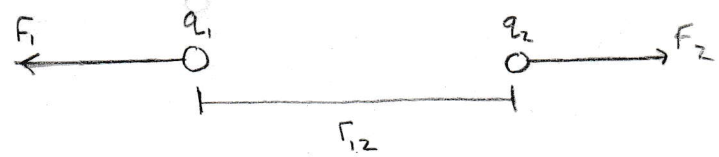
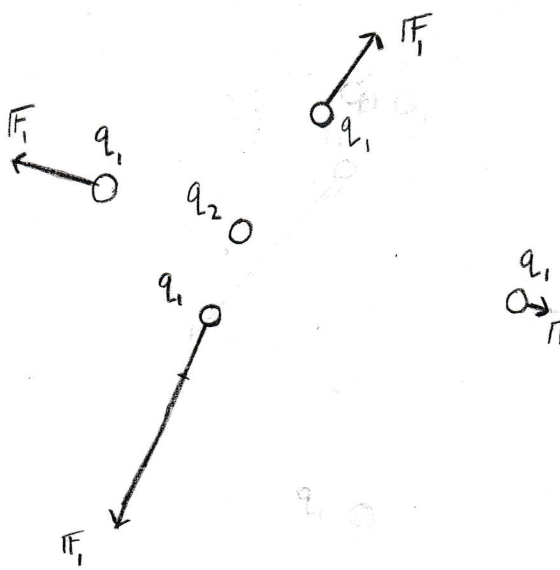
DRAWING #7: THE ELECTRIC FIELD I

ELECTRICAL FORCES ARE LONG-RANGE FORCES (CONCEPTUAL PROBLEM: ACTION AT A DISTANCE; INSTANTANEOUS FORCES):




IMAGINE $q_2$ WAS FIXED, AND $q_1$ WAS MOVED TO VARIOUS POINTS THROUGHOUT SPACE. WHAT WOULD $\mathbb{F}$ LOOK LIKE?

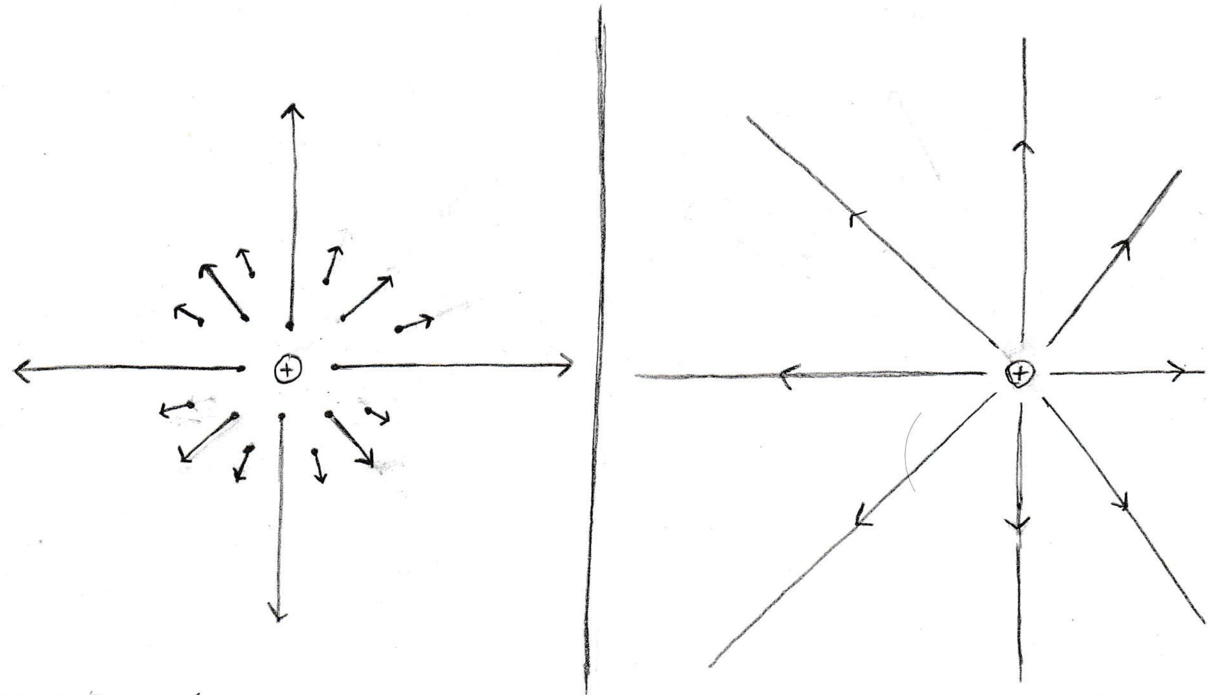



WE COULD IMAGINE DOING THE SAME WITH ANOTHER CHARGE $q_1' = 2q_1$, FINDING $\mathbb{F}_1' = 2\mathbb{F}_1$.

