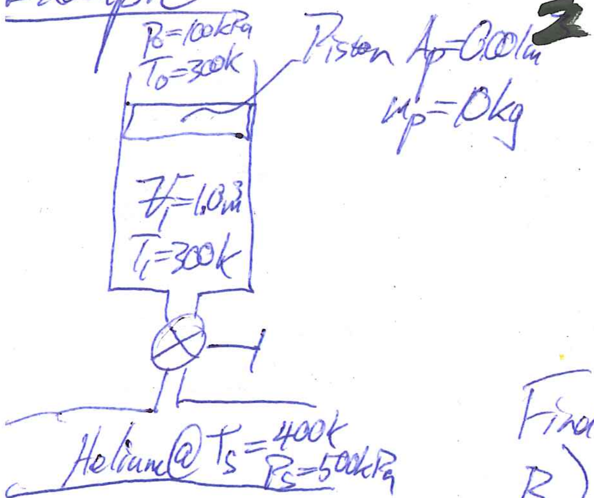
Entropy Balance for Open systems

→ Recall that we can transfer ~~energy~~ entropy across a system boundary in 2 ways: 1) heat transfer, 2) entropy transfer with mass

Example:



Valve is opened & 0.5 kg of mass flows into the cylinder before valve is closed. Eventually the helium cools back down to $T_0=T_2$

Helium is ideal, $R=2077\,J/kg\text{-}K$ $C_p=5200\,J/kg\text{-}K$

Find A) Heat transferred during equilibration
B) Total entropy generated

How do we start?

Mass Balance: $m_{in}=\overset{0}{m_{out}}+\Delta m_{stored} \Rightarrow m_{in}=m_2-m_1$

$$m_1=\frac{P_1 V_1}{R T_1}$$

$$P_1=P_0+\frac{m_p g}{A_p}=100{,}000Pa+\frac{10kg}{}\left|\frac{9.81m}{s^2}\right|\frac{}{0.001m^2}\left|\frac{N s^2}{kg\,m}\right. \Rightarrow P_1=198{,}100Pa$$

$$\hookrightarrow m_1=\frac{198100N}{m^2}\left|\frac{1m^3}{}\right|\frac{kg\text{-}K}{2077N\text{-}m}\left|\frac{}{300K}\right. \Rightarrow m_1=0.3179kg$$

$$m_2=m_{in}+m_1=0.5kg+0.3179kg=$$
$$m_2=0.8179kg$$

Energy Balance: $E_{in}=E_{out}+\Delta E_{stored}$

$$\Rightarrow \overset{0}{Q_{in}}+\overset{0}{W_{in}}+m_{in}\left(h_{in}+\cancel{\frac{v_{in}^2}{2}}+\cancel{gz_{in}}\right)=Q_{out}+\overset{OK}{W_{out}}+\cancel{m_{out}\left(h_{out}+\frac{v_{out}^2}{2}+gz_{out}\right)}+m_2u_2-m_1u_1$$

$$m_{in}h_{in}=Q_{out}+W_{out}+m_2u_2-m_1u_1$$

$m_2 C_v T_2$, $m_1 C_v T_1$ $\}\ C_v=C_p-R$

$\hookrightarrow m_{in}C_pT_{supply}$

$$\hookrightarrow W_{out}=\int_{V_1}^{V_2}P\,dV \Rightarrow W_{out}=P(V_2-V_1)$$ (P constant)

$$V_2=\frac{m_2RT_2}{P_2}=\frac{0.8179kg\,(2077\,J/kg\text{-}K)\,300K}{198100Pa} \Rightarrow V_2=2.573m^3$$

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$$Q_{out} = 0.5kg\left(\frac{5200J}{kg\text{-}k}\right)(400k) - 198100Pa(2.573m^3 - 1m^3) - 0.8179kg\left(3123\frac{J}{kg\text{-}k}\right)300k$$
$$+ 0.3179kg\left(3123\frac{J}{kg\text{-}k}\right)300k$$

$$\Rightarrow \boxed{Q_{out} = 260{,}000J}$$

Total Entropy Generated? => Entropy Balance: $S_{in} + S_{Gen} = S_{out} + \Delta S_{stored}$

$$m_{in}s_{in} + S_{Gen} = \frac{Q}{T_0} + m_2 s_2 - m_1 s_1$$ substituting in mass balance

$$(m_2 - m_1)s_{in} + S_{Gen} = \frac{Q}{T_0} + m_2 s_2 - m_1 s_1 \Rightarrow$$

$$\Rightarrow S_{gen} = \frac{Q}{T_0} + m_2(s_2 - s_{in}) - m_1(s_1 - s_{in}) \Rightarrow$$

$$\Rightarrow S_{gen} = \frac{Q}{T_0} + m_2\left[C_p \ln\left(\frac{T_2}{T_{in}}\right) - R\ln\left(\frac{P_2}{P_{in}}\right)\right] - m_1\left[C_p \ln\left(\frac{T_1}{T_{in}}\right) - R\ln\left(\frac{P_1}{P_{in}}\right)\right]$$

$$\Rightarrow S_{gen} = \frac{260{,}000J}{300k} + 0.8179kg\left[5200\frac{J}{kg\text{-}k}\ln\left(\frac{300}{400}\right) - 2077\frac{J}{kg\text{-}k}\ln\left(\frac{198100}{500{,}000}\right)\right]\ldots$$

$$\ldots - 0.3179kg\left[5200\frac{J}{kg\text{-}k}\ln\left(\frac{300}{400}\right) - 2077\frac{J}{kg\text{-}k}\ln\left(\frac{198100}{500{,}000}\right)\right]$$

$$\Rightarrow \boxed{S_{gen} = 1080\frac{J}{K}}$$