Modeling, Estimation, and Control of Tip-steerable Needles with Torsional Dynamics

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Abstract

Needle insertions serve a critical role in a wide variety of medical interventions. Steerable needles provide a means by which to enhance existing percutaneous procedures and afford the development of entirely new ones. Here, we present a new time-varying model for the torsional dynamics of a steerable needle, along with a new controller that takes advantage of the model. The torsional model incorporates time-varying mode shapes to capture the changing boundary conditions caused during insertion of the needle into the tissue. Extensive simulations demonstrate the improvement over a model that neglects torsional dynamics, and illustrates the possible effect of torsional model order on efficacy. Pilot feedback control experiments, conducted in artificial tissue (plastisol) under stereo image guidance, validate the overall approach: our results substantially out-perform previously reported experimental results on controlling tip-steerable needles.

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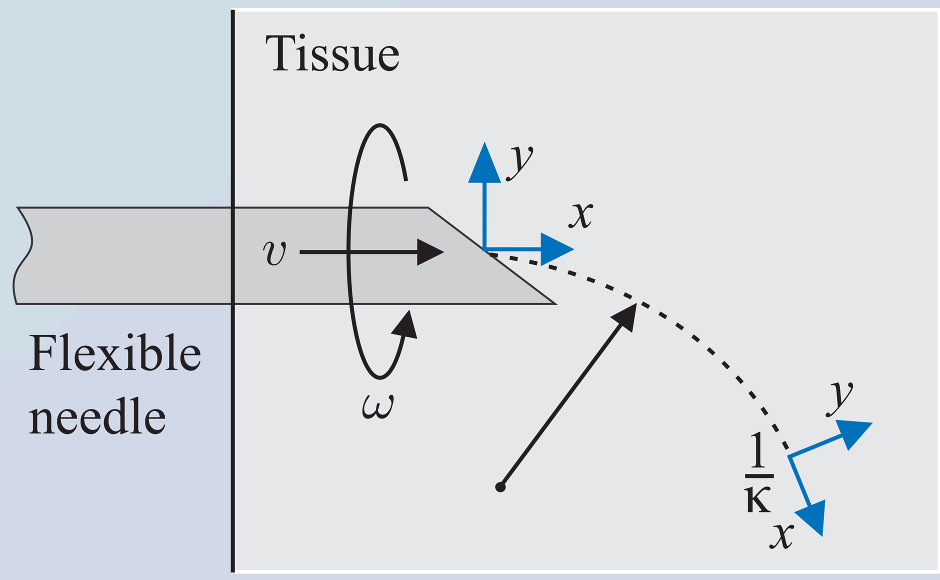
Needle Parameters

Parameter	Value
Density (ρ)	$6.45 \times 10^3\ \frac{\text{kg}}{\text{m}^3}$
Polar moment of inertia (J)	$2.3572 \times 10^{-14}\ \text{m}^4$
Shear modulus (G)	2.72×10^{10} Pascals
Viscous drag (β)	$2.23 \times 10^{-2}\ \text{N} \cdot \text{m} \cdot \text{s}$
Radius of curvature ($1/\kappa$)	0.073 m
Needle length (L)	0.3 m

