Statistical theory of parity nonconservation in compound nuclei

S. Tomsovic
Department of Physics, Washington State University, Pullman, Washington 99164

Mikkel B. Johnson, A. C. Hayes, and J. D. Bowman
Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 22 November 1999; published 10 October 2000)

We present the first application of statistical spectroscopy to study the root-mean-square value of the parity nonconserving (PNC) interaction matrix element M determined experimentally by scattering longitudinally polarized neutrons from compound nuclei. Our effective PNC interaction consists of a standard two-body meson-exchange piece and a ''doorway'' term to account for 0^- spin-flip excitations. Strength functions are calculated using realistic single-particle energies and a residual strong interaction adjusted to fit the experimental density of states for the targets, ^{238}U for $A\sim 230$ and ^{104,105,106,108}Pd for $A\sim 100$. Using the standard Desplanques-Donoghue-Holstein estimates of the weak PNC meson-nucleon coupling constants, we find that M is about a factor of 3 smaller than the experimental value for ^{238}U and about a factor of 1.7 smaller for Pd. The significance of this result for refining the empirical determination of the weak-coupling constants is discussed.

PACS number(s): 24.60.Dr, 24.80.+y

I. INTRODUCTION

Studies of the weak interaction in strongly interacting systems have been of great interest in nuclear physics for a number of reasons. First of all, on the experimental side, observables may be obtained cleanly from measurements because (1) the symmetries respected by the weak interaction are different from those respected by the strong interaction, allowing the weak interaction sector to be uniquely disentangled from the purely strong interaction sector, and (2) the strength of the weak-interaction is sufficiently small that its matrix element between definite eigenstates of the strong interaction Hamiltonian may be extracted perturbatively. On the theoretical side, knowledge of the weak-interaction observables is a rich source of insights into the behavior of many-body systems. In those cases where the weak interaction is known, the experimental observable provides an opportunity to learn about the nuclear many-body wave function, giving an often unique glimpse at a specific feature of the eigenstate of the nuclear Hamiltonian. If, on the other hand, the weak interaction is not completely known (as is the case for the hadronic weak interaction), the measurements offer the possibility of learning about the weak interaction itself. To realize this goal, the main theoretical task is to calculate the properties of the nuclear many-body system as accurately as possible with a Hamiltonian that is known to describe the initial and final states in the matrix element.

Experimental information likely to shed light on the nature of the hadronic weak interaction is available in both light and heavy nuclei. Measurements of weak-interaction observables in these cases are driving the development of theoretical formulations of the many-body problem along two principal lines distinguished by the significance placed on the interpretability of individual matrix elements. The shell model formulation of the many-body problem is an example of a nonstatistical approach, and here one envisions individual matrix elements, such as those of operators describing transitions between discrete nuclear levels, to be calculable and hence meaningful. In light nuclei [1] the data have been available for many years, and these data are being analyzed using such a shell-model approach.

Our particular interest is in heavy nuclei. In this case [2], measurements based on longitudinally polarized neutron scattering and focusing on the hadronic parity nonconserving (PNC) interaction have become available only in the last 5–10 years, and development of the microscopic statistical approach is being stimulated by the availability of this data. The main focus of the latest experiments [3] has been the strength function of the effective PNC interaction in nuclei in the mass $A\sim 230$ and mass $A\sim 100$ regions of the periodic table. The root-mean-square (rms) matrix element M of this interaction is the physically meaningful quantity, as evidenced by the fact that the nuclear spacing between the resonances involved is on the order of 10–100 eV; note that it is not possible to obtain individual many-body matrix elements from this data. The values of M have been obtained, and these provide a new means to study the parity violating interaction in nuclei based on statistical theory [2].

The particular formulation on which we base our paper is that of statistical spectroscopy [4]. Statistical spectroscopy has proved very successful for calculations of strength functions of positive parity operators, but it has not been developed as fully for negative parity operators, especially for the case of parity violation. In this paper we will develop the theory in a study of parity violation for the targets measured in Ref. [3], namely, ^{238}U for $A\sim 230$ and ^{104,105,106,108}Pd for $A\sim 100$. The statistical densities that we will need are calculated using realistic single-particle energies and a residual strong interaction adjusted to fit the experimental density of states in the two elements of interest here. The determination of the smoothed strength and level densities is well established, and the theory has been shown to be well justified through numerical and theoretical evaluations in the early work [4]. The status of statistical spectroscopy in the other nuclei measured in Ref. [3] will be explored more fully in a later paper [5]. On the other hand, the application of statis-

tical spectroscopy to parity violation raises new issues, and it is the main purpose of this paper to identify the new issues and show how statistical spectroscopy may be implemented in view of them.

We discuss the statistical observables in Sec. II, emphasizing their advantage for study of the weak PNC interaction in compound nuclei. In Sec. III and the Appendix, we emphasize the microscopic foundation of the statistical theory and also collect ingredients that will be used in this paper for specific calculations. The limited aim of our calculations of M, presented in Sec. IV, is to benchmark the theory, i.e., provide reference points for future calculations based on standard π plus ρ meson exchange interactions with strengths taken from the existing literature (specifically we use the ''DDH best'' set; see Ref. [1]). Since we are not adjusting parameters to fit data, we leave open the question of whether the data prefer some alternative parameter set [5].

In Sec. V we discuss sources of uncertainty in such a calculation and indicate additional studies that would establish a more refined theoretical error analysis. Finally, in Sec. VI we present a summary of our paper and draw our conclusions.

II. STATISTICAL AND NONSTATISTICAL LIMITS

Experiments are often able to determine individual matrix elements $\langle \Psi_I|O|\Psi_K\rangle$ of some operator O between stationary states $|\Psi_I\rangle$ and $|\Psi_K\rangle$ of the nucleus. Microscopic, nonstatistical calculations of such matrix elements are often quite successful. However, such calculations of individual matrix elements are not guaranteed to be stable. For example, a basic problem arises in calculating weak-interaction operators O (e.g., the axial charge operator or the hadronic PNC interaction) within the shell-model framework. This is because the matrix elements

$$\langle \Psi_I|O|\Psi_K\rangle = \sum_{i,k} a_I^{*i}\langle \phi_i|O|\phi_k\rangle a_K^k, \tag{1}$$

which we have expressed in terms of the expansion coefficients a_I^i of the nuclear eigenstates $|\Psi_I\rangle$ in a many-body basis $|\phi_i\rangle$,

$$|\Psi_I\rangle = \sum_i a_I^i|\phi_i\rangle, \tag{2}$$

often involve strong cancellations, so the predictions are sensitive to small changes in the Hamiltonian of the strong interaction.

It is instructive to see how this sensitive interplay comes about. In order to avoid the problem of diagonalizing the complete many-body Hamiltonian, one assumes that the important pieces of the nuclear wave functions are the low-lying components, which one isolates by making a truncation into a ''model space.'' Because the components in the model space correspond to just a piece of the wave function, one must make a correction, either by applying a renormalization to the hadronic weak interaction to account for the excitations out of the model space not accounted for by the effective interaction or by enlarging the model space. In the shell-model theory of Ref. [6], this was done by enlarging the space. Enlarging the space was accompanied by corresponding changes in the coefficients a_I^i and a_K^k, leading to dramatic changes in the matrix elements, illustrated very clearly in Ref. [6]. It is the conclusion of this and other investigations that because of the sensitive interplay among various components, the nuclear wave function must be calculated very precisely, especially by including multiple particle-hole (p-h) admixtures and carefully eliminating spurious center-of-mass motion. To avoid this strong model dependence, it was pointed out that the PNC matrix element could be calculated from the measured analog first-forbidden decay of ^{18}Ne.

With more computer power the individual matrix elements may of course be calculated in light nuclei with sufficient stability. However, for heavy nuclei, where one has many more degrees of freedom, chaotic behavior may occur, and then the problem becomes much more serious. In this case eigenstates of the Hamiltonian have so many components, with amplitudes roughly equal in size and with phases that occur in a nearly random fashion that the individual matrix elements become impossible to interpret in principle. Even small changes in the Hamiltonian (or approximation scheme) can dramatically affect the wave function and hence the expectation value of an observable [7]. (The effective dimensionality, or number of significant Slater determinant components in the eigenstates, is far more stable.) Although a large amount of information is irretrievably lost in the transition from simple to complex systems in this fashion, significant physically meaningful information does remain. Indeed, it was the original motivation in developing the theory of statistical spectroscopy to identify this information and develop methods to extract it. The chaotic behavior of the nuclear wave function suggests application of the central limit theorem for this purpose and leads to the notion that matrix elements of operators may be thought of as elements of a statistical distribution.

For chaotic nuclei, one is thus led to identify as the observable the square of the matrix element of the PNC interaction averaged over N_I and N_K states of appropriate symmetry occurring in a small window of energy,

$$M^2 \equiv \frac{1}{N_I N_K}\sum_{I,K}|\langle \Psi_I|O|\Psi_K\rangle|^2, \tag{3}$$

which can be cast into a form very similar to Eq. (1) using the statistical properties of the wave function. To see this, first use Eq. (2) to rewrite Eq. (3) as follows:

$$M^2 = \sum_{i_1,k_1}\sum_{i_2,k_2}\left(\frac{1}{N_I}\sum_I a_I^{*i_2}a_I^{i_1}\right)\langle \phi_{i_1}|O|\phi_{k_1}\rangle \times\langle \phi_{i_2}|O|\phi_{k_2}\rangle^*\left(\frac{1}{N_K}\sum_K a_K^{*k_2}a_K^{k_1}\right). \tag{4}$$

Since the phases of the components are random when the statistical approach is valid, we may write

$$\frac{1}{N_K}\sum_K a_K^{k_2*}a_K^{k_1}=\delta(k_1,k_2)\frac{1}{N_K}\sum_K |a_K^{k_2}|^2, \tag{5}$$

and if the a_K^k $(1\leqslant k\leqslant n$, where n is the effective dimensionality) in the averaging interval are additionally assumed to be elements of an ensemble of Gaussian distributed random variables, we may write, for each k_2,

$$\frac{1}{N_K}\sum_K |a_K^{k_2}|^2=\frac{1}{n}. \tag{6}$$

Typically, N_I and N_K are rather small in comparison to the effective dimensionality n. Equation (6) is the stronger assumption. In Ref. [8], the space is partitioned with the different partitions being described by separate ensembles. If we adopt the weaker statistical assumption of Eq. (5), Eq. (4) becomes

$$M^2=\frac{1}{N_I N_K}\sum_{I,K}\sum_{i,k}|a_I^{*i}\langle\phi_i|O|\phi_k\rangle a_K^k|^2. \tag{7}$$

Comparing Eq. (7) to Eq. (1), we see that now the different terms in the sum over i,k occur with the *same* sign, so that the problem with delicate cancellations that would prevent a meaningful calculation of an individual matrix element is avoided. Thus, observables amenable to a statistical analysis are robust against approximations that do not change the overall scale characterizing the distribution of the a_I^i.

In actual situations where the statistical theory is applicable, only the low-lying components a_K^k of the many-body wave function are statistical in the sense of Eq. (5). We look at a more systematic method for separating the statistical and nonstatistical behavior in Sec. III below, and find that when a more careful separation is made, we arrive at the same conclusion.

