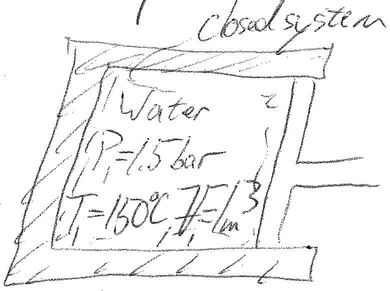
Closed System Energy Balances:

- Last week we covered property models of fluids, the week before we discussed systems, Energy & balances
- Today we start puting everything together: Balances, Energy, & fluid properties

Example: Water is compressed in a series of steps:



1→2) First it is compressed @ constant temperature until liquid water droplets begin to form. This process is maintained @ constant temperature by heat transfer to the surroundings.

2→3) The piston is pushed in farther until $V_3 = 0.05 m^3$ again @ constant temperature

Find: the heat transfer from 2→3 to maintain the constant temperature.

☆ How do we begin? Lets establish some general steps for problem solving.

Step 1: Draw & Define the system.

Step 2: List assumptions about the problem

1) Pure water (real fluid model), 2) sealed piston (closed system), 3) rigid cylinder, 4) no ΔPE, ΔKE, or ΔME, 5) process is incremental

Step 3: Apply Balances & Simplify (if needed)

Mass Balance: $M_{in}^{0} + M_{produced}^{0} = M_{out}^{0} + M_{destroyed}^{0} + \Delta M_{stored} \Rightarrow \Delta M_{stored} = M_2 - M_1 = 0$

$M_2 = M_1$

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Energy Balance: $E_{in} + \overset{0}{\cancel{E_{produced}}} = E_{out} + \overset{0}{\cancel{E_{destroyed}}} + \Delta E_{stored} \Rightarrow$

$\Rightarrow \underbrace{\cancel{Q_{in}}}_{0(5)} + W_{in} = Q_{out} + \underbrace{\cancel{W_{out}}}_{0(5)} + \Delta U + \underbrace{\cancel{\Delta PE}}_{0(4)} + \underbrace{\cancel{\Delta KE}}_{0(4)} + \underbrace{\cancel{\Delta XE}}_{0(4)} \Rightarrow$

$\Rightarrow W_{in} = Q_{out} + \Delta U \Rightarrow \quad m(u_3 - u_2)$

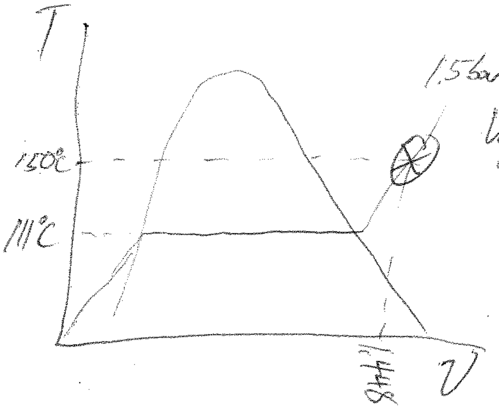
$\hookrightarrow PV_{work} \Rightarrow PV_{work} = \ominus \int_{V_2}^{V_3} P\,dV$

PV work in compression must be negative because $V_3 << V_2$ & work is going into the system. So we know that the work term must be positive & this sign is needed.

$\Rightarrow -\int_{V_2}^{V_3} P\,dV = Q_{out} + m(u_3 - u_2)$

this is about as simplified as we can make it.

Step 4: Apply fluid model, Draw diagram, & fix the states



State 1: $P_1 = 1.5$ bar, $T_1 = 150°C$, $V_1 = 1 m^3$

Vapor Table A-5: $T_{1,sat} = 111.3°C$ so we must be in the superheated vapor region

Go to Table A-6 to find v & u

@ 100 kPa & 150°C $v = 1.9367\ m^3/kg$, $u = 2{,}582.9\ kJ/kg$

@ 200 kPa & 150°C $v = 0.95986$, $u = 2571.1\ kJ/kg$

We need to Interpolate!

$$\frac{P - P_{low}}{P_{high} - P_{low}} = \frac{v - v_{low}}{v_{high} - v_{low}} \Rightarrow \frac{150-100}{200-100} = \frac{v - 1.9367}{0.95986 - 1.9367} \Rightarrow v_1 = 1.448\ m^3/kg$$