$$\kappa = JG \qquad \eta = \rho J$$

Needle Tip Kinematics

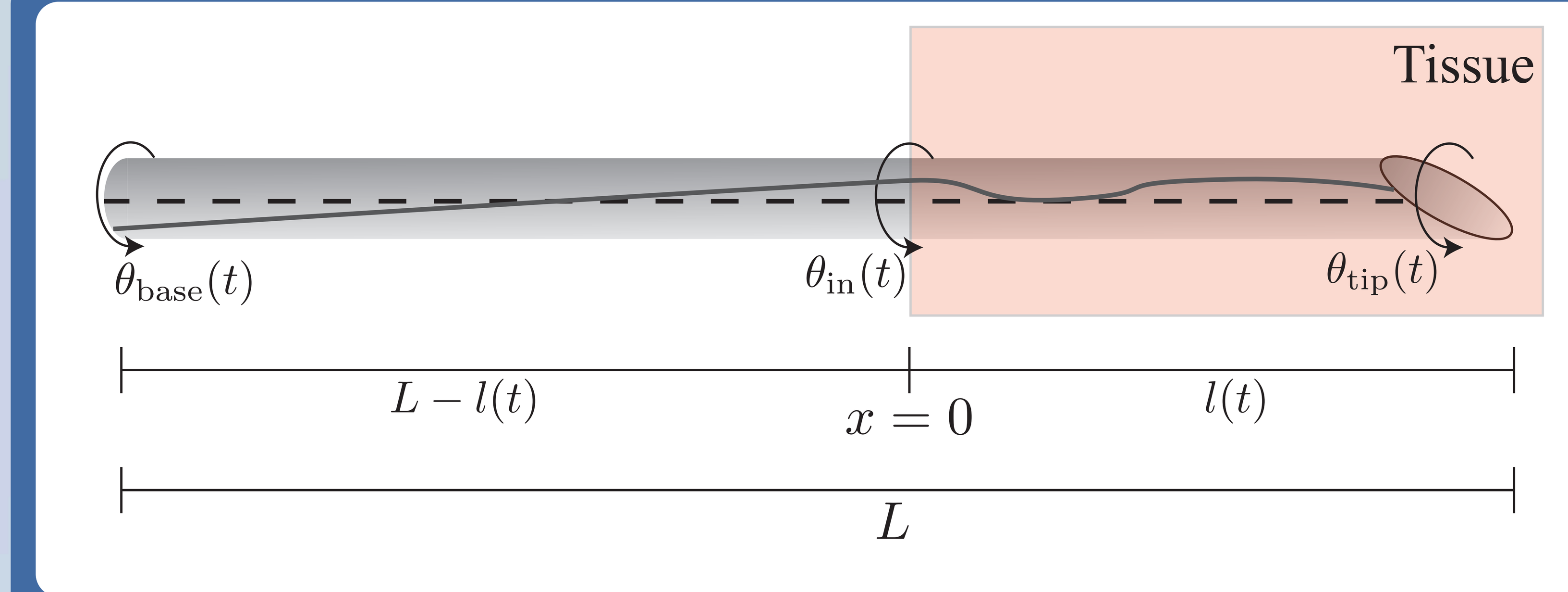


R. J. Webster III, J. S. Kim, N. J. Cowan, G. S. Chirikjian, and A. M. Okamura. Nonholonomic modeling of needle steering. Int. J. Robot. Res., 25(5/6):509–526, May 2006

Torsional Dynamics of Tip-Steerable Needles

The portion of the needles outside the tissue is modeled as a time-varying ideal torsional spring with spring constant:

$$K_{\text{spring}} = \frac{JG}{L - l(t)}$$



The portion of the needles inside the tissue is modeled using the time varying partial differential equation (PDE) from the Newton-Euler formulation:

$$\eta \frac{\partial^2 \theta}{\partial t^2} + \beta \frac{\partial \theta}{\partial t} - \kappa \frac{\partial^2 \theta}{\partial x^2} = \delta(x)\tau_{\text{in}}(t)$$

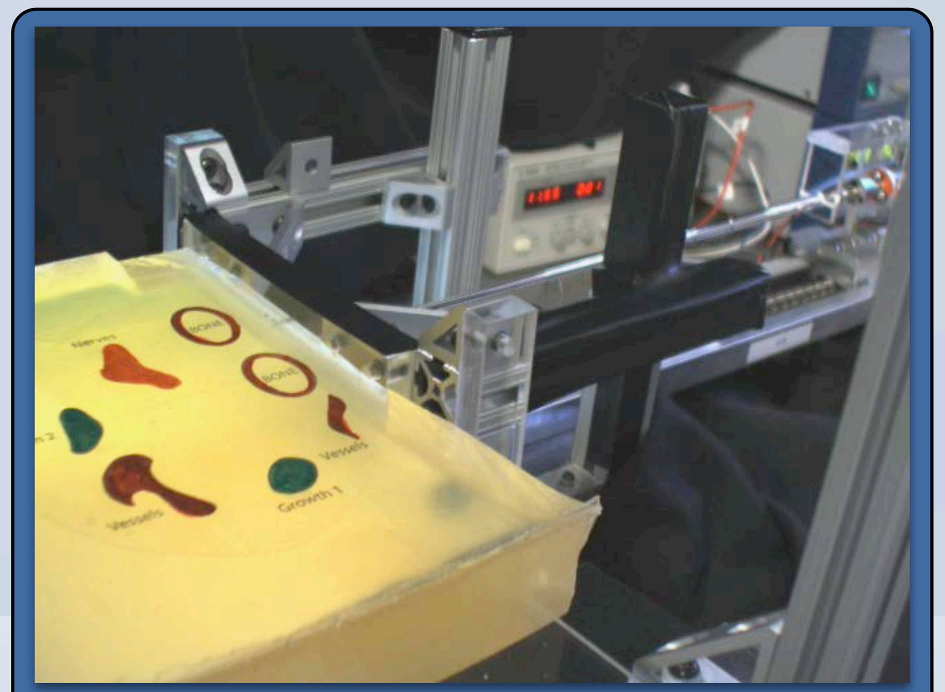
K. B. Reed, A. M. Okamura, and N. J. Cowan. Modeling and control of needles with torsional friction. IEEE Trans. Biomed. Eng., 56(12):2905–2916, Dec. 2009.