III. FORMULATION OF STATISTICAL THEORY FOR PARITY VIOLATION

The specific problem we will be addressing in trying to understand the TRIPLE data of Ref. [3] is how to calculate M when the operator O in Eq. (7) is the interaction V^{PNC}, corresponding to the hadronic parity-violating (PV) interaction between two particles in free space; we base our numerical results in this paper on a meson-exchange form for the weak PNC interaction with the coupling strengths presented by Desplanques, Donoghue, and Holstein (DDH) [9]. Although these coupling strengths were estimated by DDH from empirical nonleptonic hyperon decay data, they also incorporated some estimates of various contributions such as the strong-interaction enhancements to the symmetry breaking; as a result, the couplings still contain a considerable amount of uncertainty. Because of these uncertainites and the fact that the DDH interaction has not been fitted to nuclear PNC data, one does not expect to be able to explain the TRIPLE measurements without some further refinement of the coupling parameters.

In our formulation, we will be using the fact that statistical methods [4] provide a relatively straightforward and model-independent means to average over the important components (i,k) in Eq. (7), namely, those that mix chaotically among themselves by the action of the strong interaction. These components correspond to the independent-particle configurations formed by placing valence particles in single-particle orbitals that lie within a few MeV of each other (within the spreading width) outside a closed shell, in other words, to components within a model space. The central limit theorem is used to express the strength functions in terms of bivariate Gaussian functions characterized by a few parameters as described in Sec. III A below. Statistical mixing of single-particle states lying farther than a typical spreading width (a few MeV) away from these is weak [10] and naturally suppressed in the statistical formulation.

However, statistical mixing is unfortunately only part of the problem of understanding the TRIPLE data. Nonstatistical mixing of states lying outside the model space is expected to occur (see Sec. III B 3), and the real question at issue in applying statistical spectroscopy to parity violation is how to deal in a satisfactory way with important components of the wave function that lie just $1\hbar\omega$ outside the model space, for example, the giant resonances. One solution to this problem is provided by effective interaction theory [11].

Effective interaction theory provides bookkeeping techniques for building the contribution of the (i,k) components lying outside the model space in Eq. (7) into an *effective* operator $\overline{O}$ acting entirely within the model space. More generally, it prescribes not only how one can calculate microscopically various effective operators $\overline{O}$ corresponding to actual observables O, but also the effective Hamiltonian $\overline{H}$ whose eigenfunctions $|\overline{\Psi}_K\rangle$,

$$\overline{H}|\overline{\Psi}_K\rangle=E_K|\overline{\Psi}_K\rangle, \tag{8}$$

correspond to the true stationary states $|\Psi_K\rangle$. Once the model space has been set up, matrix elements of the Hermitian, energy-independent operators $\overline{H}$ and $\overline{O}$ may be calculated diagrammatically according to the rules of degenerate many-body perturbation theory [12]. The connection to the observable given in Eq. (1) is

$$\langle\overline{\Psi}_I|\overline{O}|\overline{\Psi}_K\rangle=\langle\Psi_I|O|\Psi_K\rangle. \tag{9}$$

An application of effective interaction theory for the calculation of PV observables in the sense outlined here was followed in Ref. [13].

Once one has obtained the effective operator $\overline{V}^{PNC}$ corresponding to V^{PNC} by making use of this theory, one may apply the powerful methods of statistical spectroscopy directly to $\overline{V}^{PNC}$. The expression for M^2 in Eq. (7) may be expressed in the model space by expanding $|\overline{\Psi}_K\rangle$ as in Eq. (2) to obtain

$$|\overline{\Psi}_K\rangle=\sum_k \overline{a}_K^k|\phi_k\rangle, \tag{10}$$

where the sum now runs over independent-particle configurations k lying purely in the model space. (Note that the coefficients $\overline{a}_K^k$ do not necessarily have a simple connection to a_K^k.) By substituting Eqs. (9) and (10) into Eq. (1), we find

$$\langle \Psi_I|O|\Psi_K\rangle = \langle \overline{\Psi}_I|\overline{O}|\overline{\Psi}_K\rangle = \sum_{i,k} \overline{a}_I^{*i}\langle \phi_i|\overline{O}|\phi_k\rangle \overline{a}_K^k. \tag{11}$$

Similarly, the model-space counterpart to Eq. (7) is found by substituting Eq. (11) into Eq. (3) and following the analogous reasoning leading from Eq. (4) to Eq. (7),

$$M^2 = \frac{1}{N_I N_K} \sum_{I,K} \sum_{i,k} |\overline{a}_I^{*i}\langle \phi_i|\overline{O}|\phi_k\rangle \overline{a}_K^k|^2. \tag{12}$$

The connection between M in Eq. (12) measured in a region of excitation energy E, the effective parity-violating interaction, and the strength function $S_{\overline{O}}$ defined below in Sec. III A (with $\overline{O}=\overline{V}^{PNC}$) is

$$\begin{aligned} M^2 &\equiv M^2(E_1,E_2,\Gamma_1,\Gamma_2) \\ &= D(E_1,\Gamma_1)D(E_2,\Gamma_2)S_{\overline{O}}(E_1,\Gamma_1,E_2,\Gamma_2), \end{aligned} \tag{13}$$

where $D(E,\Gamma)$ is the mean spacing between neutron resonances and Γ is the set of conserved quantum numbers labeling the states of interest (i.e., angular momentum, parity, isospin, etc.).

As we have emphasized, in Eqs. (11) and (12) the sums run only over independent-particle configurations i,k lying within the model space. By comparing these equations we arrive at a conclusion similar to that found at the end of Sec. II, namely, that the delicate cancellations that occur among amplitudes $\overline{a}_I^{*i}\langle \phi_i|\overline{O}|\phi_k\rangle \overline{a}_K^k$ in calculations of individual matrix elements are absent when one calculates the mean-square matrix elements in effective interaction theory. However, we also see that some residual interferences may creep back into the statistical problem through the calculation of $\overline{O}$, whose various terms may have fluctuating signs. Since these signs do not arise from statistical behavior, one may expect that this method of calculating mean-square matrix elements is the more robust procedure when statistical behavior is indicated.

Note how the statistical and nonstatistical aspects of the many-body physics cooperate to produce physical observables in this formulation: the statistical behavior is confined to the model space and handled within statistical spectroscopy, whereas the nonstatistical behavior (including any $1\hbar\omega$ collectivity that may arise from giant resonances, for example) is concentrated in the effective operator terms $\overline{O}-O$. These two aspects are interwoven at the end when M is evaluated in Eq. (12) or Eq. (13).

A. Statistical spectroscopy of French and collaborators

From a statistical point of view, spectroscopy in many-body systems reduces to two largely independent physical features: fluctuations and the secular behavior of system-dependent properties such as level densities, expectation values, and strength densities. Random matrix theories, first introduced by Wigner [14], and their growing number of generalizations especially with regards to mesoscopic, many-body electron systems [15], address the physics of fluctuation properties. This is the most widely recognized aspect of a statistical approach, but is not our concern here. Instead, we shall only describe an implementation of statistical spectroscopy that describes system-dependent large-scale secular behaviors that arise in the construction of the strength function of $\overline{V}^{PNC}$ in chaotic regimes of heavy nuclei. Just as Weyl (see Ref. [16]) was able to show that to leading order in wave number the only surviving information in the count of modes in a cavity was its volume, generalized central limit theorems (CLTs) and moment methods, as developed by French and co-workers over the past 30 years [4], identify the analogous surviving information in a microscopic approach to many-body spectroscopy and provide the means to calculate secular behaviors. In this context, convergence of the CLTs improves with the dilute limit of many valence particles occupying a much larger number of available single-particle states. It should be noted also that experience has shown good convergence to the CLT forms even in ds-shell-model examples [4] that involve spaces much smaller and less dilute than those appropriate for medium and heavy nuclei.

The matrix element M defined in Eq. (3) is an example of a strength density $S_{\overline{O}}(E_1,\Gamma_1;E_2,\Gamma_2)$, which may be expressed as a trace. In statistical spectroscopy, we consider the trace decomposed into its sum over particle configurations, $\mathbf{M}=(m_1,m_2,m_3,\ldots)$ where each m_i is the number of particles in a subset of single particle levels and $\mathbf{N}=(N_1,N_2,N_3,\ldots)$. The grouping is not restricted to a particular scheme, but for the present discussion each subset is taken to be a j orbital.

In general, strength densities are bivariate functions of energy. Using mainly the notation of French and co-workers [4]—i.e., $\langle\langle\ \ \rangle\rangle$ indicates the trace and $\langle\ \ \rangle$ indicates the expectation value ($\langle\langle\ \ \rangle\rangle/N$)—the expression is

$$S_{\overline{O}}(E_1,\Gamma_1;E_2,\Gamma_2) = \langle\langle \overline{O}^\dagger P_{\Gamma_2}\delta(E_2-\overline{H})\overline{O}P_{\Gamma_1}\delta(E_1-\overline{H})\rangle\rangle, \tag{14}$$

where $\overline{H}$ is the effective nuclear Hamiltonian, P_Γ is the projector onto the Γ subspace, and $\overline{O}$ is, for example, the effective parity-violating interaction. It is useful to perform a partitioning decomposition of Eq. (14):

$$\begin{aligned} &S_{\overline{O}}(E_1,\Gamma_1;E_2,\Gamma_2) \\ &\quad = \sum_{\mathbf{M},\mathbf{M}'} \langle\langle \overline{O}^\dagger P_{\Gamma_2}\delta(E_2-\overline{H})P_{\mathbf{M}'}\overline{O}P_{\Gamma_1}\delta(E_1-\overline{H})P_{\mathbf{M}}\rangle\rangle \\ &\quad = \sum_{\mathbf{M},\mathbf{M}'} S_{\overline{O}}(E_1,\Gamma_1;E_2,\Gamma_2;\mathbf{M},\mathbf{M}'). \end{aligned} \tag{15}$$

Parity is taken care of trivially since the configurations respect it and one can just restrict the $\mathbf{M},\mathbf{M}'$ sums to the appropriate parities. For the model spaces of the medium and

heavy nuclei considered in this paper, only the angular momentum decomposition remains to be taken into account. If $\overline{O}$ were an electromagnetic transition operator that coupled different angular momenta, we would have to modify the angular momentum decomposition given below. However, for the case we are interested in here, $\overline{O}$ is a parity-violating interaction and is a J scalar. In Eq. (14), P_{Γ_2} commutes with $\overline{O},\overline{H}$ and can be translated next to P_{Γ_1}. The strength expression vanishes unless $\Gamma_2=\Gamma_1=\Gamma$. There are two additional simplifications. For the purposes of the angular momentum decomposition relevant to the parity violation calculations, $E_1=E_2=E$. Finally, as a consequence of statistical spectroscopy, the spreading widths are nearly constant. Therefore, $S_{\overline{O}}(E,E,\Gamma)$ is proportional to the J-decomposed density of states, where the constant of proportionality $C(j|E)$ is the conditional probability density of finding the angular momentum j given E. We find

$$S_{\overline{O}}(E_1,\Gamma_1;E_2,\Gamma_2)=S_{\overline{O}}(E_1,E_2;\Gamma)\approx C(j|E)S_{\overline{O}}(E,E)$$
$$=C(j|E)\sum_{\mathbf{M},\mathbf{M}'} S_{\overline{O}}(E,E;\mathbf{M},\mathbf{M}'). \quad (16)$$

