Bound on Time-Reversal Noninvariance in the Nuclear Hamiltonian

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(Received 4 March 1985)

Energy-level fluctuations are calculated by random-matrix methods, for time-reversal noninvariant (TRNI) Hamiltonians. The theory, applied to the data, shows that in the neutron and proton resonance regions (i.e., for heavy and intermediate nuclei, respectively), the local rms off-diagonal multiparticle H(TRNI) matrix element $\lesssim D/10$, with D the level spacing. Spectral averaging methods reduce this to a bound on the nucleon-nucleon TRNI vs TRI interactions. First calculations, for ^{169}Er, give a 1% upper bound.

PACS numbers: 24.60.Ky, 05.40.+j, 11.30.Er

We derive an upper limit for the magnitude of the time-reversal noninvariant (TRNI) part of the nucleon-nucleon interaction by pursuing a suggestion of Wigner[1] that TRNI should be detectable in nuclear energy-level and strength fluctuations. The possibility of detection arises, for the spectrum, because the von-Neumann–Wigner level repulsion is larger for a TRNI Hamiltonian, and, for the strengths, because the locally renormalized strength distribution[2] has a χ_1^2 form (the Porter-Thomas law) when time-reversal invariance (TRI) is good and χ_2^2 when completely broken. The following simple model-independent argument shows that it is possible to detect a TRNI rms matrix element, connecting neighboring levels, which is somewhat smaller than the level spacing $D(E)$. Since the eigenstates of the TRI Hamiltonian may be taken as real, and, in that representation, H(TRNI) is purely imaginary, the essential transition, as H(TRNI) is "turned on," is the admixing of an imaginary amplitude into a real eigenstate. It follows, by perturbation theory for the interactions with nearby levels, that the transition is essentially completed when $\Lambda^{1/2} \equiv |\langle E'|H(\mathrm{TRNI})|E\rangle/D(E)| \simeq 1$. To determine a proper bound on the multiparticle TRNI matrix element, and to reduce it to a bound on the TRNI nucleon-nucleon interaction, three things are necessary: (1) a (random-matrix) theory for the Λ dependence of fluctuation measures; (2) a good analysis of the experimental data, based on an assumption of TRI; (3) an understanding, which comes via spectral averaging methods, of the nuclear structure involved. The second is already available[3]; the others are outlined here.

The natural ensemble[4,5] is $\{H_\alpha:v^2\} = \{H(S:v^2) + i\alpha H(A:v^2)\}$, where α is real and may be taken nonnegative; the $H(S)$ and $H(A)$ are respectively d-dimensional real symmetric and real antisymmetric matrices. Their distinct matrix elements are chosen independently by zero-centered–Gaussian laws, with matrix-element variances $\sigma_{ij}^2(S) = v^2(1+\delta_{ij})$, this then forming a "Gaussian orthogonal (v^2)" ensemble [GOE(v^2)], and $\sigma_{ij}^2(A) = v^2(1-\delta_{ij})$. The cases $\alpha = 0$ and 1 describe respectively the GOE appropriate to TRI and the unitary ensemble (GUE) appropriate when no information about TRI is available.

In this model the transition parameter $\Lambda = \alpha^2 v^2/D^2$ determines, for a many-dimensional system (just as α^2 does for the two-dimensional), the rate of change with α, near $\alpha = 0$, of local fluctuation measures. It is irrelevant that the H_α matrices are quite unrealistic, corresponding, as they do, to many-body interactions.[6] We demonstrate that in Fig. 1 by showing results for a more complicated embedded ensemble; it is clear also from the original analysis of the data,[3] and is understood formally in terms of the local nature of the process involved. The value of the (large) dimensionality d is also irrelevant for the Λ dependence of the measures,[7] since they rapidly approach their asymptotic values.