The PDE is solved using proper orthogonal decomposition (an assumed solution of torsional sine and cosine mode shape and coefficients) and reduced to a finite dimensional system of ordinary differential equations through subsequent Galerkin projection. Then, the tip velocity of the torsional dynamics formulation is the body fixed rotational velocity input with state reduction for control to a plane.

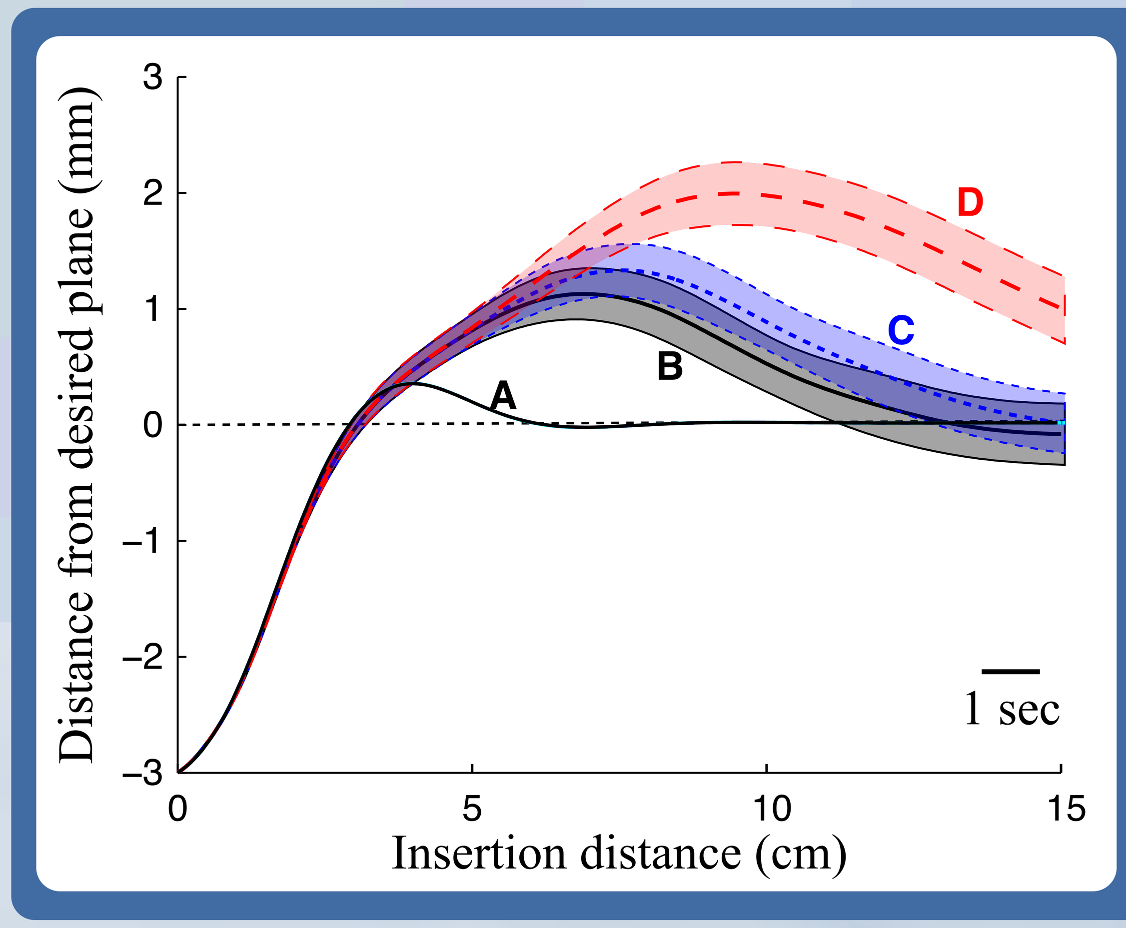
$$\dot{\mathbf{r}} = \begin{bmatrix} v \sin(r_2) \\ \kappa v \sin(r_3) \\ -v\kappa \cos(r_3) \tan(r_2) + C(t) r_{4 \cdots n} + D(t)\theta_{\text{base}} \\ A(t) q + B(t)\theta_{\text{base}} \end{bmatrix}$$