To the level of our statistical approximations, $C(j|E)$ is well approximated by a partitioned version of Bethe's form [17] with an energy-dependent spin cutoff factor $\sigma_j^2(E,\mathbf{M})$ evaluated as

$$C(j|E)=\left(\begin{array}{c}\sum N_i\\ \sum m_i\end{array}\right)^{-1}\sum_{\mathbf{M}} C(J|E;\mathbf{M})\prod_i \binom{N_i}{m_i},$$

$$C(j|E;\mathbf{M})=\left(\frac{(2j+1)^2}{8\pi\sigma_j^6(E,\mathbf{M})}\right)^{1/2}\exp\left(-\frac{(2j+1)^2}{8\sigma_j^2(E,\mathbf{M})}\right),$$

$$3\sigma_j^2(E,\mathbf{M})\approx\langle J^2\rangle^{\mathbf{M}}+\langle J^2(H-E_{\mathbf{M}})\rangle^{\mathbf{M}}\frac{E-E_{\mathbf{M}}}{\sigma_{\mathbf{M}}^2}+\cdots, \quad (17)$$

where

$$E_{\mathbf{M}}=\sum_i m_i\epsilon_i \quad (18)$$

and

$$\sigma_{\mathbf{M}}^2=\langle V^2\rangle^{\mathbf{M}}=\sum_{\substack{r\le s\\ t\le u}}\frac{(N_r-m_r)(N_s-m_s-\delta_{rs})m_t(m_u-\delta_{tu})}{(N_r-\delta_{rt}-\delta_{ru})(N_s-\delta_{st}-\delta_{su}-\delta_{rs})N_t(N_u-\delta_{tu})}\sum_{\Gamma}[\Gamma]|V_{rstu}^{\Gamma}|^2 \quad (19)$$

(below we define V_{rstu}^{Γ}), with ϵ_i the single-particle energy of the ith orbital including any contribution to the energy from the part of the two-body interaction which transforms like a one-body operator. It is further supposed that the single-particle basis diagonalizes the full one-body part of the Hamiltonian. The V_{rstu}^{Γ} are the matrix elements of the remaining two-body interaction. The counting factor $[\Gamma]$ reduces to $2j+1$ above.

In addition, only those terms survive in Eq. (16) in which the fundamental matrix elements of $\overline{O}$ appear as absolute squares. We thus have

$$S_{\overline{O}}(E_1,E_2;\mathbf{M},\mathbf{M}')$$
$$=\prod_{i,j}\binom{N_i}{m_i}\binom{N_j}{m_j}|\langle\mathbf{M}'|\overline{O}|\mathbf{M}\rangle|^2\rho(E_1,E_2;\mathbf{M},\mathbf{M}') \quad (20)$$

to evaluate. If $\overline{O}$ is a two-body operator, the matrix element can be deduced from Eq. (19) with the $\{r,s,t,u\}$ sums suppressed and, if not, from the appropriate analogous equations for other rank operators. The theoretical expression for $\rho(E_1,E_2;\mathbf{M},\mathbf{M}')$, which can be derived assuming statistical properties of many-body wave functions and has been confirmed in numerical tests, is that of a unit normalized bivariate Gaussian,

$$\rho(E_1,E_2;\mathbf{M},\mathbf{M}')=\frac{1}{2\pi\sigma_{\mathbf{M}}\sigma_{\mathbf{M}'}\sqrt{1-\xi^2}}$$
$$\times\exp\left(-\frac{1}{2(1-\xi^2)}\left\{\frac{(E_1-E_{\mathbf{M}})^2}{\sigma_{\mathbf{M}}^2}-\frac{2\xi(E_1-E_{\mathbf{M}})(E_2-E_{\mathbf{M}'})}{\sigma_{\mathbf{M}}\sigma_{\mathbf{M}'}}+\frac{(E_2-E_{\mathbf{M}'})^2}{\sigma_{\mathbf{M}'}^2}\right\}\right), \quad (21)$$

which to specify completely requires the evaluation of two centroids $E_{\mathbf{M}},E_{\mathbf{M}'}$, two variances $\sigma_{\mathbf{M}}^2,\sigma_{\mathbf{M}'}^2$, and a normalized correlation coefficient ξ. In principle, these moments are given by

$$S_{00}=\langle\langle\overline{O}^{\dagger}P_{\mathbf{M}'}\overline{O}P_{\mathbf{M}}\rangle\rangle,$$

$$\xi=\langle\langle\overline{O}^{\dagger}\overline{H}P_{\mathbf{M}'}\overline{OH}P_{\mathbf{M}}\rangle\rangle/(S_{00}\sigma_{\mathbf{M}}\sigma_{\mathbf{M}'}),$$

$$E_{\mathbf{M}}=\langle\langle\overline{O}^{\dagger}P_{\mathbf{M}'}\overline{OH}P_{\mathbf{M}}\rangle\rangle/S_{00},$$

$$E_{\mathbf{M}'}=\langle\langle\overline{O}^{\dagger}\overline{H}P_{\mathbf{M}'}\overline{O}P_{\mathbf{M}}\rangle\rangle/S_{00},$$

$$\sigma^2_{\mathbf{M}}=\langle\langle \overline{O}^{\dagger} P_{\mathbf{M}'} \overline{OH}^2 P_{\mathbf{M}}\rangle\rangle/S_{00},$$

$$\sigma^2_{\mathbf{M}'}=\langle\langle \overline{O}^{\dagger} \overline{H}^2 P_{\mathbf{M}'} \overline{O} P_{\mathbf{M}}\rangle\rangle/S_{00}. \quad (22)$$

However, the centroids and variances can be replaced by the simpler expressions given in Eqs. (18) and (19) with almost no loss in accuracy of the approximation.

The correlation coefficient ξ is quite important. As it approaches zero, strength is distributed uniformly in the sense that the many-body mean-square matrix element is a constant independent of (E_1,E_2). In the opposite limit of $\xi \rightarrow 1$, the mean-square matrix elements of $\overline{O}$ are concentrated along the diagonal and their behavior approaches being inversely proportional to the level density; see below. Although ξ depends on $(\mathbf{M},\mathbf{M}')$, the fluctuations from configuration to configuration are small and the errors made in using a single averaged value negligible compared to other approximations. In fact, we use Gaussian ensembles with operators of fixed particle rank to evaluate ξ since the strength density is not extremely sensitive to small variations. In the dilute limit of N (number of single-particle states) $\rightarrow\infty$, $\overline{H}$ a k-body operator, $\overline{O}$ a k'-body operator, and m valence particles,

$$\xi=\left(\frac{m-k'}{k}\right)\Bigg/\left(\frac{m}{k}\right)\approx 1-kk'/m. \quad (23)$$

The finite N expression is more complicated but analytically known for a Gaussian ensemble. It is this expression that we use and which can be found in Ref. [8]. It is not sufficiently illuminating to give the expression here. Typical values for ξ in medium and heavy nuclei are ~ 0.7–0.9.

The criterion for choosing an appropriate model space is more complicated in the strength forms than for the level density since it depends on the joint contribution of $(\mathbf{M},\mathbf{M}')$ and the matrix elements connecting configurations. Nevertheless, one uses a similar logic and the most important configurations are the same as in the level density calculation, i.e., the single-particle orbitals strongly mixed by the residual two-body interaction. If O connects to important configurations outside the model space, statistical methods must be extended to incorporate perturbative effects as discussed in the following subsection.

B. Model space and effective interactions

Having made these general remarks, we next discuss the choice of model space and present the contributions to the effective one-body and two-body parity-violating interactions that we will use in this work. In the theory, there exist also three- and higher-body contributions to the effective operator $\overline{O}$, but there have been no calculations of these terms, so we will omit them.

1. Model space and strong interaction

There are two practical issues with regard to the choice of the strong interaction in statistical spectroscopy. One is the specification of the one-body part of the interaction, which specifies the model space and single-particle energies. The other is the choice of the spreading interaction that is crucial for getting the level density correct. Once these two elements are specified, then the partitioning of the model space and the calculation of the bivariate densities in Sec. III A are completely specified.

By ''spreading interaction,'' one means that part of $\overline{H}$ that spreads the configurations across the eigenstates. To determine it from a given $\overline{H}$ requires removing those parts responsible for shifting the centroids of levels. In the language of Ref. [4], the spreading interaction is the irreducible rank-2 part of $\overline{H}$. The shell-model or mean-field contribution, which determines the single-particle spectrum, is the rank-1 contribution of $\overline{H}$. Note that a somewhat different definition of the one- and two-body parts of an effective interaction is used in microscopic shell-model studies, where the single-particle energies are often taken to be those appropriate to the beginning or end of the shell. The residual interaction for multiple numbers of particles in a shell can then both spread levels and shift their centroids. One must keep this distinction in mind when comparing the shell-model and statistical approaches to avoid confusion.

In our applications of statistical spectroscopy, we choose both the mean-field and spreading interactions phenomenologically. By so doing, we thus depart from a purely theoretical implementation of the theory. The mean-field part is identified with the shell-model potential. By taking the single-particle energies to be those corresponding to such a phenomenological potential, a smooth A dependence is imposed. The spreading interaction is adjusted phenomenologically to reproduce the level density in the excitation region of nuclei of interest where the weak spreading width is to be calculated. The energy dependence of the level densities for nuclei measured by TRIPLE have been extensively studied and experimentally tabulated [18], and improved level densities have been obtained also in the region of interest for M in Ref. [3]. We used these results for our numerical studies.

The choice of model space is determined mainly by the considerations discussed in Sec. III, namely, that chaotic mixing occurs within the model space. Since the strong spreading width of states is on the order of 2 MeV, a model space spanned by single-particle levels lying within 2 or 3 MeV of each other is expected to be sufficient for those nuclei in which there are at least two or three particles (or holes) in the valence space for both neutrons and protons.

For the nuclei of interest in this paper, it is a relatively straightforward matter to choose the model space subject to these criteria by looking at shell-model level schemes. In spherical nuclei, the single-particle orbitals will correspond to eigenstates in a spherical potential and in deformed nuclei to Nilsson levels. Since statistical spectroscopy has always been applied (even in regions of deformation, for example, the $A\sim 230$ region) exclusively in the spherical basis, we will do our present calculations in this basis as well, in accordance with our goal to benchmark the theory. We estimate the importance of deformation by including it through an effective one-body PNC interaction (see Sec. V B).

TABLE I. $0\hbar\omega$ model space for mass region $A\sim230$. Note that we assume neutron (N) closure at $N=126$ and proton (P) closure at $Z=82$. The single-particle energies are given for the daughter ^{239}U.

Particle	Orbit	(n,l)	Parity
P	$h_{9/2}$	(0,5)	−
P	$i_{13/2}$	(0,6)	+
P	$f_{7/2}$	(1,3)	−
N	$i_{11/2}$	(0,6)	+
N	$j_{15/2}$	(0,7)	−
N	$g_{9/2}$	(1,4)	+
N	$d_{5/2}$	(2,2)	+

In our work, the single-particle energies of the active orbits for ^{239}U, given in Table I, were taken from Ref. [8] and for Pd were calculated [19] using a Woods-Saxon potential,

$$U=Vf(r)+V_{ls}\mathbf{l}\cdot\mathbf{s}\frac{r_0^2}{r}\frac{d}{dr}f(r)$$

$$f(r)=\left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1}, \tag{24}$$

with realistic parameters [20]

$$R=r_0A^{1/3},\quad r_0=1.27\ \text{fm},\quad a=0.67\ \text{fm},$$

$$V=\left(-51+33\tau_z\frac{N-Z}{A}\right)\ \text{MeV},\quad V_{ls}=-0.44V. \tag{25}$$

The resulting energies for the daughter ^{107}Pd are given in Table II. The model space is spanned by the set of all such configurations (N in number) that can be formed by putting m valence particles in a few (active) single-particle orbits lying in the immediate neighborhood of the nearest closed neutron and proton subshells (we refer to this as a $0\hbar\omega$ space). Note that there are both positive and negative parity levels included. The appearance of the intruder states of opposite parity is a feature particular to heavy nuclei: for the light nuclei for which parity violation has been considered, there are no such intruder states.