THIS SUGGESTS A MATHEMATICAL WAY TO DESCRIBE THE REGION AROUND $q_2$, INDEPENDENT OF $q_1$. WE CAN DO SO USING A VECTOR FIELD (THE ELECTRIC FIELD $\mathbb{E}$):

$$\mathbb{E}(x,y,z) = \frac{\mathbb{F}(x,y,z)}{q} \qquad (q>0)$$

# DRAWING #8: THE ELECTRIC FIELD 2, AND ELECTRIC FORCE




NOTE: $\mathbb{E}$ IS A <u>VECTOR FIELD</u> (AN ASSIGNMENT OF A VECTOR TO <u>EACH POINT</u> IN SPACE)

WE CAN USE OUR EQUATION FOR $\mathbb{E}$ TO FIND THE FORCE ON A CHARGED PARTICLE PLACED AT ANY POINT IN SPACE:

$$\mathbb{F}_q(x,y,z) = q\,\mathbb{E}(x,y,z)$$

FARADAY'S INTERPRETATION WAS THAT A CHARGE WOULD ALTER THE SPACE AROUND IT ($\mathbb{E}$); OTHER CHARGES THEN RESPOND TO THIS ALTERATION...

... THOUGH, THE MOST CORRECT WAY IS ALSO THE MOST ABSTRACT: CONSIDER $\mathbb{E}$ AS SIMPLY A MATHEMATICAL FUNCTION OF POSITION (AND TIME).

DRAWING #1: THE ELECTRIC FIELD

RECALL OUR DEFINITION OF THE ELECTRIC FIELD $\mathbb{E}$:

$$\mathbb{E}(x,y,z) = \frac{\mathbb{F}(x,y,z)}{q} \qquad (q>0)$$

(UNITS: N/C)

THE ELECTRIC FIELD DESCRIBES THE FORCE FELT PER UNIT CHARGE AT A POINT, EVEN WHEN THERE ARE NO CHARGES PRESENT.

REMEMBER, THE MOST CORRECT WAY OF THINKING ABOUT THE ELECTRIC FIELD IS ALSO THE MOST ABSTRACT: CONSIDER $\mathbb{E}$ AS SIMPLY A MATHEMATICAL FUNCTION OF POSITION (A VECTOR FIELD) (AND TIME).

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KEEP IN MIND THAT $\mathbb{E}$ IS A VECTOR:

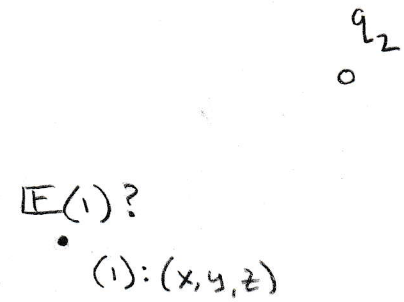
$$E_x(x,y,z) = \frac{F_x(x,y,z)}{q}$$

$$E_y(x,y,z) = \frac{F_y(x,y,z)}{q}$$

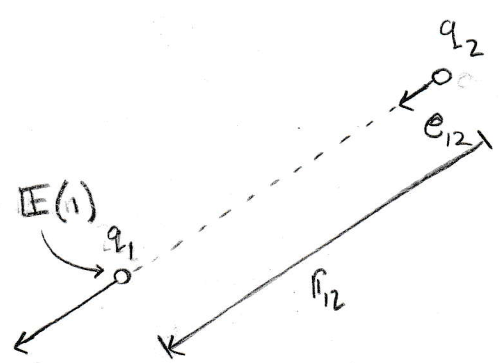
$$E_z(x,y,z) = \frac{F_z(x,y,z)}{q}$$

DRAWING #2: THE ELECTRIC FIELD DUE TO A POINT CHARGE I

SUPPOSE WE WANTED TO KNOW THE ELECTRIC FIELD AT SOME LOCATION $(1):(x,y,z)$ AROUND A POINT CHARGE $q_2$:




IMAGINE PLACING A TEST CHARGE $q_1$ AT $(1)$, AND USING COULOMB'S LAW:




$$e_{12} = \frac{r_{12}}{r_{12}}$$

$$\mathbb{F}_1 = k_e \frac{q_1 q_2}{r_{12}^2} e_{12}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2} e_{12}$$

NOTE: A TEST CHARGE IS A CHARGE THAT IS SMALL ENOUGH SUCH THAT THE CHARGE CONFIGURATION THAT WE ARE LOOKING AT IS NOT DISTURBED.

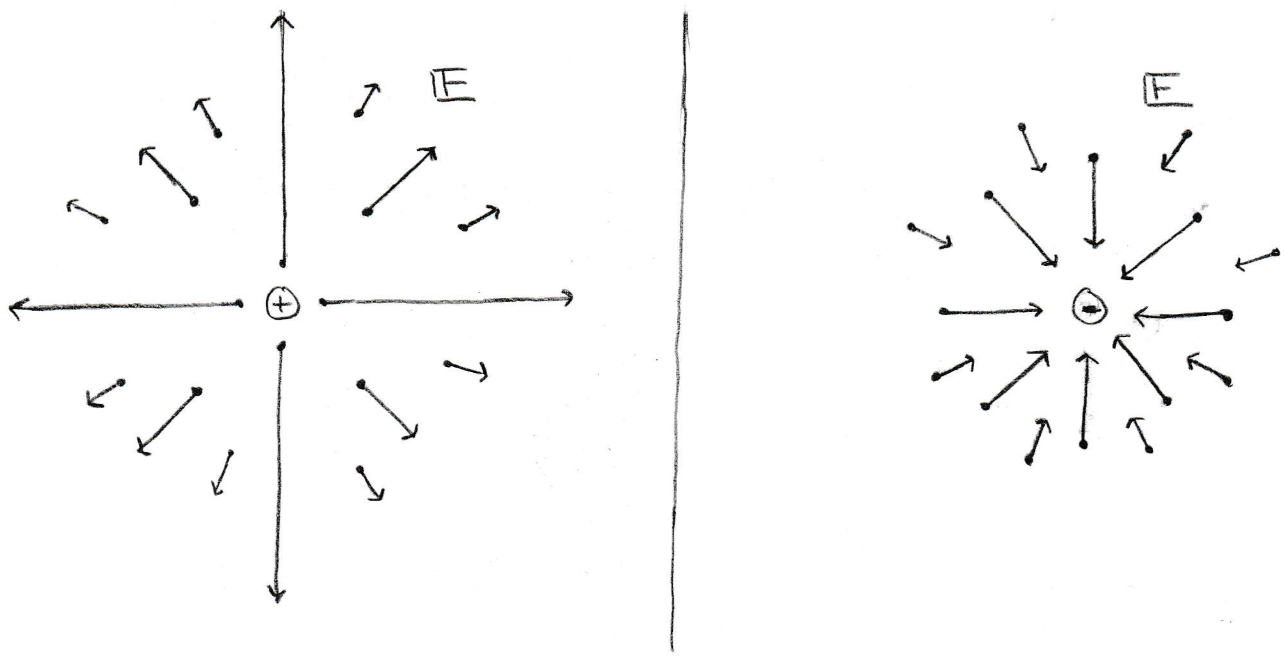
$$\mathbb{E}(1) = \frac{\mathbb{F}(1)}{q_1}$$