Two elementary facts enable us to find the $\alpha = 0$

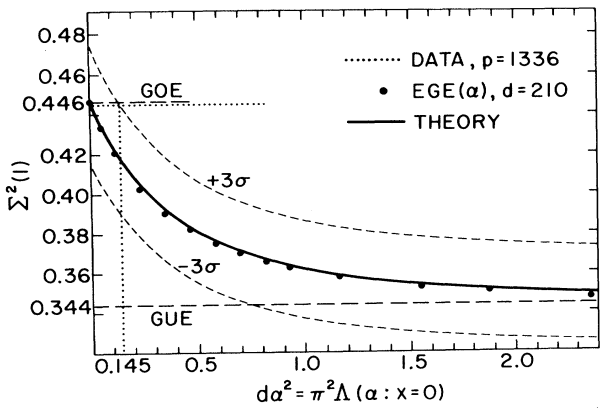

FIG. 1. The GOE→ GUE transition curve for $\Sigma^2(1)$, as given by Eq. (2) and by exact theory. Shown also, as described in the text, are the $\pm 3\sigma$ lines drawn for the set of all neutron-resonance levels (the last group in the table), and the lines showing the datum point (0.445) and the corresponding $\pi^2\Lambda$ bound (0.145). Points are indicated for fifty-member embedded Gaussian ensembles (Refs. 6 and 9) [EGE(α)] of two-body interactions acting in a four-particle 210-dimensional space; the agreement shown continues to $d\alpha^2 = \infty$ (0.347 vs 0.344) and is in accord with the arguments which give rise to the transition parameter Λ.

transition-curve slopes. The first is that the sum of two independent similar ensembles (identical to within a scaling factor) is similar, the variances v^2 adding. The second is that the second-order perturbation energies arising from the antisymmetric matrix $i\alpha H(A)$ are identical with those from the real symmetric $\alpha\hat{H}(A)$, where [in the $H(S)$-diagonal basis] $\hat{H}_{ij}(A)=\hat{H}_{ji}(A)=H_{ij}(A)$ for $i<j$ and $\hat{H}_{ii}(A)=H_{ii}(A)=0$. Except for its missing diagonal elements $\{\hat{H}(A)\}$ is a GOE(v^2); introducing, then, the ensemble of real diagonal matrices $\{\mathscr{W}\}=\{w_{ii}\delta_{ij}\}$, with the w_{ii} distributed as independent zero-centered Gaussians of variances $2v^2$, we see that $\{\hat{H}_\alpha=H(S)+\alpha[\hat{H}(A)+\mathscr{W}]\}$ is a GOE($(1+\alpha^2)v^2$). Manipulating these results, we find for the corresponding eigenvalues of H_α and $\hat{H}_\alpha$, and then for the variance V of the motion of the i th level in $\{H_\alpha;v^2\}$,

$$\hat{E}_{\alpha:i}(v^2)=E_{\alpha:i}(v^2)+\alpha w_{ii}+\ldots,\qquad \overline{w_{ii}}=0,\quad \overline{w_{ii}^2}=2v^2;$$
$$V[E_{\alpha:i}(v^2)]=V[E_{0:i}(1+\alpha^2)v^2)]-2\alpha^2v^2+\ldots, \tag{1}$$

the αw_{ii} being independent. We see immediately that, to lowest order in Λ, the k th nearest-neighbor spacing variances are $\sigma^2(k;\Lambda)=\sigma^2(k;0)-4\Lambda$, this confirming our identification of Λ as the transition parameter and demonstrating the increase with Λ of the spectrum rigidity. Using (1) we can in fact write the slopes, at $\Lambda=0$, of all spectral measures and correlation functions of all orders. For the "number variance" $\Sigma^2(r)$, the variance of the number of levels in an interval which contains r levels on the average, it turns out that

$$\Sigma^2(r:\Lambda)=\Sigma^2(r:0)-4[1-Y_2(r)]\Lambda+\ldots,$$

where $Y_2(r)$ (which $\simeq 1/\pi^2r^2$ for $r\gtrsim 1$) is the Dyson two-level cluster function[8] for the GOE.