TABLE II. $0\hbar\omega$ model space for mass region $A\sim100$. Note that we assume neutron (N) closure at $N=50$ and proton (P) closure at $Z=28$. The single-particle energies are given for the daughter ^{107}Pd.

Particle	Orbit	(n,l)	Parity	Orbital energy (MeV)
P	$g_{9/2}$	(0,4)	+	−7.94
P	$p_{1/2}$	(1,1)	−	−9.04
P	$f_{5/2}$	(0,3)	−	−11.2
P	$p_{3/2}$	(1,1)	−	−10.7
N	$d_{5/2}$	(1,2)	+	−8.44
N	$g_{7/2}$	(0,4)	+	−7.34
N	$s_{1/2}$	(2,0)	+	−6.63
N	$d_{3/2}$	(1,2)	+	−5.96
N	$h_{11/2}$	(0,5)	−	-4.85

In the following two subsections, Secs. III B 2 and III B 3, we discuss three different pieces of $\overline{V}^{PNC}$, a standard one-body piece, a standard two-body piece, and a higher-order correction. The latter will be seen to be peculiar to heavy nuclei, where the relative importance of the standard one-body and two-body PNC interactions is different from what it is in light nuclei.

2. Lowest-order effective one- and two-body PNC potentials

The standard two-body piece of the effective PNC interaction in the nucleus $V_{Std}^{PNC(2)}$ that we use is related to the free-space two-body PNC interaction as discussed in Ref. [1] and in the Appendix. The standard one-body interaction $V_{Std}^{PNC(1)}$ is, in physical terms, that part of the effective PNC interaction acting on nucleons outside the core (the valence nucleons) that originates from the core nucleons. Since $V_{Std}^{PNC(1)}$ is just an average of $V_{Std}^{PNC(2)}$ over the core of the nucleus [1], we have a relatively simple connection between the two. Referring to the Fermi gas model [see Eq. (20) of Ref. [1]], we obtain

$$V_{Std}^{PNC(1)}=\frac{\rho}{\rho_0}\sum_i\ (C_0+C_1\tau_{zi})\boldsymbol{\sigma}_i\cdot\mathbf{p}_i\,, \tag{26}$$

where ρ is the density of the core and ρ_0 is the central density in nuclei. The coefficients C_0 and C_1 are given in Eq. (A3) of the Appendix. For $V_{Std}^{PNC(1)}$ as well as $V_{Std}^{PNC(2)}$ we retain only the contributions from F_π and F_0, the weak π-NN and (isoscalar) ρ-NN coupling constants, respectively, since these are the dominant contributions from the DDH parameter set.

Next, we want to discuss the higher-order pieces of this interaction. The situation can become quite involved, since higher-order pieces of the effective one-body and two-body parity violating interactions can be formed from both $V_{Std}^{PNC(1)}$ and $V_{Std}^{PNC(2)}$. It is our aim in this paper to limit our attention to issues that have been discussed already in the literature, so we will not enumerate the other terms in any systematic fashion. This limits us to only one type of higher-order term, discussed in the next subsection.

3. Higher-order contributions arising from $V_{Std}^{PNC(1)}$

One of the differences between parity violation in light and heavy nuclei is the relative importance of single-particle transitions in PNC observables. Large shell-model calculations (see, for example, Refs. [21,22]) indicate that the PNC transitions in light nuclei are largely single particle in nature [1,23]. However, in heavy nuclei, because of the appearance of intruder states of opposite parity in the $0\hbar\omega$ model space, $\overline{V}^{PNC(2)}$ can connect states within this space, and many-body mixing plays a much more important role. The fact that there are generally no active single-particle states that are coupled by $V_{Std}^{PNC(1)}$ (i.e., that there are no opposite-parity single-particle levels with the same total angular momentum inside the $0\hbar\omega$ model space in a heavy spherical nucleus, even though there are intruder states) means that $V_{Std}^{PNC(1)}$ acts indirectly. Its contribution to the effective PNC interaction

may be enumerated in second- and higher-order perturbation theory; in the theory of this paper, these are the major contributors to the difference of the operators $O-\overline{O}$ defined in Eq. (9) and accounts fully for the effect of $V_{Std}^{PNC(1)}$ on M. In a deformed nucleus the situation is somewhat different; see Sec. V.

The leading perturbative contribution of $V_{Std}^{PNC(1)}$ to $\overline{V}^{PNC}$ has been discussed in Ref. [24]. In an effort to use physical ideas to identify the important correction, Auerbach [25] developed the idea of ''doorway'' states for the PNC interaction. In this approach, the mixing of $V_{Std}^{PNC(1)}$ is mediated by the isoscalar and isovector 0^- spin-flip giant resonances. The disadvantage of Auerbach's formulation is that it is highly phenomenological. A microscopic expression $V_{Dwy}^{PNC(2)}$ for the doorway contribution to the effective two-body PNC interaction was obtained in Ref. [13] by identifying the doorway state as a collective one-particle one-hole 0^- phonon in the Tamm-Dancoff approximation (TDA). The result brought together the results of Refs. [24,25], showing that the effect of the doorway is to renormalize the perturbative contribution of Ref. [24] by a factor (ω_0/ω), where ω_0 is the energy of the unperturbed 0^- state, and ω is the energy of the TDA phonon. The result for $V_{Dwy}^{PNC(2)}$ consists of one isoscalar contribution, given in Eq. (4.9) of Ref. [13], and two isovector contributions, given in Eqs. (4.10) and (4.11) of that paper. The renormalization amounts to a suppression of the isovector 0^- part $V_{Dwy}^{PNC(2)}$ by about a factor of about 3 with little change in the isoscalar 0^- part (which was found to be much smaller for other reasons).

Desplanques has extended the study by considering the 0^- resonances in the random-phase approximation (RPA) [27] (see also Ref. [26]). This result shows that the multiparticle multihole correlations are very important. To get a nonvanishing contribution from the collective excitations in the RPA, it is necessary to include nonlocal terms in the residual strong interaction. The combined effect of these two considerations is to undo most of the suppression arising from the one-particle one-hole TDA correlations found in Ref. [13]. Desplanques found, essentially, that the doorway result changes by the following substitutions:

$$\omega_0/\omega \rightarrow \frac{1}{1+\lambda_p}, \quad \text{isoscalar},$$

$$\omega_0/\omega \rightarrow \frac{1}{1+\lambda_p'}, \quad \text{isosvector}, \tag{27}$$

where λ_p and λ_p' are defined in terms of the Fermi-liquid parameters as

$$\lambda_p = \frac{1}{3}G_1 - \frac{10}{3}H_0 + \frac{4}{3}H_1 - \frac{2}{15}H_2,$$

$$\lambda_p' = \frac{1}{3}G_1' - \frac{10}{3}H_0' + \frac{4}{3}H_1' - \frac{2}{15}H_2', \tag{28}$$

and where the effective mass of the nucleon M^* is set to its free-space value (the dominant momenta in the phonon are momenta above the Fermi momentum p_F). Using the Fermi-liquid parameters of Ref. [28], we calculate that $\lambda_p = -0.21$ and $\lambda_p' = 0.27$.

We use here the RPA treatment [27] of the O^- resonances rather than the TDA treatment. This is implemented by using Eq. (A2) (see the Appendix) with the replacements in Eq. (27). The isovector interaction, which is the only significant piece to this doorway result, continues to be suppressed in the RPA, but the renormalization is about a factor of 2 smaller than it was in Ref. [13].

IV. CALCULATIONS

In this section we describe our calculation of M and compare the results to the values of M deduced from scattering longitudinally polarized neutrons from the target nuclei ^{238}U and ^{104,105,106,108}Pd. These nuclei are good first cases to consider, because our calculation is relatively straightforward for proton and neutron midshell occupancy, and because the empirical values of the weak spreading width Γ_w are rather well determined [3] for these cases. The relationship between Γ_w and M is most precisely stated if we affix a subscript J on M to specify the total angular momentum of the CN states over which the PNC interaction is averaged and introduce D_J, the theoretical level spacing for s-wave levels of total angular momentum J in the region of the spectrum where the measurement has been performed. Then,

$$\Gamma_w = 2\pi |M_J|^2 / D_J, \tag{29}$$

where the label J has been omitted from Γ_w because of the expectation that this quantity is insensitive to the details of the distribution of CN levels. The resulting values for M_{expt} are given in Table III. Also shown in the table is the spacing D_0 of all s-wave levels, which is the same as D_J only for $J=0$ targets. In the case of the daughter ^{106}Pd there are two relevant intermediate states, of $J=2$ and 3, and in this case the quantity D_0 is the combined level density for the two states. To determine value of M_{expt} in this case required making an assumption about the relationship between D_0 and the two values of D_J.

We report the results of our calculation of M based on the theory developed in earlier sections of this paper in Table IV. The contributions to M arise from two sources: the standard two-body parity-violating interaction $V_{Std}^{PNC(2)}$ (Sec. III A 2) and the doorway piece $V_{Dwy}^{PNC(2)}$ (Sec. III A 3). To evaluate the matrix element M corresponding to $V_{Dwy}^{PNC(2)}$, we replace $\omega_0/\omega_{iv} \rightarrow 0.78$ in Table VII in accordance with Eq. (27). Details of our calculation of the matrix elements of $V_{Dwy}^{PNC(2)}$ and $V_{Std}^{PNC(2)}$ are given in the Appendix. The values of M corresponding to these two pieces of the interaction separately are given in the second and third columns of Table IV, respectively. In the case of the daughter ^{106}Pd, there are two different sets of s-wave resonances corresponding to $J=2$ and $J=3$ as discussed earlier; the individual values of M_J differ by less than 10%, so we have given only the average value M in this case in Table IV. The small value of M for ^{106}Pd reflects a smaller level spacing seen in Table III and the near constancy of Γ_m in the Pd isotopes.

TABLE III. The empirical weak spreading width Γ_w^{expt} [3] (preliminary) for the daughters ^{239}U and ^{105,106,107,109}Pd. Also shown are the neutron number N, the proton number Z, the theoretical level spacings D, and the value of M calculated according to Eq. (29).

Nucleus	(N,Z)	Γ_w^{expt} $(10^7$ eV)	D_0 (eV)	M_{expt} (meV)
^{239}U	(146,92)	$1.35^{+0.97}_{-0.64}$	20.75 (20.9)	$0.67^{+0.24}_{-0.16}$
^{105}Pd	(59,46)	$1.40^{+6.91}_{-0.99}$	165 (165±61)	$2.2^{+2.4}_{-0.9}$
^{106}Pd	(60,46)	$0.086^{+0.098}_{-0.042}$	13.1 (13.3±0.7)	$0.20^{+0.10}_{-0.07}$
^{107}Pd	(61,46)	$0.18^{+0.91}_{-0.15}$	159 (159±24)	$0.79^{+0.88}_{-0.36}$
^{109}Pd	(63,46)	$0.81^{+4.45}_{-0.62}$	159 (159±24)	$1.6^{+2.0}_{-0.7}$

The fourth column of Table IV contains the value of M corresponding to the sum of Dwy and Std. These results must be added in quadrature to obtain the combined value of M. Note that in doing this one must allow for the possibility that the two pieces can interfere. We can easily verify from the values given in Table VII that in practice there is only a small interference between the Std and Dwy contributions. We find the interference is constructive for $A\sim230$ but destructive for $A\sim100$.

