTIAL DESCRIBES THE AMOUNT OF ELECTRICAL POTENTIAL ENERGY THAT A CHARGE WOULD HAVE IF PLACED AT ANY POINT IN SPACE --- i.e., IT IS A PROPERTY OF THE ELECTRIC FIELD ITSELF.

THE IDEA OF ELECTRIC POTENTIAL WILL ALLOW US TO:

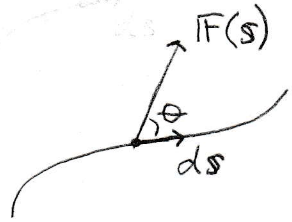
- WRITE THE ELECTRIC FIELD (A VECTOR) IN TERMS OF A SCALAR
- DERIVE OUR SECOND (FINAL) FUNDAMENTAL FIELD EQUATION OF ELECTROSTATICS

DRAWING #2: WORK

THE ELECTRIC POTENTIAL (AND ELECTRIC POTENTIAL ENERGY) IS RELATED TO THE WORK DONE ON A POINT CHARGE AS IT MOVES IN THE PRESENCE OF AN ELECTRICAL FORCE(S).

RECALL THAT <u>WORK W</u> IS THE RESULT OF A FORCE $\mathbb{F}$ ON A POINT THAT MOVES THROUGH A DISTANCE.

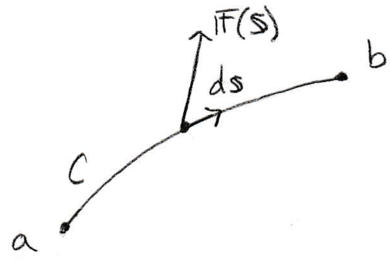
OVER A SMALL DISTANCE ds, THE AMOUNT OF WORK dW DONE <u>BY</u> A FORCE $\mathbb{F}(s)$:



ds: INFITESIMAL DISPLACEMENT VECTOR ALONG PATH

$$dW = \mathbb{F}(s)\cdot ds$$
$$= F(s)\cos\theta\, ds$$

THEREFORE, THE TOTAL AMOUNT OF WORK DONE <u>BY</u> A FORCE OVER THE TRAJECTORY C OF THE POINT IS:



$$W = \int_C dW$$
$$= \int_C \mathbb{F}(s)\cdot ds$$

NOTE: THIS INTEGRAL IMPLIES THAT THE AMOUNT OF WORK DONE <u>MAY</u> DEPEND ON THE PATH TAKEN.

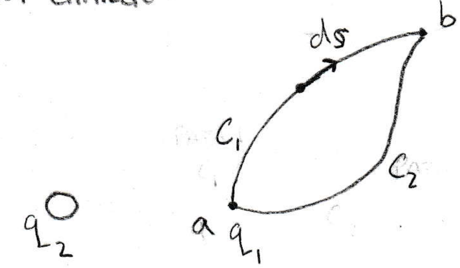
SOMETIMES, WE'RE INTERESTED IN THE WORK THAT MUST BE DONE <u>AGAINST</u> A FORCE:

$$W = \int_C (-\mathbb{F}(s))\cdot ds$$
$$= -\int_C \mathbb{F}(s)\cdot ds$$

DRAWING #3: THE WORK DONE BY (THE ELECTRIC FORCE OF) A POINT CHARGE I

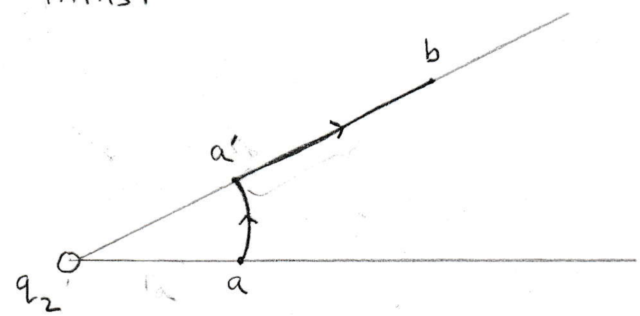
LET'S CONSIDER THE WORK DONE BY THE ELECTRICAL FORCE ON A POINT CHARGE.

