4.B-7

A rigid tank contains both air and a block of copper, as shown in Figure 4.B-7. The internal volume of the tank (the volume available for air) is V = 1 m^3. The initial pressure of the air in the tank is P_1 = 5000 Pa and the initial temperature of both the air and the copper block is T_1 = 200 K.

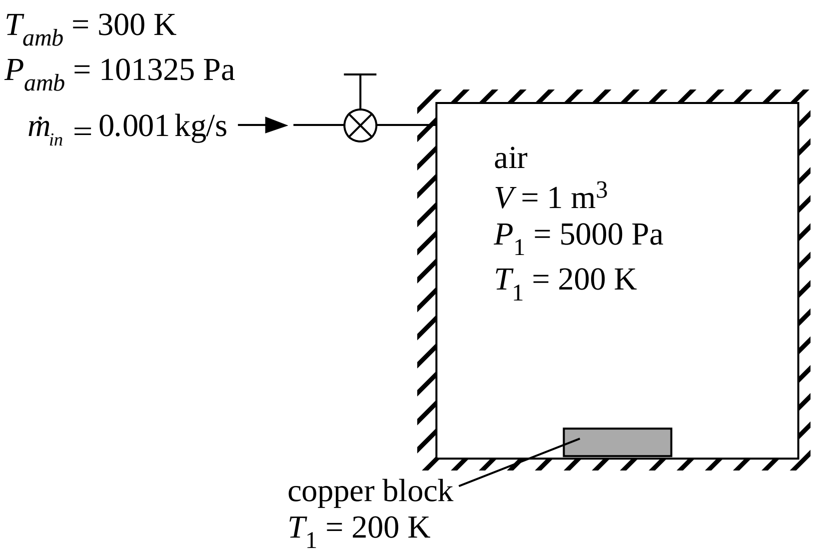

Figure 4.B-7: A rigid tank containing air and a block of copper.

At some time the tank valve starts to leak. Air from ambient conditions, T_{amb} = 300 K and P_{amb} = 101325 Pa, flow through the valve and enters the tank. The valve leaks for *time* = 15 minutes at a constant mass flow rate of $\dot{m}_{in}$ = 0.001 kg/s. After 15 minutes the leak stops and the temperature of both the air in the tank and the block of copper is T_2 = 280 K. You may assume that the contents of the tank are perfectly insulated from the outside environment (i.e., it is adiabatic). Model air as an ideal gas with constant specific heat capacities: R = 287 J/kg-K, c_V = 717 J/kg-K, and c_P = 1004 J/kg-K. Model copper as an incompressible substance with specific heat capacity c_b = 370 J/kg-K.

a.) What is the mass of air that is initially in the tank?

The initial state of the air in the tank is specified by the temperature and pressure. The specific volume is:

$$v_1 = \frac{R\,T_1}{P_1} = \frac{287\text{ N-m}}{\text{kg-K}}\left|\frac{200\text{ K}}{}\right|\frac{\text{m}^2}{5000\text{ N}} = 11.48\text{ m}^3/\text{kg} \quad (1)$$

The specific internal energy is:

$$u_1 = c_V\,T_1 = \frac{717\text{ J}}{\text{kg-K}}\left|\frac{200\text{ K}}{}\right| = 143400\text{ J/kg} \quad (2)$$

The mass of air in the tank is:

$$m_1 = \frac{V}{v_1} = \frac{1\,\text{m}^3}{} \left| \frac{\text{kg}}{11.48\,\text{m}^3} \right| = 0.0871\,\text{kg} \tag{3}$$

b.) What is the pressure in the tank after 15 minutes?

The mass entering the tank is obtained from:

$$m_{in} = \dot{m}_{in}\, time = \frac{0.001\,\text{kg}}{\text{s}} \left| \frac{900\ \text{s}}{} \right| = 0.9\,\text{kg} \tag{4}$$

A mass balance on the tank is shown in Figure 2 and leads to:

$$m_{in} = m_2 - m_1 \tag{5}$$

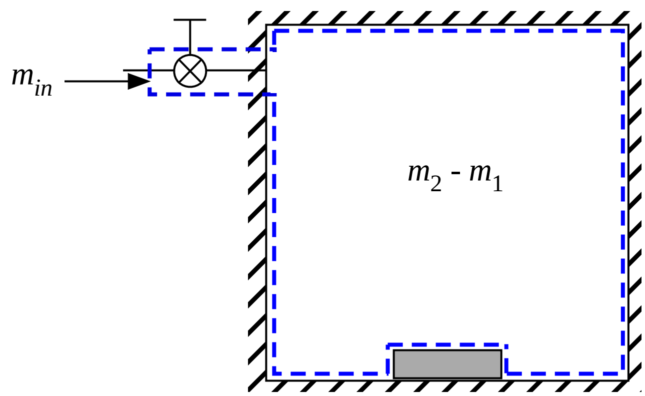

Figure 2: Mass balance.

Solving Eq. (5) for the final mass of air in the tank leads to:

$$m_2 = m_1 + m_{in} = 0.0871\,\text{kg} + 0.9\ \text{kg} = 0.9871\,\text{kg} \tag{6}$$

The final specific volume of the air in the tank is:

$$v_2 = \frac{V}{m_2} = \frac{1\,\text{m}^3}{0.9871\,\text{kg}} = 1.013\,\text{m}^3/\text{kg} \tag{7}$$

State 2 is specified by the specific volume and temperature. The final pressure is:

$$P_2 = \frac{R\,T_2}{v_2} = \frac{287\ \text{N-m}}{\text{kg-K}} \left| \frac{280\ \text{K}}{} \right| \frac{\text{kg}}{1.013\,\text{m}^3} = 79324\,\text{Pa} \tag{8}$$

and the specific internal energy is:

$$u_2 = c_V\, T_2 = \frac{717\ \text{J}}{\text{kg-K}}\left|\frac{280\ \text{K}}{}\right| = 200760\ \text{J/kg} \tag{9}$$

c.) What is the heat transfer from the air to the copper block during the leak process?