In the last column of Table IV we show the empirical value of M taken from Table III. We see that for $A\sim100$ the experimental values of M fluctuate around the theoretical values. As the weak spreading widths are expected to be much more stable than M (the theoretical spreading widths are stable to within about $\pm10\%$), we compare in Table V the theoretical and experimental spreading widths in the two mass regions considered. Given that the spreading widths are quadratic in M, we see from Table VI that the M in the mass $A\sim230$ region is about a factor of 3 smaller than the empirical value and in the mass $A\sim100$ region about a factor of 1.7 smaller. The agreement between theory and experiment is remarkably close and encourages further applications with the goal of refining the empirical determination of the weak coupling parameters.

Earlier calculations [29,24,13] in the mass region $A\sim230$ share some features in common with the present approach. Reference [29] described the nucleus as a chaotic Fermi gas; i.e., the basis states of the nucleons were taken to be plane waves. The framework of statistical spectroscopy was used to calculate M with the strength function of the PNC interaction normalized to the strength function calculated in a finite nucleus [8]. When plane waves are used, the selection rules are much weaker than with shell-model states, and for this reason both $V_{Std}^{PNC(2)}$ and $V_{Std}^{PNC(1)}$ contributed *directly* to M. In the present work $V_{Std}^{PNC(1)}$ contributes directly only if the nuclear core is deformed (see Sec. V B). Otherwise, $V_{Std}^{PNC(1)}$ contributes indirectly through spin-flip excitations. As discussed in Sec. III B 3, $V_{Std}^{PNC(2)}$ contributes directly because of the presence of opposite-parity intruder states. In Ref. [29], M was found to be several times larger than the empirical value (see also Ref. [13]), whereas in the present work it appears to be several times smaller. It would thus appear that the nuclear matter treatment overestimates the result of statistical spectroscopy in a finite nucleus for the contribution of our one-body PNC interaction for the daughter ^{239}U.

Reference [24] employed a $0\hbar\omega$ model space like ours, so $V_{Std}^{PNC(1)}$ contributed indirectly, through what can be thought of as the perturbative piece of the doorway process. The average over states was defined by a Lorentzian function approximating the spreading width and normalized to an empirical estimate of the number of ''principal'' components of the nuclear wave function. Good agreement between measured and calculated values of M were reported. In Ref. [24] the indirect contribution of $V_{Std}^{PNC(1)}$ dominated the contribution of $V_{Std}^{PNC(2)}$, and the latter was neglected. In our work, the matrix elements of $V_{Std}^{PNC(2)}$ are similar to those in Ref. [24] (note that a different representation of the two-body PNC interaction was used in our work). However, in contrast to Ref. [24], the matrix elements of $V_{Std}^{PNC(2)}$ are comparable to the sizes of the matrix elements of our $V_{Dwy}^{PNC(2)}$, as can be inferred from Table IV. Although we have used different parametrizations of the effective strong interaction, we do not believe that this can fully account for the large differences we find.

It was suggested in Ref. [24] that the contribution of $V_{Std}^{PNC(1)}$ to M, relative to that of $V_{Std}^{PNC(2)}$, would follow an

TABLE IV. Theoretical values of M for the effective parity-violating interaction. Contributions are shown separately for the standard (Std) and doorway (Dwy) pieces of the two-body interaction. A comparison of the experimental value of M given in Table III is also shown.

Nucleus	M_{Std} (meV)	M_{Dwy} (meV)	$M_{Std+Dwy}$ (meV)	M_{expt} (meV)
^{239}U	0.116	0.177	0.218	$0.67^{+0.24}_{-0.16}$
^{105}Pd	0.70	0.79	1.03	$2.2^{+2.4}_{-0.9}$
^{106}Pd	0.304	0.357	0.44	$0.20^{+0.10}_{-0.07}$
^{107}Pd	0.698	0.728	0.968	$0.79^{+0.88}_{-0.36}$
^{109}Pd	0.73	0.72	0.97	$1.6^{+2.0}_{-0.7}$

TABLE V. Comparison of theoretical weak spreading width to experiment. The value of Γ_w^{expt} for Pd is obtained by statistically combining the results for all values given in Table III; the theoretical value of Γ_w^{the} is, to within a few percent, the same for all Pd isotopes.

Nucleus	$\Gamma_w^{the}(10^7$ eV)	$\Gamma_w^{expt}(10^7$ eV)
^{239}U	0.143	$1.35^{+0.97}_{-0.64}$
Pd	0.40	$1.73^{+1.65}_{-0.84}$

$A^{1/3}$ law. We see from Table IV that the relative value of the doorway contribution does increase with A as predicted in Ref. [24], but a more detailed estimate suggests that it increases by more like a power of $A^{1/2}$. The existence of the different rate of increase of the standard and doorway contributions shows that the A dependence of the measurements may provide an empirical means to determine the size of the doorway contribution.

V. DISCUSSION

In this section we will address the implementation of statistical spectroscopy and various sources of uncertainty in the calculations of the previous section. Most of the uncertainties we discuss are amenable to further analysis in large-basis shell-model studies, which would be valuable to gain additional confidence in the theoretical results.

The theory of level densities and strength distributions within statistical spectroscopy applies to individual systems and operators, and as such is not inherently an ensemble theory. The central limit theorem results and correction expansions are all based on having many strongly interacting particles dilutely spread among even more single-particle states. The inputs required for all calculations are low-order operator moments. If it were possible to calculate exact moments up to products of approximately, say, six operators decomposed for the symmetries involved, we would have an essentially ''perfect'' implementation of statistical spectroscopy. It would be extremely accurate and would work even far out in the tails of the distributions. Instead, we have made a number of practical compromises in order to complete this first implementation for the parity-violation problem. Some of the compromises can and should be eliminated in future, planned, improved calculations.

Most of the compromises we have chosen are motivated by the following considerations: (i) the necessary angular momentum decomposed moments cannot be written down analytically; (ii) even the nondecomposed moments (scalar moments) are not worked out beyond a product of four operators—the expressions for products of three or four operators, though they exist, are almost prohibitively unwieldy; and, (iii) the exact operators may not be known in the model spaces—for example, the exact, residual, strong operator in a model space appropriate for heavy nuclei. We have addressed these points by (i) treating the angular momentum as a statistical variable in the spirit of Bethe's ansatz for the spin cutoff factor; (ii) using partitioning of the moments to improve accuracy instead of incorporating polynomial corrections to the lowest-order central limit theorems—this alleviates much of the need for higher moments; and (iii) supplementing exact moments with ensemble calculations and nuclear modeling—the idea being that in chaotically behaving systems whole classes of operators deviate very little from certain ensemble results (subject to very few restrictions). This also reduces the need for higher-order moments.

To be more specific about ensemble results, we have in mind two-body random operator ensembles which are a subclass of embedded Gaussian ensembles [30]. The basic ensemble would have random k-body matrix elements embedded in an m-particle space distributed over N single-particle levels; the two-body matrix element ($k=2$) is the most important one. Some of the moment simplifications that we are making in this paper [see the comments after Eq. (22)] are ensemble results. Expectation value relations such as

$$\langle OH\rangle = \langle O\rangle\langle H\rangle,$$

$$\langle O^2H^2\rangle = \langle O^2\rangle\langle H^2\rangle, \tag{30}$$

which express a factoring approximation true in systems that behave statistically, have been found to hold to a considerable accuracy (see Refs. [4,8]). Operators O or their higher powers, which effectively contain a piece proportional to H (or a function of H), will not satisfy relations of this kind.

In the following subsections, we address issues related to the nuclear modeling we have done, specifically the size of the model space, deformation, correlations between $\overline{O}$ and $\overline{H}$ including spurious center of mass motion, and the choice of weak coupling parameters. We critique each uncertainty below in Secs. V A–V D. In Sec. V E we discuss how the shell model could be used to gain additional confidence in the models. All of the statistical spectroscopy compromises that were discussed above could (and should) be incorporated into the shell-model testing.

A. Size of the model space

The physical motivation for choosing a $0\hbar\omega$ model space was discussed in Sec. III. However, to establish that the optimal selection of active orbits has been made, it must be verified that the amount of parity mixing as determined by our statistical methods is stable under enlargements of the model space. For this purpose, we added a spherical $1f_{5/2}$ neutron and $1g_{7/2}$ proton orbit to the model space for ^{239}U. Adding these two orbitals, we found that the calculation of M decreased by about 10% when the residual strong and weak interactions were chosen as described above. Note that according to the theory of Sec. III, the effective interactions should be changed as the model space is enlarged. Although we have not made such compensating changes, we expect that the effect is small for the small enlargements considered here. Our results thus indicate that the estimates based on the model space given in Table I and Table II are sufficiently accurate for the purposes of comparing to experiment, where the errors are greater than 10%.

The reason for the stability is that the neutron and proton valence orbits are not close to being either fully occupied or

completely empty. In the opposite situation, choosing the optimal model space becomes a more delicate issue. There are a number of cases measured in Ref. [3] in which this is true for one of the shells (for example, Nb and Cs), and in these cases the choice of the optimal model space requires more effort. The data show that the spreading width is anomalously small in these cases, and it will be an interesting test of the theory to be able to reproduce this trend.

B. Deformation

Deformation contributes in two possible ways: if the nucleus has a permanent deformation, the one-body PNC potential develops a nonspherical component arising from the interaction of a valence nucleon with the deformed core. Otherwise, the two-body effective interaction may have a contribution that arises as one of the nucleons interacts with a core nucleon, exciting it and causing a core-deformation fluctuation. A second nucleon may then experience a long-range interaction with the first through the resulting core polarization. We estimate the effect of permanent deformation in this section.

1. Permanent deformation

The fact that the empirical and theoretical values of M more nearly agree for $A\sim 100$ than they do in the region of $A\sim 230$ could point to an important contribution of permanent deformation, since ^{238}U is a highly deformed nucleus but Pd is not. It is easy to show that there can be nonvanishing matrix elements of the deformed piece of $V_{Std}^{PNC(1)}$ for opposite-parity spherical orbits in which the j differ by zero or *two* units (and for opposite-parity deformed orbits in which the z-projection quantum numbers Ω are the same). For the spherical orbits in Table I there are pairs of orbits for both protons and neutrons that satisfy this requirement and have rather large matrix elements. Thus, for our calculations in the $A\sim 230$ region we will have direct contributions from $V_{Std}^{PNC(1)}$ even in the $0\hbar\omega$ model space, similar to the situation in nuclear matter [29].

