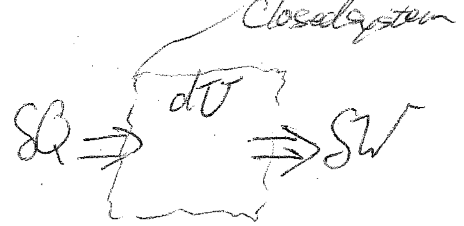
On Friday we derived the property entropy & applied it to a closed system with real fluid properties.

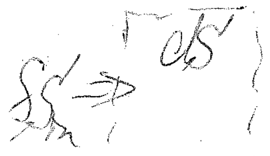
Today we will calculate the change in entropy of incompressible substances & ideal gases. To do this we must first show how entropy is related to other thermodynamic properties.

Lets start with a differentially small piece of fluid



<u>1st Law Balance</u>: $\delta E_{in} = \delta E_{out} + dE_{stored} \Rightarrow$

$\Rightarrow \delta Q = \delta W_{w} + dU \rightarrow P d\mathcal{V} = \text{work} = \delta W$



<u>2nd Law Balance</u>: $\delta S_{in} + \cancel{\delta S_{gen}} = \cancel{\delta S_{out}} + dS_{stored}$

$\frac{\delta Q}{T_b}$ If in equilibrium & differentially small $T_b = T$

$$\frac{\delta Q}{T} = dS_{stored} \Rightarrow \delta Q = T dS$$

Substituting

$T dS = P d\mathcal{V} + dU$ or specifically $T ds = P dv + du$ ← One of famous Tds relations

If we know T, P, v & u we can calculate the change in entropy directly for <u>any</u> fluid

$$ds = \frac{P}{T} dv + \frac{du}{T}$$

how integrating: $$\int_{s_1}^{s_2} ds = \int_{v_1}^{v_2} \frac{P}{T}dv + \int_{u_1}^{u_2} \frac{du}{T}$$

But what do we set s_1, v_1 & u_1 = to?

~~Some specified reference state~~

We can't measure entropy absolutely, so we define it relative to a specified reference state ⇒ another reason not to mix fluid models or sources!

When we use our tables, EES, or REFPROP someone has chosen this reference state for us, & carried out the integrals.

→ When we have an incompressible substance or ideal gas, the integrations are sufficiently simple that we can do them.

ΔS for incompressible Substance What do we remember about incompressible substances?

1) v is constant ⇒ incompressible 2) $u = u(T) \Rightarrow C = \frac{du}{dT} \Rightarrow du = CdT$

if $$T ds = P \overset{0(1)}{dv} + \overset{CdT}{du} \Rightarrow T ds = CdT \Rightarrow ds = \frac{CdT}{T} \Rightarrow$$

$$\Rightarrow \int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} \frac{C}{T} dT \Rightarrow \underline{s_2 - s_1 = C \ln\left(\frac{T_2}{T_1}\right)}$$

← If C is constant?

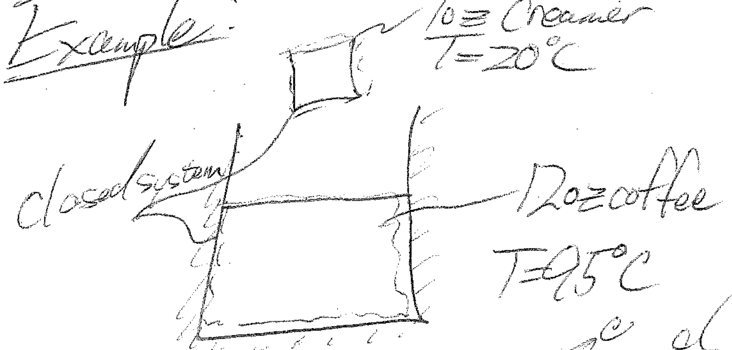
Question: If I told you an incompressible substance was isentropic, what would the temperatures do?

$$\hookrightarrow \Delta S = 0 \quad C\ln\left(\frac{T_2}{T_1}\right) = 0 \Rightarrow \ln\left(\frac{T_2}{T_1}\right) = 0 \Rightarrow \exp\left(\ln\left(\frac{T_2}{T_1}\right)\right) = \exp(0) \Rightarrow$$

$$\Rightarrow \frac{T_2}{T_1} = 1 \Rightarrow \underline{T_2 = T_1}$$

Incompressible substances with no ΔS are isothermal.

Example:



What is the temperature of the coffee after creamer is added?

Was entropy generated?