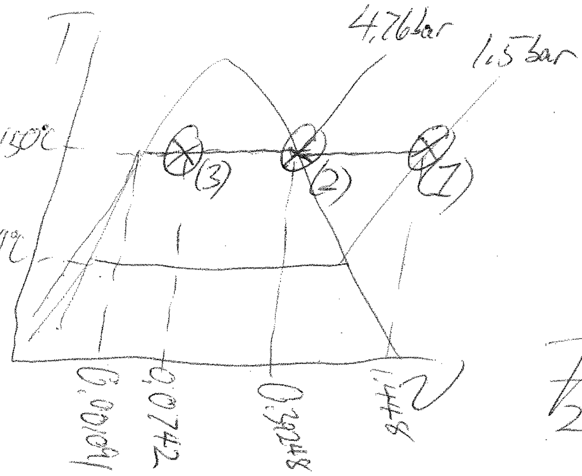
$$\frac{P - P_{low}}{P_{high} - P_{low}} = \frac{u - u_{low}}{u_{high} - u_{low}} \Rightarrow \frac{150-100}{200-100} = \frac{u - 2582.9}{2571.1 - 2582.9} \Rightarrow u_1 = 2580\ kJ/kg$$

We can find the mass of water in the tank now:

$$M = \frac{V_1}{v_1} = \frac{1m^3}{} \Big| \frac{kg}{1.448 m^3} \Rightarrow \underline{M = 0.690 kg}$$

Now find state 2: compressed @ constant T until droplets form: $X_2 = 1.0$

$T_2 = 150°C$



Which table? Saturated water A-4 @ $T_2 = 150°C$

we see that $P_2 = 4.76 bar$ $\quad v_{g,2} = 0.39248\ m^3/kg$

$u_{g,2} = 2559.1\ kJ/kg$

$$V_2 = ? = M v_{g,2} = 0.690kg(0.39248\ m^3/kg) \Rightarrow \underline{V_2 = 0.271 m^3}$$

Now find state 3: compressed @ constant T until $V_3 = 0.05 m^3$, $T_3 = 150°C$

What can we find? we know that $v_{f,2} = 0.001091\ m^3/kg$ are we above this?

$$v_3 = \frac{V_3}{M} = \frac{0.05 m^3}{0.690 kg} \Rightarrow \underline{v_3 = 0.0742\ m^3/kg}$$ Yes we're in the vapor dome

But how to find u_3? → find the quality: $X_3 = \frac{v_3 - v_f}{v_g - v_f} \Rightarrow \frac{0.0742 - 0.001091}{0.39248 - 0.001091} \Rightarrow X_3 = 0.187$

$$X_3 = \frac{u_3 - u_f}{u_g - u_f} \Rightarrow u_3 = u_f + x(u_g - u_f) \Rightarrow u_3 = 631.66 + 0.187(1927.4) \Rightarrow \underline{u_3 = 992.08\ kJ/kg}$$

Volume of just the liquid? $x = 1 - \frac{M_f}{M} \Rightarrow M_f = (1-x)M \Rightarrow (1-0.187)0.690kg$

$\boxed{M_f = 0.561 kg}$

$$V_{liq} = M_f v_f = \frac{0.561 kg}{} \Big| \frac{0.00109 m^3}{kg} \Rightarrow \underline{V_{liq} = 0.000612 m^3}$$

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Step 5: Now solve balance: ~~State~~ $-\int_{V_2}^{V_3} P\,dV = Q_{out} + m(u_3 - u_2)$

State 2→3 process occurs @ constant pressure

$-P(V_3 - V_2) = Q_{out} + m(u_3 - u_2) \Rightarrow Q_{out} = P(\overset{mv_2}{V_2} - \overset{mv_3}{V_3}) + m(u_2 - u_3)$

$Q_{out} = 476{,}000\,Pa(0.271\,m^3 - 0.05\,m^3) + 0.60\,kg(2{,}559{,}100\,\frac{J}{kg} - 992{,}080\,\frac{J}{kg})$

$Q_{out} = 1{,}186{,}000\,J \Rightarrow \boxed{Q_{out} = 1.186\,MJ}$

→ Heat left the system as the sign is still positive

makes sense with engineering judgement