We find an estimate for the contribution to M arising from the deformation by using a one-body potential similar to that given in Eq. (26) and using a harmonic oscillator basis, replacing ρ by a density having a deformed Woods-Saxon shape with radius R and diffuseness a. The contribution of the one-body PV potential arising from deformation is found by expanding the density to lowest order in the deformation parameter β_2, leading to the following shape:

$$\delta\rho(r)/\rho_0 = -\frac{R\beta_2}{a} g(r) Y_{20}(\hat{r}), \tag{31}$$

where

$$g(r) = \frac{\rho(r)}{\rho_0}\frac{e^{(r-R)/a}}{1+e^{(r-R)/a}}. \tag{32}$$

Taking matrix elements between the single-particle orbitals, we obtain

$$\begin{aligned}\langle n_f l_f j_f m_i|\delta V^{PNC(1)}|n_i l_i j_i m_i\rangle &= \frac{-im\omega_0 R\beta_2}{a}(C_0+C_1, C_0-C_1)\\ &\times\langle n_f l_f j_f m_i|\frac{1}{2}[g(r)\sigma\cdot p+\sigma\cdot p g(r)]\\ &\times Y_{20}(\hat{r})|n_i l_i j_i m_i\rangle,\end{aligned} \tag{33}$$

where at $\rho=\rho_0$, $C_0+C_1=3.53\times10^{-8}$ and $C_0-C_1=-3.15\times10^{-9}$ for (proton,neutron) orbits, and where the radial quantum number n is the number nodes in the radial wave function, l is the orbital angular momentum, j is the total angular momentum, m is the projection of the total angular momentum along the z axis, and

$$\begin{aligned}\langle n_f l_f j_f^{\pm} m_i|\frac{1}{2}[g(r)\sigma\cdot p+\sigma\cdot p g(r)]Y_{20}(\hat{r})|n_i l_i j_i m_i\rangle &= (-1)^{m_i-3/2}\sqrt{(2j_f+1)(2j_i+1)}\begin{Bmatrix} j_i & 2 & j_f\\ l_f\pm1 & \frac{1}{2} & l_i\end{Bmatrix}\sqrt{\frac{5(2l_i+1)}{4\pi}}\begin{pmatrix} j_i & 2 & j_f\\ m_i & 0 & -m_i\end{pmatrix}\sqrt{2(l_f\pm1)+1}\begin{pmatrix} l_i & l_f\pm1 & 2\\ 0 & 0 & 0\end{pmatrix}\\ &\times\langle n_f l_f|\frac{1}{2}[g(r)\sigma\cdot p+\sigma\cdot p g(r)]|n_i l_i\pm1\rangle,\end{aligned} \tag{34}$$

with $j_f^{\pm}\equiv l_f\pm\frac{1}{2}$.

Matrix elements of Eq. (33) do not vanish in the model space of Table I. There are two nonvanishing matrix elements: one for the pair proton orbits $(h_{9/2}, i_{13/2})$ and one for the pair of neutron orbits $(j_{15/2}, i_{11/2})$. The matrix element of $\delta V^{PNC(1)}$ of Eq. (33) is about 2.4 times smaller for neutrons than for protons. Averaging the square of Eq. (33) over the quantum number m_i (noting that there are $2j_<+1$ nonvanishing matrix elements), we find that the magnitude of the average proton matrix element is 0.011 eV.

We have estimated M corresponding to these values bynormalizing to the calculation in Ref. [8] along the lines explained in Ref. [29], which gives

$$M_{Def}^2(\mathrm{keV}^2) = 2.6\alpha_p^2, \tag{35}$$

where

$$\alpha_p^2 = \frac{(1.2)^2}{A_v} \frac{\mathrm{Tr}[(\delta V^{PNC(1)})^2]}{\mathrm{Tr}[(U_2)^2]}. \tag{36}$$

The factor of $1/A_v$ arises because the trace is taken over the squares of the one- and two-body matrix elements within the model space; the square of the matrix elements is weighted by the relative number of valence nucleon pairs, and the factor of 1.2^2 represents the relative normalization of the strength function for a one- and two-body operator. The two-body interaction U_2 has no diagonal matrix elements but is otherwise modeled after the surface delta-function interaction [31]. We estimated in this fashion that in ^{238}U, $M_{Def} = 0.004$ meV.

Compared to the values of M arising from other sources in Table IV, we see that the contribution of deformation to M is completely negligible. A more quantitative calculation would entail a calculation using a deformed model space of Nilsson orbits, which would require no new theoretical development because statistical spectroscopy as formulated in Sec. III B applies in either a spherical or deformed basis.

2. Core polarization

Core polarization as described at the beginning of this section has been studied phenomenologically, and its contribution to the effective nucleon-nucleon interaction is known to act dominantly in $T=1$ and be largely independent of spin [32]. Just as in the case of the doorway model [13] the effective parity-violating interaction results from the commutator with the residual strong interaction and $\boldsymbol{\sigma}\cdot\mathbf{r}$. This commutator does not vanish for the core-polarization piece of the effective nucleon-nucleon interaction, and the resulting contribution to the effective PNC interaction deserves quantitative study in the future.

C. Correlations between *O* and *H*

There are several classes of correlations that we wish to mention. One arises when O couples strongly to a mode of motion generated by H lying predominantly outside the model space. In such a case, one would miss important physics by averaging O over only those states within the model space. To deal with this situation properly, one must use effective interaction theory and consider instead of O its effective operator counterpart $\overline{O}$. The bookkeeping rules of the theory ensure that corrections necessary to represent the effect of this correlation are included in $\overline{O}$.

Such a correlation is quite important for parity violation, where the isoscalar and isovector 0^- spin-flip resonances couple strongly to the one-body piece of the parity-violating interaction, so in this case O is identified with $V_{Std}^{PNC(1)}$. In order to deal with this situation in the context of parity violation, the doorway model was introduced, as discussed in Sec. III A 3. We showed that the effect of the 0^- spin-flip excitations is quite important.

Yet another type of significant correlation occurs when the operator O and Hamiltonian H commute with each other or where both commute with a third operator R. Let us consider the latter case, so that H and O may be characterized by the same quantum numbers Γ that are the good quantum numbers of R. Values of the mean-square average of O over distinct ensembles of states, each characterized by its own value of Γ, would in general be different. Thus, it would be misguided to average O over an ensemble of states with mixed Γ, since H does not produce such mixtures. If a statistical ensemble of mixed Γ were formed, either because one was not aware of the symmetry R or because of some unfortunate approximation, this ensemble would fail to properly describe the system.

One place where such a consideration is relevant is in the description of spurious center-of-mass motion. Generally, the eigenstates of H may be grouped into sets distinguished by quantum numbers specifying the center-of-mass motion. Suppose that each such set is spanned by distinct individual-particle basis states. Then, any mean-square average of O corresponding to one set would differ from that of any other, because the corresponding statistical ensembles are different. For example, if one compares two ensemble averages of O, one in which the center-of-mass motion is pure and one in which it is a mixture of several modes, one may find different results. Fortunately, this problem turns out not to be a serious one for us. As can be seen from Tables I and II, our $0\hbar\omega$ basis does not permit any spurious center-of-mass excitations. Thus, all matrix elements of $V^{PNC(2)}$ are completely nonspurious. As is generally the case in applying perturbation theory to include excitations outside the model space, some level of spuriosity is unavoidably introduced in calculating $V^{PNC(1)}$. However, the nature of statistical calculations, where the observable is a mean-squared matrix element insensitive to relative phases, considerably reduces the magnitude of the problem compared to the standard shell-model case.

D. Choice of weak-coupling parameters

It is important to bear in mind that the relative sizes of M corresponding to $V_{Dwy}^{PNC(2)}$ and $V_{Std}^{PNC(2)}$ depend upon the choice of weak-coupling constants. The values of these coupling constants are not certain, so the relative sizes of M for the two contributions may actually be different (perhaps substantially so) than the values reported in Tables III–V when more refined choices of the coupling constants are made. As we have indicated, we use the DDH parameter set for our calculations given in these tables. A more detailed breakdown of the operators involved as well as a careful statistical study of the additional empirical information available from TRIPLE and from other sources should allow extraction of a consistent set of values for F_π and F_0 from experiment. The application of our statistical theory to the remaining nuclei of the TRIPLE data set requires a more elaborate study because, as we have remarked, in cases not examined here the valence

shells contain fewer nucleons and are therefore more sensitive to details requiring further refinement, such as the size of the model space and possibly the single-particle energies.

Note that the size of the *Dwy* contribution in our calculations is determined predominantly by our choice of the weak pion-nucleon coupling constant. If instead of the DDH value for this coupling parameter we would rely on empirical values inferred from ^{18}F [1], *Dwy* would be considerably smaller. Likewise, the value we found for the *Std* component of the force may change substantially. For example, if the measured anapole moment of ^{133}Cs is taken to determine the weak π and ρ coupling constants, with F_π constrained by the measurement in ^{18}F, the value of the weak ρ coupling constant would be substantially larger [33], as would our value of the *Std* contribution in Table IV. The interplay between the role of the *Std* and *Dwy* contributions clearly depends on the region in F_π-F_0 space of interest, the details of which will be reported elsewhere.

E. Shell-model tests

There are a number of useful tests of the validity of the statistical model for parity violation that could be carried out within large-basis standard shell-model calculations. We give one example: how to model parity mixing, as manifested in *M*, as it develops between states within the $0\hbar\omega$ model space and those lying outside it. We have assumed in this paper that the mixing occurs through the doorway mechanism as modified by Desplanques [27]. The assumption can be evaluated in a toy model (based on large but finite spaces and schematic interactions) as follows: On the one hand, the chosen interaction may be diagonalized numerically (thus obtaining a model-exact result) using a large-basis shell-model code. This same problem may then be examined in a model space applying a version of effective interaction theory. Various questions may be answered by comparing results obtained from the exact diagonalization and from effective interaction theory. Limiting cases would be instructive to consider.

For example, if excited states are initially well separated from the model space by a large gap, the leading terms in the perturbative expansions discussed in Sec. III will dominate. In the opposite limit of a very small gap, which corresponds to the well-defined case of statistical mixing in the full space, the calculation again becomes simple. The more difficult question of theoretical interest is how to handle the intermediate cases, where the gap is neither small nor large. It is in this regime where the doorway model has been conjectured to be applicable. It would be interesting to (1) confirm the doorway model in specific numerical cases; it may even be possible to motivate extensions of it valid over larger gap sizes by making selective summations over specific classes of diagrams to account for multiple p-h excitations; (2) investigate the relative importance of effective three-body PNC interactions, which have so far been neglected in all theoretical studies. In the most favorable outcome, the doorway result we have already evaluated describes this mixing over gaps comparable to shell spacings. Well-posed questions in such a model could lead to a better understanding of the theoretical uncertainties in applying statistical spectroscopy to PNC.

VI. SUMMARY AND CONCLUSION

In this paper, we have made the first extension of statistical spectroscopy to the case of parity violation and the weak spreading width in nuclei as measured by the TRIPLE Collaboration [3]. We have emphasized that statistical strength-function methods are advantageous in this case because the observables are expressed in terms of matrix elements of the effective PNC interaction averaged over *squares* of wave function components in a basis of independent-particle model states. We have also stressed that the corresponding theoretical results are less sensitive to theoretical uncertainties in familiar shell-model calculations even though the nuclear wave functions are very complicated.

In extending statistical spectroscopy to parity violation, we find that it is essential to include corrections to the underlying parity violating interaction to account for nonstatistical correlations. Of particular importance are the spin-flip correlations between states separated by $n\hbar\omega$, with $n \geqslant 1$. In this regard, effective interaction theory is particularly useful, and the $n\hbar\omega$ corrections have been incorporated through an effective operator. In calculating the effective operator of the PNC interaction we have implemented in the RPA approximation suggested by Desplanques [27].