$$= k_e \frac{q_2}{r_{12}^2} e_{12}$$

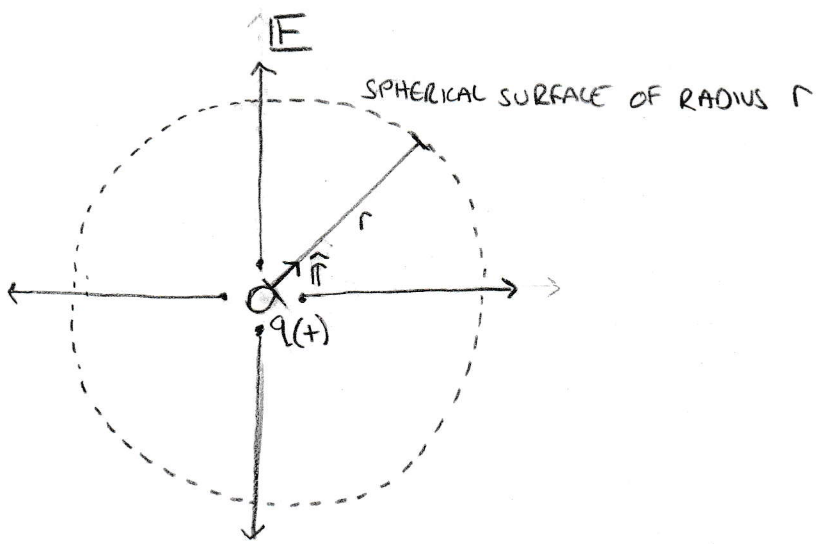
$$= \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_{12}^2} e_{12}$$

DRAWING #3: THE ELECTRIC FIELD DUE TO A POINT CHARGE 2

THE ELECTRIC FIELD OF POSITIVE AND NEGATIVE POINT CHARGES:




DUE TO THE SPHERICAL SYMMETRY:



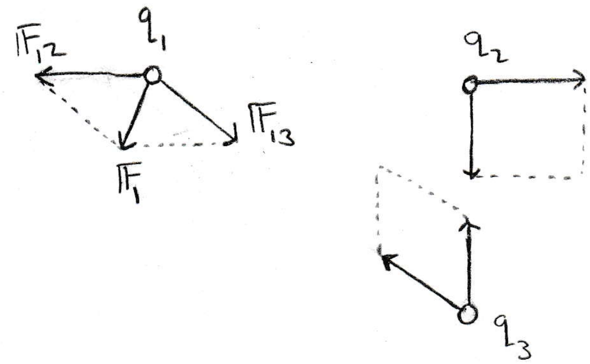

$$\mathbb{E}(\mathbb{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbb{r}}$$

# DRAWING #4: THE ELECTRIC FIELD DUE TO MULTIPLE POINT CHARGES

RECALL THE <u>SUPERPOSITION PRINCIPLE</u>:

$$\mathbb{F}_1 = \sum_{i=2}^{N} \mathbb{F}_{1i}$$

$\mathbb{F}_{1i}$ : THE FORCE ON PARTICLE 1, DUE TO PARTICLE $i$

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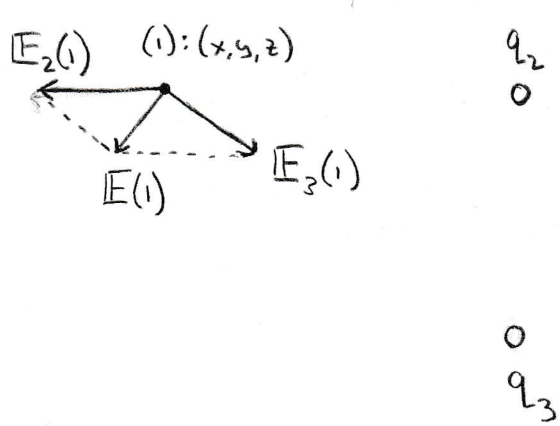
THE ELECTRIC FIELD DUE TO MULTIPLE POINT PARTICLES IS THEREFORE:

$$\mathbb{E}(1) = \frac{\mathbb{F}_1}{q_1} \qquad (q_1 > 0)$$

$$= \sum_{i=2}^{N} \frac{\mathbb{F}_{1i}}{q_1}$$

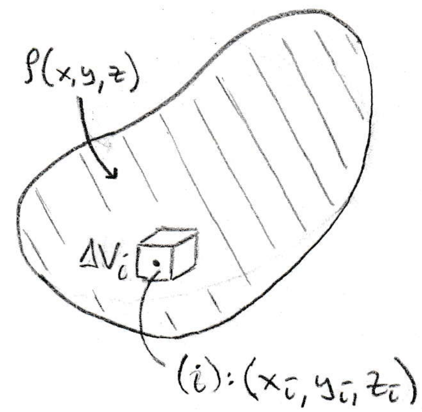
$$= \sum_{i=2}^{N} \mathbb{E}_i(1)$$



# DRAWING #5: CONTINUOUS CHARGE DISTRIBUTION

OFTEN CHARGED OBJECTS CONTAIN A SIGNIFICANT NUMBER OF EXCESS ELECTRONS....

