Long-range wave propagation through random media

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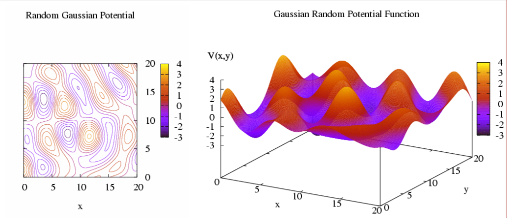


The potential function V(x,y) modeling a Gaussian random medium is governed by

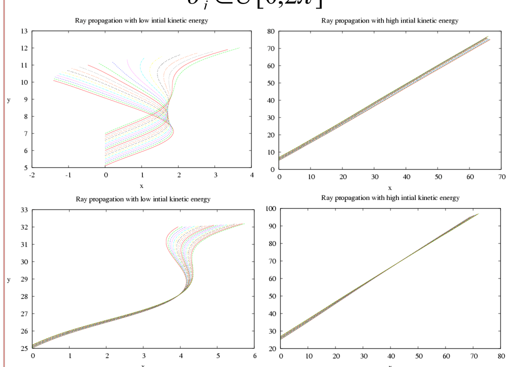
$$V(x,y) = \sqrt{\frac{2}{N}} \sum_{j=1}^{N} Cos[xCos(\theta_j) + ySin(\theta_j) + \phi_j]$$

where

$N = 10000$ superimposed plane waves

$$\phi_j \in U[0,2\pi]$$

$$\theta_i \in U[0,2\pi]$$



The rays propagate according to the Hamiltonian

$$H = \frac{p_x^2 + p_y^2}{2m} + V(x,y)$$

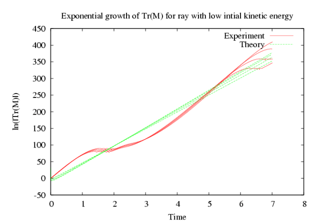
With two degrees of freedom, Hamilton's equations become

$$\frac{dx}{dt} = p_x \qquad \frac{dy}{dt} = p_y$$

$$\frac{dp_x}{dt} = -\frac{\partial V(x,y)}{\partial x} \qquad \frac{dp_y}{dt} = -\frac{\partial V(x,y)}{\partial y}$$

where the location of the ray is given by (x,y) and its momentum by p_x and p_y.

As a ray travels through a medium, it encounters variations which cause slight changes in velocity. Over long ranges, these variations cause the ray to become chaotic. Our goal is to better understand the distribution of the lyapunov exponent ν to determine transition timing between stable and chaotic propagation.



The stability matrix evolves according to $\frac{d}{dt}M = KM$

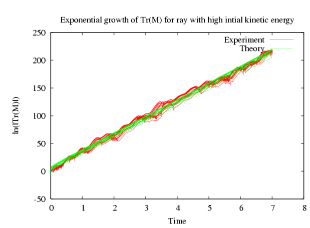
where

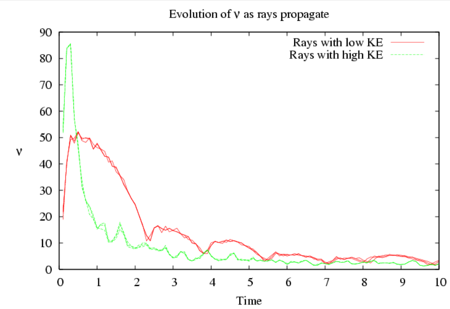
$$K = \begin{pmatrix} 0 & 0 & -\frac{\partial^2 V(x,y)}{\partial x^2} & -\frac{\partial^2 V(x,y)}{\partial x \partial y} \\ 0 & 0 & -\frac{\partial^2 V(x,y)}{\partial y \partial x} & -\frac{\partial^2 V(x,y)}{\partial y^2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

At each time *t*, the trace of the stability matrix Tr(*M*) is calculated. After data collection, a least squares linear regression is performed using the relationship

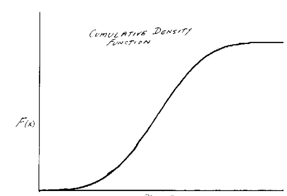
$$\ln[|\mathrm{Tr}(M)|] = \alpha + \omega \ln(t) + \nu t$$

to find the lyapunov exponent ν.

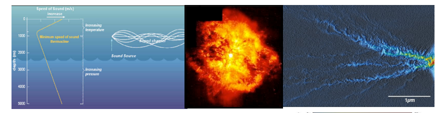




By calculating ν at various times in the ray's propagation, the evolution of ν is apparent. This calculation is only valid for long ranges, as shown by the transients for t ≤ 5. As the rays propagates further, all ν converge to the same value, ν_0.



For limited ranges, the distribution of the lyapunov exponent follows a log normal distribution. Eventually all the rays transition to chaotic propagation; however the long normal distribution allows for a tiny percentage of rays to be stable as they propagate to infinity.



Applications (from left): Sound moving in the ocean tends to chaos to due changes in pressure, temperature and salination. Likewise, light traveling from stars to the earth propagates through varying pressure. Electrons also tend to chaos when shot through a randomized potential.

This work was supported by the National Science Foundation's REU program under grant numbers PHY-0649023 and PHY-0555301.