We have evaluated the weak-spreading width for $A \sim 230$ in the ^{238}U target and for $A \sim 100$ in the ^{104,105,106,108}Pd targets using the standard estimates for the weak-coupling parameters of Desplanques, Donoghue, and Holstein [9]. The theoretical results are in qualitative agreement with the experimental results: the measured *M* in the Pd isotopes differs by about a factor of 1.7 and in U by a factor of 3. This observation supports the hope that ambiguities present in the values of the underlying weak meson-nucleon coupling parameters may be settled by using the weak-spreading width measured via neutron scattering from compound nuclei [3].

We have investigated the sensitivity of our results to various improvements in the theory, such as the sensitivity to enlargements of the model space and to permanent deformation. Permanent deformation seems to be a small correction, and our estimates indicate that results for *M* are stable to 10% accuracy under enlargements of the space. The reason for the stability is that these nuclei are in regions of the periodic table where large numbers of neutrons and protons occupy the shells. When, on the other hand, shells are nearly empty (or nearly full), choosing the optimal model space becomes a more delicate issue. We have indicated that the explanation of the anomalously small spreading width in the Nb and Cs cases [3] will constitute an interesting test of the theory. Because of the extended space required, these calculations require considerably more effort and have therefore not been considered in this paper. We have suggested additional toy-model calculations that might provide insight and improve confidence in the theoretical results.

ACKNOWLEDGMENTS

The work of M.B.J. and J.D.B. was supported in part by the U.S. Department of Energy under Contract No. W-7405-ENG-36 and that of S.T. by National Science Foundation Grant No. PHY-9800106. S.T. expresses his gratitude for the hospitality extended by the Physics Division of Los Alamos National Laboratory.

APPENDIX: MATRIX ELEMENTS OF THE EFFECTIVE PARITY-VIOLATING INTERACTION

In this appendix we give explicit expressions for the effective two-body PNC interaction and their matrix elements in an harmonic oscillator basis that have been used for our numerical calculations.

We may represent V^{PNC} in general as

$$V^{PNC}=\sum_{\alpha} V_{\alpha}^{PNC}, \tag{A1}$$

where V_{α}^{PNC} are terms depending on spin operators $\theta^{S(\alpha)}$ and isospin operators $\theta^{T(\alpha)}$. The free-space PNC interaction in the meson-exchange model is given in Table VI. Although there have been numerous estimates of the strengths of the weak meson-nucleon coupling constants in the literature, for our numerical work we use the DDH ''best'' values of F_{π} and F_0 (corresponding to the values $f_{\pi}=0.454\times10^{-6}$ and $h_{\rho}^{0}=-1.14\times10^{-6}$). As we stated earlier, we omit the other coupling constants, since they are quite small in the DDH analysis.

The interplay between the two-body strong and weak interaction is approximately described [1] by the two-body correlation function $f(r)$ given by Ref. [34]. The modification in $V_{Std}^{PNC(2)}$ arising from the short-range correlation function occurs by multiplying the initial and final nuclear wave function by $1+f(r)$, where $f(r)$ describes the suppression of the relative wave function at short distances due to the action of the nucleon-nucleon interaction. Otherwise, the representation of $V_{Std}^{PNC(2)}$ is the same as that for the free-space interaction as given in Eq. (A1) and Table VI.

Our representation of $V_{Dwy}^{PNC(2)}$ is

$$V_{Dwy}^{PNC(2)}=\sum_{\alpha=7,9,11,13}\theta^{S(\alpha)}\cdot\mathbf{v}_1^{\alpha}(r)V_0^{\alpha}(R)\,\theta^{T(\alpha)} + \sum_{\alpha=15,17}\theta^{S(\alpha)}\cdot\mathbf{V}_1^{\alpha}(R)v_0^{\alpha}(r)\,\theta^{T(\alpha)}, \tag{A2}$$

where a dependence on R, the center-of-mass position of the two nucleons, is now possible because of the presence of a third body (the nucleus). The definition of the various quantities is given in Table VII. The pieces associated with the isovector 0^- resonance are proportional to C_1, which is dominated by F_{π}. These, in particular Eq. (4.10) of Ref. [13], are much more important than the isoscalar piece associated with the isoscalar 0^- resonance, which is proportional to C_0 and dominated by F_0 [the numerical values of C_{π} and C_{ρ} in Eq. (4.10) of Ref. [13] are misquoted there and should rather be identified with the values of c_{ρ}^{s} and c_{π}^{v} given in Eq. (A3), given next]. These constants are given in the local density approximation by

$$C_0=c_{\rho}^{s}F_0+c_{\pi}^{s}F_{\pi}\frac{N_c-Z_c}{A_c},$$

$$C_1=c_{\pi}^{v}F_{\pi}+c_{\rho}^{v}F_0\frac{N_c-Z_c}{A_c}, \tag{A3}$$

where $c_{\rho}^{s}=7.61\times10^{-3}$, $c_{\pi}^{s}=1.74\times10^{-2}$, $c_{\pi}^{v}=1.74\times10^{-2}$, and $c_{\rho}^{v}=1.3\times10^{-3}$, and where N_c and Z_c are the neutron and proton numbers of the core ($A_c=N_c+Z_c$). In our evaluation of $V_{Dwy}^{PNC(2)}$ in this paper, we have taken $\rho=\rho_0$.

Next, we give an explicit expression for the matrix elements of $V_{Std}^{PNC(2)}$ and $V_{Dwy}^{PNC(2)}$. We express the matrix element of the effective interaction $\overline{V}^{PNC(2)}$ in second-quantized notation as

$$\overline{V}^{PNC(2)}=\frac{1}{4}\sum_{\beta j\beta' j' J}(2J+1)^{1/2} \times\langle\beta_1 j_1;\beta_2 j_2||\tilde{V}_J^{PNC(2)}||\beta_1' j_1';\beta_2' j_2'\rangle \times\{(a_{\beta_1 j_1}^{\dagger}a_{\beta_2 j_2}^{\dagger})^J(\tilde{a}_{\beta_2' j_2'}\tilde{a}_{\beta_1' j_1'})^J\}_0^0, \tag{A4}$$

where the sum runs over all quantum numbers (β,j), where β corresponds to the radial quantum number n, the orbital angular momentum, the spin, and the isospin of the nucleon. Here $a_{\beta_1 j_1}^{\dagger}$ is a creation operator, and $\tilde{a}_{\beta_1 j_1}$ is related to the corresponding annihilation operator by $\tilde{a}_{jm}=(-1)^{j+m}a_{j,m}$. The density matrix corresponds to initial and final pairs of particles (of total angular momentum J) coupled to $\Delta J=0$. The matrix element is antisymmetrized,

$$\langle\beta_1 j_1;\beta_2 j_2||\tilde{V}_J^{PNC(2)}||\beta_1' j_1';\beta_2' j_2'\rangle =\langle\beta_1 j_1;\beta_2 j_2||\overline{V}_J^{PNC(2)}||\beta_1' j_1';\beta_2' j_2'\rangle -(-1)^{j_1+j_2-J}\langle\beta_2 j_2;\beta_1 j_1||\overline{V}_J^{PNC(2)}||\beta_1' j_1';\beta_2' j_2'\rangle, \tag{A5}$$

and reduced in angular momentum only (we calculate separately matrix elements for neutron pairs, proton pairs, and neutron-proton pairs).

A general expression for $\overline{V}_J^{PNC(2)}$ is

$$\langle \beta_1 j_1;\beta_2 j_2||\overline{V}_J^{PNC(2)}||\beta_1' j_1';\beta_2' j_2'\rangle = \sum_\alpha \langle N_{\beta_1} N_{\beta_2}|T^{(\alpha_T)}|N_{\beta_1'} N_{\beta_2'}\rangle \sum_{\substack{nl\\NL}} \sum_{\substack{n'l'\\N'L'}} \sum_{S\mathcal{L}} \sum_{S'\mathcal{L}'} [(2j_1+1)(2j_2+1)(2j_1'+1)(2j_2'+1)$$

$$\times(2\mathcal{L}+1)^2(2S+1)^2(2\mathcal{L}'+1)(2S'+1)]^{1/2} \begin{Bmatrix} l_1 & l_2 & \mathcal{L} \\ \frac{1}{2} & \frac{1}{2} & S \\ j_1 & j_2 & J \end{Bmatrix} \begin{Bmatrix} l_1' & l_2' & \mathcal{L}' \\ \frac{1}{2} & \frac{1}{2} & S' \\ j_1' & j_2' & J \end{Bmatrix}$$

$$\times\langle n'l',N'L',\mathcal{L}'|n_1'l_1',n_2'l_2',\mathcal{L}'\rangle\langle nl,NL,\mathcal{L}|n_1l_1,n_2l_2,\mathcal{L}\rangle(-)^{J+\mathcal{L}'+S} \begin{Bmatrix} \mathcal{L} & \mathcal{L}' & 1 \\ S' & S & J \end{Bmatrix}$$

$$\times[3(2l+1)(2L+1)]^{1/2} \begin{Bmatrix} \mathcal{L} & \mathcal{L}' & 1 \\ l & l' & k_1 \\ L & L' & k_2 \end{Bmatrix} \langle S\|S_\lambda^{S(\alpha_S)}\|S'\rangle\langle nl\|v_{k_1}^\alpha\|n'l'\rangle\langle NL\|V_{k_2}^\alpha\|N'L'\rangle, \tag{A6}$$

where the functional dependence of α_S, α_T, k_1, and k_2 on α as well as the values of v_{k1}^α, V_{k2}^α, $S^{(\alpha_S)}$, and $T^{(\alpha_T)}$ are given in Tables VI and VII for the standard and doorway pieces of the effective interaction, and we have made a Moshinsky transfor-

TABLE VI. Free-space parity-violating interaction of Desplanques, Donaghue, and Holstein [10].

(a) Definition of $V_\alpha^{PNC} \equiv \theta^{T(\alpha)}\boldsymbol{\theta}^{S(\alpha)}\cdot\mathbf{v}^\alpha$ for the standard PNC interaction.

α	α_S	α_T	$\theta^{T(\alpha)}=T^{(\alpha_T)}$	$\theta^{S(\alpha)}=\mathbf{S}^{(\alpha_S)}$	v^α
1	2	3	$t_{10}^{(3)}$	$\mathbf{S}^{(2)}$	$\frac{\sqrt{2}}{M}(F_\pi\mathbf{u}_\pi^{(-)}+H_1\mathbf{u}_\rho^{(-)})$
2	1	4	$F_0t_{00}^{(4)}-\frac{1}{2\sqrt{3}}F_1t_{10}^{(2)}-\frac{1}{2\sqrt{3}}F_2t_{20}^{(6)}$	$\mathbf{S}^{(1)}$	$-\frac{\sqrt{3}}{M}\mathbf{u}_\rho^{(+)}$
3	3	4	$F_0t_{00}^{(4)}-\frac{1}{2\sqrt{3}}F_1t_{10}^{(2)}-\frac{1}{2\sqrt{3}}F_2t_{20}^{(6)}$	$\mathbf{S}^{(3)}$	$-\frac{\sqrt{6}}{M}(1+\mu_V)\mathbf{u}_\rho^{(-)}$
4	1	2	$G_0t_{00}^{(0)}+\frac{1}{2}G_1t_{10}^{(2)}$	$\mathbf{S}^{(1)}$	$\frac{1}{M}\mathbf{u}_\omega^{(+)}$
5	3	2	$G_0t_{00}^{(0)}+\frac{1}{2}G_1t_{10}^{(2)}$	$\mathbf{S}^{(3)}$	$\frac{\sqrt{2}}{M}(1+\mu_S)\mathbf{u}_\omega^{(-)}$
6	2	1	$t_{10}^{(1)}$	$\mathbf{S}^{(2)}$	$\frac{1}{2M}(G_1\mathbf{u}_\omega^{(+)}-F_1\mathbf{u}_\rho^{(+)})$

(b) Definition of operators in Table VI(a). Note that the momentum operator $\mathbf{p}$ is defined as twice the relative momentum, $\mathbf{p}=\mathbf{p}_1-\mathbf{p}_2$.