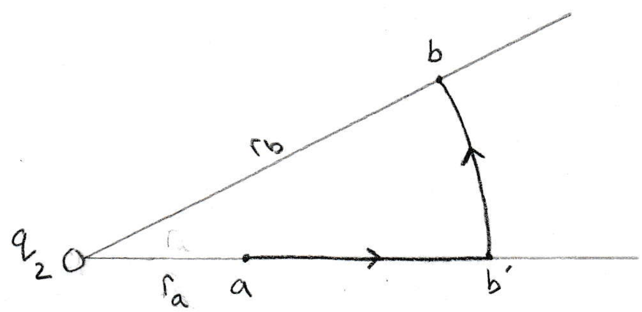
CONSIDER A POINT CHARGE THAT MOVES IN THE ELECTRIC FIELD OF (ANOTHER) POINT CHARGE:



WE WISH TO KNOW WHETHER WORK DONE ON THE PARTICLE DEPENDS ON THE PATH TAKEN.

CONSIDER TWO PATHS:





SINCE THE ELECTRIC FORCE IS RADIAL:

- THERE IS NO WORK DONE ON THE PATHS $a \to a'$ OR $b' \to b$:

$$W = \int_a^{a'} \mathbb{F}(s) \cdot ds = 0 \qquad \left(\mathbb{F}(s) \cdot ds = 0 \right)$$

- THE SAME AMOUNT OF WORK IS DONE ON THE PATHS $a \to b'$ OR $a' \to b$:

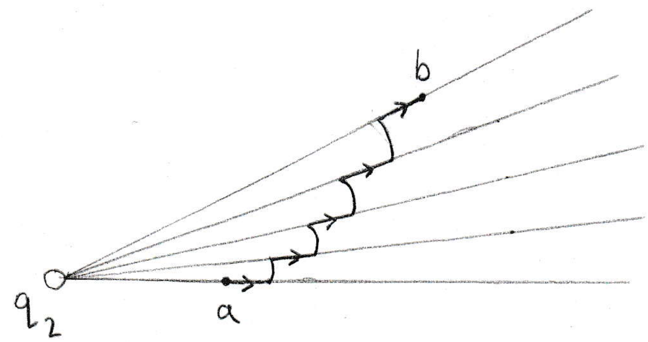
$$W = \int_a^{b'} \mathbb{F}(s) \cdot ds = \int_{a'}^{b} \mathbb{F}(s) \cdot ds$$

$$= \int_a^{b'} \left(\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \right) dr = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \int_a^{b'} dr \frac{1}{r^2}$$

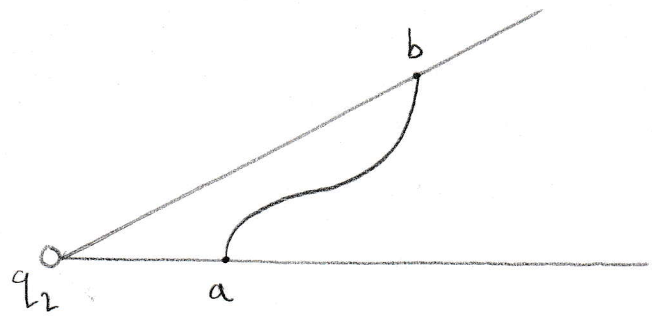
$$= \frac{1}{4\pi\varepsilon_0} q_1 q_2 \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$$

DRAWING #4: THE WORK DONE BY (THE ELECTRICAL FORCE OF) A POINT CHARGE 2

THE PRIOR RESULT CAN BE EXTENDED FOR ANY PATH a→b:



IN THE LIMIT OF INFITESIMAL DISPLACEMENTS (A SMOOTH PATH):



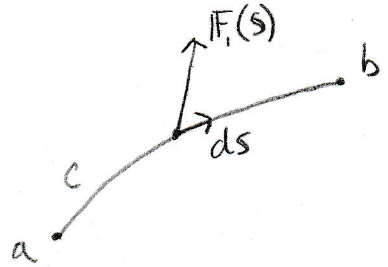
THEREFORE:

<u>THE WORK DONE BY THE ELECTRICAL FORCE OF A POINT CHARGE ON (ANOTHER) POINT CHARGE AS IT MOVES BETWEEN TWO END POINTS DOES NOT DEPEND ON THE PATH TAKEN.</u>

$$W = \int_{\substack{a \\ \text{ANY PATH}}}^{b} \mathbb{F}(s) \cdot ds$$

DRAWING #5: THE WORK DONE BY (THE ELECTRICAL FORCE OF) MULTIPLE POINT CHARGES

CONSIDER A POINT CHARGE (1) THAT MOVES IN THE PRESENCE OF MULTIPLE POINT CHARGES:



$\mathbb{F}_1$: THE TOTAL FORCE ON (1)

THE WORK DONE ON (1) IS:

$$W = \int_{a \text{ (ANY PATH)}}^{b} \mathbb{F}_1(s) \cdot ds$$

BECAUSE ELECTRICAL FORCES OBEY THE SUPERPOSITION PRINCIPLE:

$$\mathbb{F}_1(s) = \sum_{i=2}^{N} \mathbb{F}_{1i}(s)$$

WE CAN WRITE:

$$\begin{aligned} W_1 &= \int_{a \text{ (ANY PATH)}}^{b} \mathbb{F}_1(s) \cdot ds \\ &= \int_{a \text{ (ANY PATH)}}^{b} \left(\sum_{i=2}^{N} \mathbb{F}_{1i}(s) \right) \cdot ds \\ &= \sum_{i=2}^{N} \int_{a \text{ (ANY PATH)}}^{b} \mathbb{F}_{1i}(s) \cdot ds \\ &= \sum_{i=2}^{N} W_{1i} \end{aligned}$$

THIS RESULT SHOWS THAT THE TOTAL WORK ON THE POINT CHARGE IS THE SUM OF WORK DONE BY THE ELECTRICAL FORCE OF EACH (OTHER) POINT CHARGE INDIVIDUALLY (THE SUPERPOSITION PRINCIPLE HOLDS); ADDITIONALLY, DUE TO PATH INDEPENDENCE, WE ARE FREE TO (CONVENIENTLY) CHOOSE DIFFERENT PATHS FOR EACH FORCE.

DRAWING #6: THE WORK DONE BY (THE ELECTRICAL FORCE OF) MULTIPLE POINT CHARGES 2

OUR PRIOR RESULT EXTENDS TO THE CASE OF A CHARGE DISTRIBUTION:

$$\mathbb{F}_1 = \iiint_D dx\,dy\,dz \, \frac{1}{4\pi\varepsilon_0} \frac{q_1 \rho(x,y,z)}{r_{1(x,y,z)}^2} e_{1(x,y,z)}$$

$$w = \int_{a \atop \text{ANY PATH}}^{b} \mathbb{F}_1(s) \cdot ds$$

$$= \int_{a \atop \text{ANY PATH}}^{b} \left(\iiint_D dx\,dy\,dz \, \frac{1}{4\pi\varepsilon_0} \frac{q_1 \rho(x,y,z)}{r_{s(x,y,z)}^2} e_{s(x,y,z)} \right) \cdot ds$$

$$= \frac{1}{4\pi\varepsilon_0} q_1 \iiint_D dx\,dy\,dz \, \rho(x,y,z) \int_{a \atop \text{ANY PATH}}^{b} \frac{1}{r_{s(x,y,z)}^2} e_{s(x,y,z)} \cdot ds$$

WE HAVE THE GENERAL RESULT:

<u>THE WORK DONE ON A POINT CHARGE BY/AGAINST THE ELECTRICAL FORCE AS IT MOVES BETWEEN TWO POINTS DOES NOT DEPEND ON THE PATH FOLLOWED.</u>

THIS MEANS THAT:

<u>THE ELECTRICAL FORCE IS A CONSERVATIVE FORCE.</u>

(FOR EXAMPLE, WE CANNOT GET WORK OUT OF THE ELECTRICAL FORCE BY CONSTRUCTING A CLOSED PATH.)