$\rho = 1000\ \mathrm{kg/m^3}$; $C = 4200\ \frac{J}{kg\,K}$ ← almost water

Energy Balance: $\cancelto{0}{E_{in}} = \cancelto{0}{E_{out}} + \Delta E_{stored} \Rightarrow 0 = m_{cf}(u_{cf2} - u_{cf1}) + m_{CR}(u_{CR2} - u_{CR1})$

$$m_{cf} = \forall_{cf}\rho \quad \Delta u_{cf} = C(T_{f2} - T_{f1}) \quad m_{CR} = \forall_{CR}\rho \; , \; \Delta u_{CR} = C(T_{R2} - T_{R1})$$

$$0 = \forall_{cf}\rho C(T_{cf2} - T_{cf1}) + \forall_{CR}\rho C(T_{CR2} - T_{CR1})$$

$$T_{cf2} = T_{CR2} \Rightarrow \underline{\underline{T_2 = 89.2^\circ C}} = 362.4\,K$$

Entropy Balance: $\cancelto{0}{S_{in}} + S_{Gen} = \cancelto{0}{S_{out}} + \Delta S_{stored} \Rightarrow S_{Gen} = m_{cf}(s_{cf2} - s_{cf1})$
$+ m_{CR}(s_{CR2} - s_{CR1})$

$$\Rightarrow S_{Gen} = \forall_{cf}\rho C \ln\left(\frac{T_2}{T_{cf1}}\right) + \forall_{CR}\rho C \ln\left(\frac{T_2}{T_{CR1}}\right) \Rightarrow \underline{S_{Gen} = 2.79\ J/K}$$

We know this process can happen ∴ $S_{Gen} > 0$ what if S_{Gen} was negative? → We made a mistake.

Entropy Change of an ideal gas: What do we know about ideal gases?

1) Ideal Gas Law: $Pv = RT$

2) $u = u(T) \rightarrow C_v = \left(\frac{\partial u}{\partial T}\right)_v \overset{\text{for ideal gas}}{\Rightarrow} C_v = \frac{du}{dT}$ so $\Delta u = \int_{T_1}^{T_2} C_v\, dT$ ← if C is constant $\Delta u = C_v(T_2 - T_1)$

3) $h = h(T) \rightarrow C_p = \left(\frac{\partial h}{\partial T}\right)_p \rightarrow C_p = \frac{dh}{dT}$ so $\Delta h = \int_{T_1}^{T_2} C_p\, dT \Rightarrow \Delta h = C_p(T_2 - T_1)$

4) $C_p = C_v + R$

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$$ds = \frac{P}{T}dv + \frac{du}{T} \quad du = C_v dT$$

$$Pv = RT \Rightarrow \frac{P}{T} = \frac{R}{v} \Rightarrow ds = \frac{R}{v}dv + \frac{C_v dT}{T}$$

$$\Rightarrow \int_{s_1}^{s_2} ds = R\int_{v_1}^{v_2}\frac{dv}{v} + \overset{\text{if constant}}{C_v}\int_{T_1}^{T_2}\frac{dT}{T} \Rightarrow s_2 - s_1 = R\ln\left(\frac{v_2}{v_1}\right) + C_v\ln\left(\frac{T_2}{T_1}\right)$$

Air, O_2, H_2O, etc.

Change in entropy of an ideal gas. When you use an ideal gas function in EES, this is done for you but with varying heat capacity.
won't be asked to do this on a test.

→ If you have a constant heat capacity, EES doesn't have these models so you will have to enter the above function instead of using a property lookup.

→ What if I don't know Δv, but ΔP instead?

→ we can rewrite it: $v_2 = \frac{RT_2}{P_2}$; & $v_1 = \frac{RT_1}{P_1} \Rightarrow$

$$\Rightarrow s_2 - s_1 = R\ln\left(\frac{RT_2}{P_2}\frac{P_1}{RT_1}\right) + C_v\ln\left(\frac{T_2}{T_1}\right) \Rightarrow s_2 - s_1 = R\ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{P_1}{P_2}\right) + C_v\ln\left(\frac{T_2}{T_1}\right)$$

$$\Rightarrow s_2 - s_1 = (R + C_v)\ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\left(\frac{P_2}{P_1}\right)^{-1}\right) \Rightarrow \underline{s_2 - s_1 = C_p\ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)}$$

→ What if I only know ΔP & Δv?

$$T_2 = \frac{P_2 v_2}{R}, \; T_1 = \frac{P_1 v_1}{R} \Rightarrow \underline{s_2 - s_1 = C_v\ln\left(\frac{P_2}{P_1}\right) + C_p\ln\left(\frac{v_2}{v_1}\right)}$$

These 3 results are useful for more than calculating changes in entropy.

What if we have an isentropic process? $\Delta S = 0$?

$$C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) = 0 \Rightarrow \ln\left(\frac{T_2}{T_1}\right) = \frac{R}{C_p} \ln\left(\frac{P_2}{P_1}\right) \Rightarrow$$

$$\Rightarrow \underbrace{\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\left(\frac{P_2}{P_1}\right)^{\frac{R}{C_p}}\right)}_{\text{take exponential of both sides}} \Rightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{R}{C_p}}$$

Where if $R = C_p - C_v$ & $k = \frac{C_p}{C_v}$ ← ratio of specific heats

Isentropic relation of an ideal gas

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}^{(k-1)/k}$$

Similarly →

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{k}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

We are starting to develope a bunch of these relations that are applicable only in very specific situations. Its a good idea to keep these in a quick reference note sheet **with** the necessary assumptions/situation that it is applicable