An energy balance on the air alone (excluding the copper block) is shown in Figure 3 and leads to:

$$m_{in}\, h_{in} = Q + m_2\, u_2 - m_1\, u_1 \tag{10}$$

where Q is the heat transfer from the air to the copper block and h_{in} is the enthalpy of the air entering the control volume (the air entering the valve at ambient conditions). The enthalpy of the air entering the valve is:

$$h_{in} = c_P\, T_{amb} = \frac{1004\ \text{J}}{\text{kg-K}}\left|\frac{300\ \text{K}}{}\right| = 301200\ \text{J/kg} \tag{11}$$

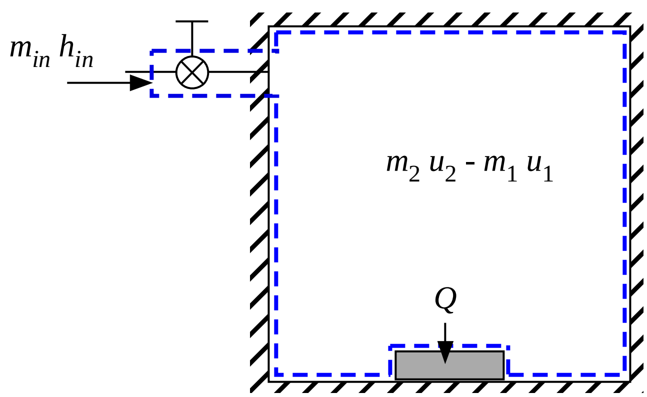

Figure 3: Energy balance on the air.

Solving Eq. (10) for the heat transfer leads to:

$$\begin{aligned} Q &= m_{in}\, h_{in} - m_2\, u_2 + m_1\, u_1 \\ &= \frac{0.9\ \text{kg}}{}\left|\frac{301200\ \text{J}}{\text{kg}} - \frac{0.9871\ \text{kg}}{}\right|\frac{200760\ \text{J}}{\text{kg}} + \frac{0.0871\ \text{kg}}{}\left|\frac{143400\ \text{J}}{\text{kg}}\right. \\ &= 85399\ \text{J} \end{aligned} \tag{12}$$

d.) What is the mass of the copper block?

An energy balance on the copper block is shown in Figure 4 and leads to:

$$Q = m_b\left(u_{b,2} - u_{b,1}\right) \tag{13}$$

The specific internal energy difference can be expressed in terms of the specific heat capacity:

$$Q = m_b\, c_b \left(T_2 - T_1\right) \tag{14}$$

Solving Eq. (14) for the mass of copper leads to:

$$m_b = \frac{Q}{c_b\left(T_2 - T_1\right)} = \frac{85399\,\text{J}}{}\left|\frac{\text{kg-K}}{370\,\text{J}}\right|\frac{}{\left(280-200\right)\text{K}} = 2.885\,\text{kg} \tag{15}$$

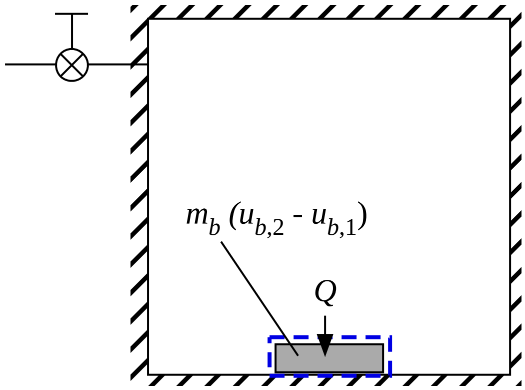

Figure 4: Energy balance on the copper block.

e.) What is the temperature of the air leaving the valve and entering the tank? Justify your answer clearly.

An energy balance on the valve is shown in Figure 5 and leads to:

$$\dot{m}_{in}\, h_{in} = \dot{m}_{out}\, h_{out} \tag{16}$$

There is no storage of mass in the valve, therefore $\dot{m}_{in} = \dot{m}_{out}$ and Eq. (16) can be reduced to:

$$h_{in} = h_{out} \tag{17}$$

For an ideal gas, the enthalpy is only a function of temperature and therefore:

$$T_{in} = T_{out} \tag{18}$$

so the temperature of the air leaving the valve is always equal to 300 K.

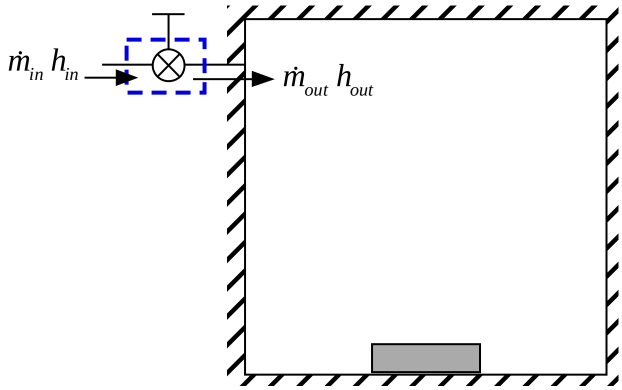

Figure 5: Energy balance on the valve.