Operator		Operator	
Spin		Isospin	
$\mathbf{S}^{(1)}$	$\boldsymbol{\sigma}(1)-\boldsymbol{\sigma}(2)$	$t_{00}^{(0)}$	1
$\mathbf{S}^{(2)}$	$\boldsymbol{\sigma}(1)+\boldsymbol{\sigma}(2)$	$t_{10}^{(1)}$	$\tau_z(1)-\tau_z(2)$
$\mathbf{S}^{(3)}$	$i\boldsymbol{\sigma}(1)\times\boldsymbol{\sigma}(2)/\sqrt{2}$	$t_{10}^{(2)}$	$\tau_z(1)+\tau_z(2)$
Space		$t_{10}^{(3)}$	$i[\boldsymbol{\tau}(1)\times\boldsymbol{\tau}(2)]_z/\sqrt{2}$
$\mathbf{u}(r)^{(-)}$	$[\boldsymbol{p},u_0]$	$t_{00}^{(4)}$	$-\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)/\sqrt{3}$
$\mathbf{u}(r)^{(+)}$	$\{\boldsymbol{p},u_0\}$	$t_{20}^{(6)}$	$[3\tau_z(1)\tau_z(2)-\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)]/\sqrt{6}$
$u_0(r)$	$e^{-mr/4\pi r}$		

TABLE VII. Effective interaction arising from the doorway contribution as formulated in Ref. [18]. Some of the notation is defined in Table VI. Note that the doorway contribution is defined for odd values of α only. $\lambda_{00}=-53.9$ MeV fm^3, $\lambda_{11}=239$ MeV fm^3, $\lambda_{01}=200$ MeV fm^3, $\lambda_{10}=59.8$ MeV fm^3, $M=4.758$ fm^{-1} (nucleon mass), and $\mu=3.897$ fm^{-1} (ρ meson mass).

α	α_S	α_T	k_1	k_2	$\theta^{T(\alpha)}=T^{(\alpha_T)}$	$\theta^{S(\alpha)}=\mathbf{S}^{(\alpha_S)}$	$v^{\alpha}_{k1}(r)$ (fm^{-1})	$V^{\alpha}_{k2}(R)$
7	3	4	1	0	$\frac{-\sqrt{6}M\mu\omega_0}{\omega_{is}\hbar c}\lambda_{11}C_0t^{(4)}_{00}$	$S^{(3)}$	$\frac{i\mu}{4\pi}e^{-\mu r}\hat{r}$	1
9	3	2	1	0	$\frac{\sqrt{2}M\mu\omega_0}{\omega_{is}\hbar c}\lambda_{10}C_0t^{(0)}_{00}$	$S^{(3)}$	$\frac{i\mu}{4\pi}e^{-\mu r}\hat{r}$	1
11	2	3	1	0	$\frac{\sqrt{2}M\mu\omega_0}{2\omega_{iv}\hbar c}(\lambda_{01}+\lambda_{11})C_1t^{(3)}_{10}$	$S^{(2)}$	$\frac{i\mu}{4\pi}e^{-\mu r}\hat{r}$	1
13	3	4	1	0	$\frac{\sqrt{2}M\mu\omega_0}{2\omega_{iv}\hbar c}(\lambda_{10}+\lambda_{11})C_1t^{(2)}_{10}$	$S^{(3)}$	$\frac{i\mu}{4\pi}e^{-\mu r}\hat{r}$	1
15	1	3	0	1	$\frac{\sqrt{2}M\mu\omega_0}{\omega_{iv}\hbar c}(\lambda_{01}-\lambda_{11})C_1t^{(3)}_{10}$	$S^{(1)}$	$\frac{i}{4\pi r}e^{-\mu r}$	$\mu\vec{R}$
17	3	1	0	1	$\frac{\sqrt{2}M\mu\omega_0}{\omega_{iv}\hbar c}(\lambda_{10}-\lambda_{11})C_1t^{(1)}_{10}$	$S^{(3)}$	$\frac{i}{4\pi r}e^{-\mu r}$	$\mu\vec{R}$

mation to the relative and center-of-mass coordinates ($\langle nl,NL,\mathcal{L}|n_1l_1,n_2l_2,\mathcal{L}\rangle$ is a Moshinsky bracket [35]). This is a generalization of the results given in Ref. [36]. Our angular momentum conventions follow Brink and Satchler [37]. We have checked our results against various special cases, and two codes have been written independently to check the calculation of $V^{PNC(2)}_{Std}$. The V^{Γ}_{rstu} of Eq. (19) is equal to the $\tilde{V}^{PNC(2)}_{J}$ in Eq. (A4) multiplied by $\sqrt{2}$ when the quantum numbers of particle 1 or 2 are equal in the initial or final states.

[1] E. G. Adelberger and W. C. Haxton, Annu. Rev. Nucl. Part. Sci. **35**, 35 (1985).

[2] J. D. Bowman, G. T. Garvey, M. B. Johnson, and G. E. Mitchell, Annu. Rev. Nucl. Part. Sci. **43**, 829 (1993).

[3] J. D. Bowman *et al.*, Phys. Rev. Lett. **65**, 1192 (1990); X. Zhu *et al.*, Phys. Rev. C **46**, 768 (1992); C. M. Frankle *et al.*, Phys. Rev. Lett. **46**, 1542 (1992); B. E. Crawford *et al.*, Phys. Rev. C **58**, 1225 (1998); J. D. Bowman *et al.* (unpublished).

[4] J. B. French and L. S. Hsu, Phys. Lett. **25B**, 75 (1967); F. S. Chang, J. B. French, and T. H. Thio, Ann. Phys. (N.Y.) **66**, 137 (1971); K. K. Mon and J. B. French, *ibid.* **95**, 90 (1975); J. P. Draayer, J. B. French, and S. S. M. Wong, *ibid.* **106**, 472 (1977); **106**, 503 (1977); J. B. French, P. A. Mello, and A. Pandey, *ibid.* **113**, 277 (1978); J. B. French and V. K. B. Kota, Phys. Rev. Lett. **51**, 2183 (1983); J. B. French, V. K. B. Kota, A. Pandey, and S. Tomsovic, Ann. Phys. (N.Y.) **181**, 198 (1988); **181**, 235 (1988).

[5] S. Tomsovic, M. B. Johnson, A. Hayes, and J. D. Bowman (unpublished).

[6] W. C. Haxton, Phys. Rev. Lett. **46**, 698 (1981).

[7] O. Bohigas and H. A. Weidenmueller, Annu. Rev. Nucl. Part. Sci. **38**, 421 (1988).

[8] S. Tomsovic, Ph.D. thesis, University of Rochester, Report No. 974, 1987 (unpublished).

[9] B. Desplanques, J. F. Donoghue, and B. R. Holstein, Ann. Phys. (N.Y.) **124**, 449 (1980).

[10] J. B. French and V. K. B. Kota, Phys. Rev. Lett. **51**, 2183 (1983).

[11] P. J. Ellis and E. Osnes, Rev. Mod. Phys. **49**, 777 (1976); B. R. Barrett and W. M. Kirson, in *Advances in Nuclear Physics 6*, edited by M. Baranger and E. Vogt (Plenum, New York, 1973), p. 219.

[12] M. B. Johnson and M. Baranger, Ann. Phys. (N.Y.) **62**, 172 (1971).

[13] M. B. Johnson and J. D. Bowman, Phys. Rev. C **51**, 999 (1995).

[14] E. P. Wigner, Ann. Math. **62**, 548 (1955); M. L. Mehta, *Random Matrices and the Statistical Theory of Energy Levels*, 2nd ed. (Academic Press, New York, 1990).

[15] See several of the courses in *Mesoscopic Quantum Physics*, Proceedings of the Les Houches Summer School of Theoretical Physics, Haute-Savoie, France, 1994, edited by E. Akkermans, G. Montambaux, J.-L. Pichard, and J. Zinn-Justin (Elsevier Science, New York, 1995), Session LXI.

[16] H. Baltes and E. R. Hilf, *Spectra of Finite Systems* (Bibliographisches Institut, Mannheim, 1976).

[17] H. A. Bethe, Phys. Rev. **50**, 332 (1936).

[18] W. Dilg, W. Schantl, H. Vonach, and M. Uhl, Nucl. Phys. **A217**, 269 (1973); A. Gilbert and A. G. W. Cameron, Can. J. Phys. **43**, 1446 (1965).

[19] S. Perez, FORTRAN code BDSTS, 1973.

[20] A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. I.

[21] W. C. Haxton, B. F. Gibson, and E. M. Henley, Phys. Rev. Lett. **45**, 1677 (1980).

[22] B. A. Brown, W. A. Richter, and N. S. Godwin, Phys. Rev. Lett. **45**, 1681 (1980).
[23] A. C. Hayes, in *Parity and Time Reversal Violation in Compound Nuclear States and Related Topics*, edited by N. Auerbach and J. D. Bowman (World Scientific, Singapore, 1996), p. 272.
[24] V. V. Flambaum and O. K. Vorov, Phys. Rev. Lett. **70**, 4051 (1993); Phys. Rev. C **51**, 1521 (1995).
[25] N. Auerbach, Phys. Rev. C **45**, R514 (1992).
[26] V. V. Flambaum and O. K. Vorov, Phys. Rev. C **49**, 1827 (1994).
[27] B. Desplanques, Phys. Rep. **297**, 1 (1998).
[28] D. Pines, K. F. Quader, and J. Wambach, Nucl. Phys. **A477**, 365 (1988).
[29] M. B. Johnson, J. D. Bowman, and H. S. Yoo, Phys. Rev. Lett. **67**, 310 (1991); M. B. Johnson, S. H. Yoo, and D. J. Bowman (unpublished).
[30] J. B. French and S. S. M. Wong, Phys. Lett. **33B**, 449 (1970); O. Bohigas and J. Flores, *ibid.* **34B**, 261 (1971); T. A. Brody, J. Flores, J. B. French, P. A. Mello, A. Pandey, and S. S. M. Wong, Rev. Mod. Phys. **53**, 385 (1981).
[31] P. W. M. Glaudemans and P. J. Brussard, *Shell-Model Applications in Nuclear Spectroscopy* (North-Holland, Amsterdam, 1977).
[32] A. Molinari, M. B. Johnson, H. A. Bethe, and W. M. Alberico, Nucl. Phys. **A239**, 43 (1974).
[33] W. S. Wilburn and J. D. Bowman, Phys. Rev. C **57**, 3425 (1998).
[34] G. A. Miller and J. E. Spencer, Ann. Phys. (N.Y.) **100**, 562 (1976).
[35] T. A. Brody and M. Moshinsky, *Tables of Transformation Brackets for Nuclear Shell-Model Calculations* (Gordon and Breach, New York, 1967).
[36] B. Castel and I. S. Towner, *Modern Theories of Nuclear Moments* (Clarendon Press, Oxford, 1990).
[37] D. M. Brink and G. R. Satchler, *Angular Momentum* (Clarendon Press, Oxford, 1968).