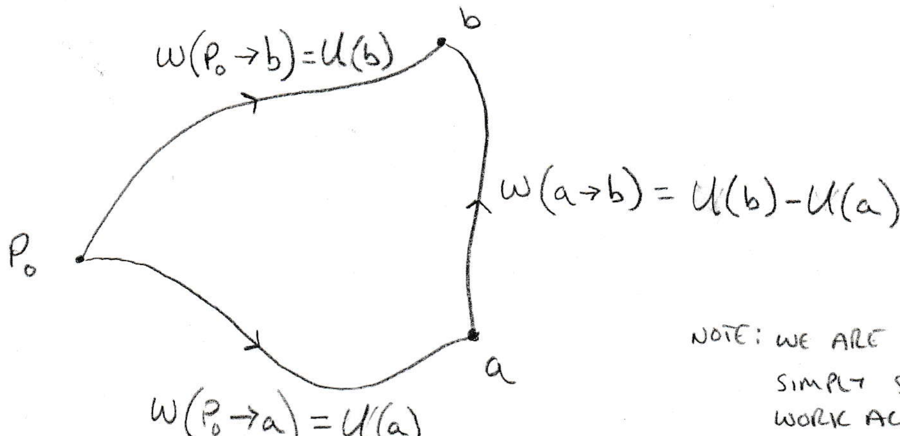
DRAWING #7: THE ELECTRIC POTENTIAL ENERGY

SINCE THE WORK IN MOVING A POINT CHARGE IS INDEPENDENT OF PATH, IT CAN BE EXPRESSED AS THE DIFFERENCE IN A SCALAR FUNCTION BETWEEN THE ENDPOINTS:

$$W = \int_{a \atop \text{ANY PATH}}^{b} \mathbb{F}(s)\cdot ds = -\big(U(b) - U(a)\big)$$

WHERE U IS THE (SCALAR) <u>ELECTRIC POTENTIAL ENERGY</u>. NOTE THAT WE <u>DEFINE</u> U AS THE WORK <u>AGAINST</u> A FORCE (HENCE, THE NEGATIVE SIGN), SO THAT POSITIVE WORK LEADS TO A REDUCTION IN POTENTIAL.

TO SEE THE ABOVE, CONSIDER CONSTRUCTING PATHS TO THE ENDPOINTS a AND b BY WAY OF A REFERENCE POINT P_0:



NOTE: WE ARE LETTING $U(a)$ SIMPLY STAND FOR THE WORK AGAINST THE FIELD IN GOING FROM P_0 TO a, FOR EXAMPLE.

SINCE ONLY THE DIFFERENCE IN IS EVER INVOLVED, WE DON'T NEED TO ACTUALLY SPECIFY P_0...

...ONCE WE HAVE CHOSEN SOME P_0 (OFTEN AT INFINITY), U IS DETERMINED AT EVERY POINT IN SPACE --- IT IS A <u>SCALAR FIELD</u> (THE ASSIGNMENT OF A SCALAR TO EVERY POINT (x,y,z)).

NOTE: THAT SINCE THE ELECTRIC POTENTIAL ENERGY IS PATH INDEPENDENT, WE ARE FREE TO ASSEMBLE THE SYSTEM IN ANY WAY MOST CONVENIENT FOR CALCULATIONS (SEE THE FOLLOWING EXAMPLES).

DRAWING #8: THE ELECTRIC POTENTIAL ENERGY OF MULTIPLE POINT CHARGES

THE ELECTRIC POTENTIAL ENERGY OF MULTIPLE POINT CHARGES CAN BE SUCCINCTLY WRITTEN:

$$U = \frac{1}{2} \sum_{i \neq j}^{N,N} \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{r_{ij}}$$
$$= \sum_{i<j}^{N,N} \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{r_{ij}}$$

(IT IS ASSUMED THAT THE ELECTRIC POTENTIAL ENERGY BETWEEN POINT CHARGES IS ZERO AT INFINITE SEPARATION)

THESE EXPRESSIONS REPRESENT THAT THAT EACH PAIR INTERACTION IS COUNTED ONCE, AND THE TOTAL ELECTRICAL POTENTIAL ENERGY IS THE SUM OF THE POTENTIAL ENERGY OF ALL THE PAIRS.

NOTE: THESE EXPRESSIONS IGNORE THE SELF-ENERGY OF EACH CHARGE $(i = j)$.

WE MAY THEREFORE CALL THIS ELECTRIC POTENTIAL ENERGY THE ELECTRIC POTENTIAL ENERGY OF INTERACTION (BETWEEN CHARGES).

TAKING THE CONTINUUM LIMIT OF THE ABOVE (A CHARGE DISTRIBUTION):

$$U = \frac{1}{2} \iint d\mathbf{r}\, d\mathbf{r}' \frac{1}{4\pi\varepsilon_0} \frac{\rho(\mathbf{r})\,\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$