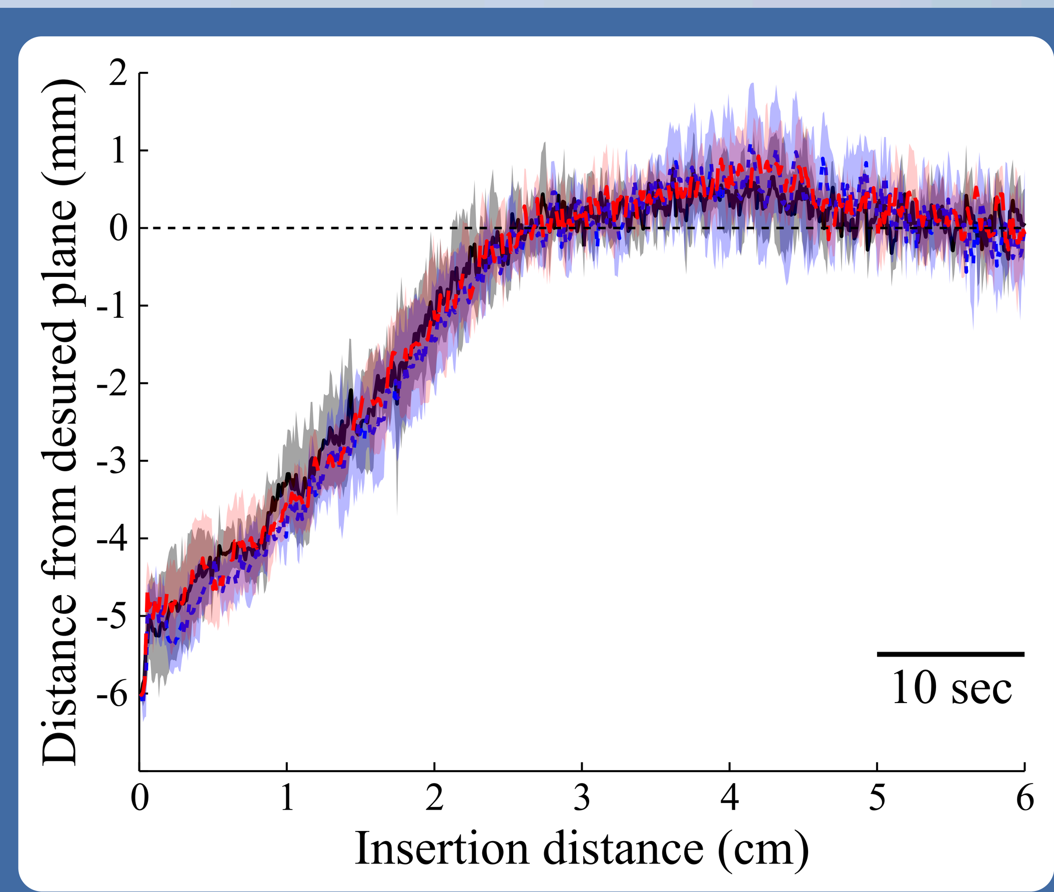
Simulations and Experiments

- Achieves convergence to a plane in approximately 2.5-3.0 cm of insertion from 6 mm initial distance (compared to previously reported results of 8 cm of insertion from 3 mm initial distance in [V. Kallem and N. J. Cowan. Image guidance of flexible tip-steerable needles. IEEE Trans. Robot., 25:191–196, 2009]).
- Experimentally, even one torsional mode is sufficient to achieve improved convergence results.
- These results used rough estimates of needle and tissue parameters from manufacturer specs. Future work will more carefully characterize system parameters.



Needle insertion robot with translational an rotational stage inserting a needle into simulated tissue (plastisol).





(Left) State feedback control simulations:
(A) Deterministic state feedback assuming full state access. This represents the best possible rate of convergence.
(B-D) Estimated state feedback using a Kalman filter for estimating the kinematic states and 1, 5, and 25 torsional state, respectively.

(Right) Physical experiments:
Preliminary trials in simulated tissue (plastisol) and a needle which acheives radius of curvature of 73 mm for different torsional model orders.
(Red) 1 torsional state
(Blue) 5 torsional state
(Black) 25 torsional state