... IN THESE CASES, IT IS OFTEN CONVENIENT TO CONSIDER THEM AS BEING DISTRIBUTED CONTINUOUSLY OVER THE OBJECT (A DISTRIBUTION).




... WE CAN DESCRIBE THE CHARGE DISTRIBUTION BY THE CHARGE DENSITY $\rho(x,y,z)$:

$$\rho(i) = \frac{\Delta q_i}{\Delta V_i}$$

THE AMOUNT OF CHARGE $\Delta q_i$ IN A SMALL VOLUME $\Delta V_i$

NOTE: A COLLECTION OF CHARGES IS A LIMITING CASE OF A CHARGE DISTRIBUTION:

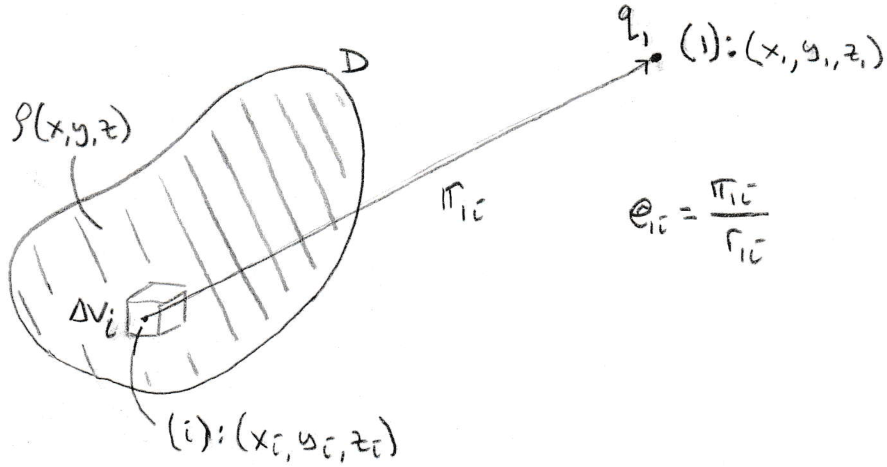
$$\rho(x,y,z) = \sum_{i=1}^{N} \delta^3(x_i, y_i, z_i)\, q_i$$

$$\delta^3(x,y,z) = \delta(x)\,\delta(y)\,\delta(z)$$

$$\delta(x) = \begin{cases} +\infty & x = 0 \\ 0 & x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} dx\, \delta(x) = 1$$

DRAWING #6: COULOMB'S LAW FOR A CHARGE DISTRIBUTION




SUPPOSE THE CHARGE DENSITY IS DIVIDED INTO N VOLUME ELEMENTS. AS LONG AS $\Delta V_i$ IS SMALL COMPARED TO $\mathbb{r}_{1i}$, WE CAN <u>APPROXIMATE</u> THE FORCE ON CHARGE $q_1$ DUE TO THE CHARGE $\Delta q_i$ IN $\Delta V_i$ AS:

$$\mathbb{F}_{1i} \approx \frac{1}{4\pi\varepsilon_0} \frac{q_1 \Delta q_i}{r_{1i}^2} e_{1i}$$

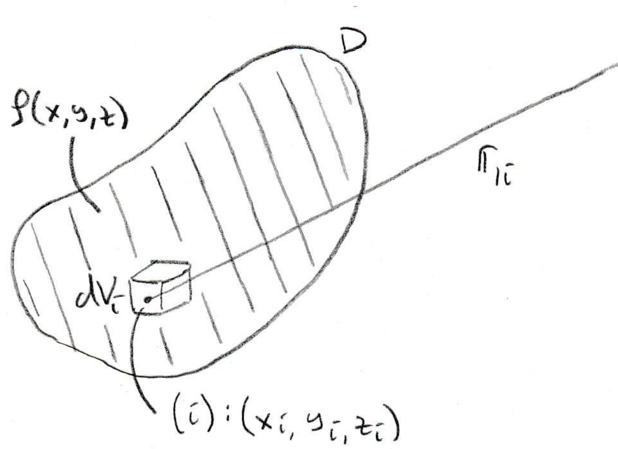
USING THE SUPERPOSITION PRINCIPLE, WE CAN WRITE THE TOTAL FORCE ON $q_1$:

$$\begin{aligned}
\mathbb{F}_1 &= \sum_{i=1}^{N} \mathbb{F}_{1i} \\
&\approx \sum_{i=1}^{N} \frac{1}{4\pi\varepsilon_0} \frac{q_1 \Delta q_i}{r_{1i}^2} e_{1i} \\
&\approx \sum_{i=1}^{N} \frac{1}{4\pi\varepsilon_0} \frac{q_1 \rho(i) \Delta V_i}{r_{1i}^2} e_{1i}
\end{aligned}$$