These results are adequate for the analysis of data. But the complete transition curves for two-point fluctuations, which represent the solution of a complicated nonlinear problem, are of considerable interest. A calculation of the two-point correlation function, based on the binary-correlation approximation,[4,9,10] leads to

$$\Sigma^2(r:\Lambda)=\Sigma^2_{\mathrm{GUE}}+\frac{1}{2\pi^2}\ln\left\{1+\frac{\pi^2r^2}{4\{\tau+2\pi^2\Lambda\}^2}\right\}. \tag{2}$$

The numerical parameter τ, of order unity, enters when we use a convergence factor to extend to infinity a finite sum over d normal-mode excitations of the semicircular density. Its value can be fixed by matching the GOE($\Lambda=0$)-GUE($\Lambda=\infty$) difference for $\Sigma^2(1)$ ($=0.446-0.344$) which gives $\tau=0.6155$, or the exact slope of $\Sigma^2(\alpha)$ at $\alpha=0$ (which gives $\tau=\frac{1}{2}$). The two values give essentially identical transition curves; comparison with the later exact theory,[11] in which results appear in terms of integrals, favors the first value which gives, for $\Sigma^2(r)$ with $r\gtrsim 1$, errors only in the third decimal place.

The usable data are taken from a recent fluctuation analysis[3] which deals with 1762 levels from the 32 nuclei. We mainly use $\Sigma^2(1)$, optimal for our purpose, and for comparison with data adopt a 3σ criterion which then defines a confidence interval about the theoretical $\Sigma^2(1)$-vs-Λ curve of Fig. 1. The data value defines a horizontal line whose intersection with the upper edge of the band gives the upper limit Λ_0. The sample error, derived[3] for $\alpha=0$ (TRI) by a combination of theory and Monte Carlo calculations, and since verified for TRNI, is

$$\sigma(p:\Lambda)\simeq 0.6(2/p)^{1/2}\Sigma^2_\Lambda(1)\sim 0.35p^{-1/2},$$

where p is the number of levels in the sample. For a run of 100 levels $3\sigma\simeq 0.1$ so that the band may span the entire GOE-GUE difference. To obtain more levels we combine the data, according to their average level spacing (a rough measure of complexity) into five exclusive groups. The results for these groups, and for two others, are given in Table I. All groups give about the same value (0.1–0.2) for the upper bound on $\Lambda^{1/2}$ (and no lower bounds are indicated). The analysis shows that to improve this bound by an order of magnitude would require $\sim 10^6$ levels.

For the strength distributions mentioned above, the $\chi^2_1\rightarrow\chi^2_2$ transition is also moderated by the parameter

TABLE I. The nuclei used for the TRNI bound, their groupings according to level spacing D, the number p of levels in each group, the values derived for $\Sigma^2(1)$, and the resultant bound on the transition parameter as determined by use of the 3σ criterion discussed in the text. All levels are $\frac{1}{2}^+$ except those arising from the odd targets ^{177}Hf ($J=3,4$), ^{179}Hf ($J=4,5$), and ^{235}U ($J=3,4$), and from the even target ^{44}Ca which gives also a $\frac{1}{2}^-$ sequence. See Ref. 3 for the primary references to the data.

D	Target nuclei	p	$\Sigma^2(1)$	$\Lambda_0^{1/2}$
<25 eV	^{154}Gd, ^{177,179}Hf ^{232}Th, ^{235,238}U	573	0.426	0.20
25–80 eV	^{152}Sm, ^{156}Gd, ^{160}Dy ^{162}Dy, ^{166}Er, ^{172}Yb ^{182}W, ^{186,190}Os	466	0.439	0.18
80–210 eV	^{114}Cd, ^{154}Sm, ^{158}Gd ^{160}Gd, ^{164}Dy, ^{168}Er ^{170}Er, ^{174,176}Yb ^{184,186}W	297	0.490	0.09
3–6 keV	^{64,66,68}Zn	213	0.444	0.22
10–16 keV	^{44}Ca, ^{48}Ti, ^{56}Fe	213	0.470	0.16
16.8 eV	^{232}Th	178	0.474	0.16
<210 eV	Groups 1+2+3	1336	0.445	0.12

Λ, so that we can reasonably expect results similar to those given by the spectra. This expectation is borne out by a calculation of the transition curve, combined with a preliminary data analysis, which gives approximately the same upper bound.

To deduce from this bound a corresponding bound for the nucleon-nucleon interaction we rewrite H_α as $h + V(\mathrm{TRI}) + i\alpha U(\mathrm{TRNI})$, with h the one-body part. For a given nucleus with h, V, and U specified in a model space fixed by an appropriate set of spherical orbits we could *in principle* (but not in practice) determine α by a huge shell-model calculation. We would proceed by first diagonalizing H_0 in the many-particle space, then calculating the U matrix in the resultant basis, and finally choosing α to match the TRNI matrix-element bound determined above. The orbital space and the h operator come from inspection of the single-particle spectrum (available, for example, from a zero-deformation Nilsson calculation); for ^{169}Er, for example, there are nine active orbits, giving, for the 21 active nucleons, a space dimensionality 6×10^{14}, which splits into $\sim 30\,000$ positive-parity configurations, ~ 1000 of which contribute significantly to the nuclear states in the resonance region. Any of the standard interaction forms could be used for V, the strength being adjusted to fit the level density in the resonance region; with the surface-delta interaction[12] we would find a reasonable interaction strength. Since the only aspect of the (antisymmetric) U which we need to know about is the rms near-diagonal matrix element in the resonance region, we in fact need no general form for U. For the purpose of the calculation we could assume that U and V have the same norms in the two-particle space but are uncorrelated in low order.

Conventional shell-model calculations being obviously impossible, we resort to spectral-averaging methods[13,14] which, with the same input as above, undertake to give the shell-model results correct to within fluctuations. They derive from the existence of central limit theorems (CLT's) in the spectroscopic model spaces. The CLT's generate essentially Gaussian configuration distributions.[15] The spectral methods give their centroids and variances, thus fixing the ground-state energy and the configuration decomposition of the states via a construction of the spectral distribution function; an extension of the standard theory has been worked out to deal with the angular momentum (most of the data are for $J=\frac{1}{2}$).

Ignoring the J extension for purposes of discussion, we remark that an approximate form (Eq. 36 of Ref. 14) for $|\langle\Psi(E')U\Psi(E)\rangle|^2$, locally averaged in the neighborhoods of E' and E, follows from the configuration decomposition of the states. We go beyond that, as far as the quadratic terms in the exact formal expansion (Eq. 42), which takes account of correlations ignored in (36).

This theory with its J extension, applied to ^{169}Er, has given a 1% value for α which, as should be clear from the fact that U and V have the same norms, is a proper bound for the local magnitude of U(TRNI) compared with V(TRI). We propose shortly to extend the TRI calculations, to apply them to other nuclei listed in Table I, to complete the analysis of strength distributions, to make comparisons with other TRNI determinations,[16] and to consider applications of the same methods to other symmetries.

We acknowledge computing help from the University of Rochester High-Energy Group. This work has been supported in part by Division de Physique Théorique, Institute de Physique Nucléaire, Orsay, France, where part of the work was done, and in part by the U. S. Department of Energy.

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[5]The brackets $\{\}$ indicate an ensemble, and $\overline{K}$ the ensemble average of K. Note that $\{H_\alpha\}$ is invariant under orthogonal transformations so that $\{H(A)\}$ is invariant under transformations which diagonalize any $H(S)$. The eigenvalue density is $d\times\rho(E)$, where

$$\bar{\rho}(E)=\frac{[4dv^2(1+\alpha^2)-E^2]^{1/2}}{2\pi dv^2(1+\alpha^2)}$$

and $\int\rho(E)\,dE=1$. The (locally averaged) level spacing is $D(E)=[\bar{\rho}(E)d]^{-1}$.

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[7]However, d is relevant in other ways. For a fixed angle on Wigner's semicircle (Ref. 5) $v/D(E)\sim d^{1/2}$ so that the critical α, marking the completion of the transition, $\propto d^{-1/2}$; this "speeding up" of the transition arises from the compression due to distant levels (the spectrum span increasing as $d^{1/2}$ instead of d). This effect survives in real systems; without it we could only determine a relative TRNI bound, $\alpha\sim 1$. But it is limited there by the effect of the single-particle spacings, and there is no analog of the "phase transition" which occurs in the random-matrix model as $d\rightarrow\infty$.

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