THE LAST LINE WE RECOGNIZE AS A <u>RIEMANN SUM</u> OVER THE CHARGE DISTRIBUTION; WE CAN MAKE OUR EXPRESSION EXACT BY TAKING MAX $\Delta V_i \to 0$:

$$\begin{aligned}
\mathbb{F}_1 &= \lim_{\max \Delta V_i \to 0} \sum_{i=1}^{N} \frac{1}{4\pi\varepsilon_0} \frac{q_1 \rho(i) \Delta V_i}{r_{1i}^2} e_{1i} \\
&= \int_D dV \frac{1}{4\pi\varepsilon_0} \frac{q_1 \rho(V)}{r_{1i}^2} e_{1V} \\
&= \iiint_D dx\,dy\,dz \frac{1}{4\pi\varepsilon_0} \frac{q_1 \rho(x,y,z)}{r_{1(x,y,z)}^2} e_{1(x,y,z)}
\end{aligned}$$

DRAWING #7: THE ELECTRIC FIELD DUE TO A CHARGE DISTRIBUTION




$$e_{1i} = \frac{\mathbb{r}_{1i}}{r_{1i}}$$

$$\mathbb{F}_1 = \frac{1}{4\pi\varepsilon_0} q_1 \int_D dV \frac{\rho(V)}{r_{1V}^2} e_{1V}$$

$$= \frac{1}{4\pi\varepsilon_0} q_1 \iiint_D dx\,dy\,dz \frac{\rho(x,y,z)}{r_{1(x,y,z)}^2} e_{1(x,y,z)}$$

GIVEN THAT WE KNOW THE FORCE ON A CHARGE AT LOCATION (1), WE CAN EASILY WRITE DOWN THE EXPRESSION FOR THE ELECTRIC FIELD AT (1) DUE TO A CHARGE DISTRIBUTION:

$$\mathbb{E}(1) = \frac{\mathbb{F}_1}{q_1}$$

$$= \frac{1}{4\pi\varepsilon_0} \int_D dV \frac{\rho(V)}{r_{1V}^2} e_{1V}$$

$$= \frac{1}{4\pi\varepsilon_0} \iiint_D dx\,dy\,dz \frac{\rho(x,y,z)}{r_{1(x,y,z)}^2} e_{1(x,y,z)}$$

REMEMBER THAT $\mathbb{E}$ IS A VECTOR; WE COULD WRITE FOR EACH COMPONENT, FOR EXAMPLE:

$$E_x(x_1, y_1, z_1) = \frac{1}{4\pi\varepsilon_0} \iiint_D dx\,dy\,dz \frac{\rho(x,y,z)\,(x_1 - x)}{\left[(x_1-x)^2 + (y_1-y)^2 + (z_1-z)^2\right]^{3/2}}$$

# DRAWING #8: REAL ELECTROSTATIC PROBLEMS

WITH OUR EXPRESSIONS FOR $\mathbb{F}$ AND $\mathbb{E}$, WE HAVE SOLVED ALL ELECTROSTATIC PROBLEMS FOR WHICH WE KNOW THE LOCATIONS OF ALL OF THE CHARGES OR THE CHARGE DISTRIBUTION(S)...

... IT IS JUST A MATTER OF SOLVING COMPLICATED INTEGRALS.

WE CAN FIND FORCES AND ELECTRIC FIELDS FROM:

- A LINE OF CHARGE
- A SHEET OF CHARGE
- ANY OTHER CHARGE DISTRIBUTION

... JUST BY DOING THE INTEGRAL(S).

LATER WE'LL CONSIDER SOME CLEVER CONCEPTS THAT WILL SIMPLIFY THE CARRYING OUT OF THESE INTEGRALS.