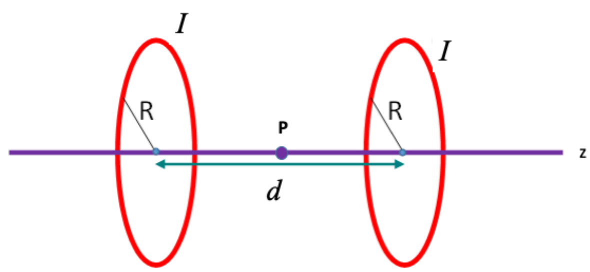
Helmholtz Coils

A system for producing a small region of space with nearly uniform magnetic field, called Helmholtz Coils, consists of two identical coils of radius R on the same (z) axis as shown here:



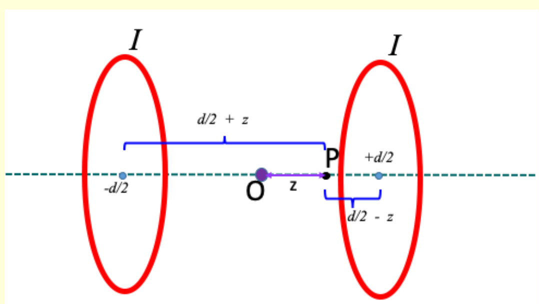
The z axis is the symmetry axis, P is the point of interest (we take it to be the origin ({0,0,0})); so the centers of the two coils are at {0,0, -d/2} and {0,0,+d/2}. [Clearly, we are taking nudges from Helmholtz to derive a useful arrangement for creating a desired **B** at P.] We are interested in **B** at P but realize that for small changes in position about P the field will not change dramatically.

We will explore the behavior of **B** as a function of z only, striving for uniformity.

(a) Derive **B** along the z axis for identical currents I running through the two coils

If you want **B** to be 'additive' for the two coils, and pointing in the +z direction, what directions should I flow in each coil?

THIS FIGURE MAY BE OF USE:



ENTER YOUR DERIVATION (OR JUST YOUR RESULTS) HERE:

(b) It's desirable to have $\vec{B}$[z] be uniform over at least a small range of z around the origin. We can use a Taylor Series for the above expression for $\vec{B}$[z] to explore this. First, Use the Series Function to generate a few terms. Here is a Quicky Tutorial on Series:

M’s Series function can be used to form a Taylor series of a function about a given point. For a one D function f[x]:

Series $[f, \{x, x_0, n\}]$
generates a power series expansion for f about the point $x = x_0$ to order $(x - x_0)^n$.

Normal[Series $[f, \{x, x_0, n\}]$]
generates a power series expansion for f about the point $x = x_0$ to order $(x - x_0)^n$ and drops the $O[x]^n$ (which cannot be evaluated).

Consider xo = 0 and n = 6

```
ClearAll["`*"]
Series[f[x], {x, 0, 6}]
Normal[Series[f[x], {x, 0, 6}]]
(*  this drops the O[x]^n Results in the desired Taylor series out to Order 6 *)
```

Now apply the above to form the Taylor Series for *B*[z], the two coil B-Field along the z axis.

In[16]:=
```
B[z_] = 1/2 II R^2 (1/(R^2 + (d/2 - z)^2)^(3/2) + 1/(R^2 + (d/2 + z)^2)^(3/2)) μzero
Series[B[z], {z, 0, 6}]
BSeries[z_] = Normal[Series[B[z], {z, 0, 6}]] (*  this drops the O[x]^n  *)
```

Out[16]= $\frac{1}{2}\left(\frac{1}{\left(1+\left(\frac{d}{2}-z\right)^2\right)^{3/2}}+\frac{1}{\left(1+\left(\frac{d}{2}+z\right)^2\right)^{3/2}}\right)$

Out[17]= $\frac{8}{\left(4+d^2\right)^{3/2}}+\frac{192\left(-1+d^2\right)z^2}{\left(4+d^2\right)^{7/2}}+\frac{1920\left(2-6d^2+d^4\right)z^4}{\left(4+d^2\right)^{11/2}}+\frac{14336\left(-5+30d^2-15d^4+d^6\right)z^6}{\left(4+d^2\right)^{15/2}}+O[z]^7$

Out[18]= $\frac{8}{\left(4+d^2\right)^{3/2}}+\frac{192\left(-1+d^2\right)z^2}{\left(4+d^2\right)^{7/2}}+\frac{1920\left(2-6d^2+d^4\right)z^4}{\left(4+d^2\right)^{11/2}}+\frac{14336\left(-5+30d^2-15d^4+d^6\right)z^6}{\left(4+d^2\right)^{15/2}}$

Examine carefully the ODD TERMS (HINT: they are missing!).

(c) Take these odd derivatives separately using M’s D (derivative) function and show that at z = 0, they disappear (seems to hold for all odd n). Reminder: D[f, {x,n}] gives the multiple derivative $\partial^n f / \partial x^n$

```
(*  Input Cell  *)
```

(d) The second derivative term in the Taylor Series, $\frac{192\ II\ R^2\ (d^2-R^2)\ z^2\ \mu zero}{(d^2+4\ R^2)^{7/2}}$, looks like an easy one to make vanish if we pick a certain d. Assign d to it's obvious value to make the second term in the Taylor Series vanish and re-write the series (BSeries[z]) with this particular d:

```
(*  Input Cell  *)
```

(e) Use Manipulate to vary d and examine the changes in the Plots of B[z] (I assumed II, R, and μzero to be all equal to 1) Take note of the shapes of the curves near d = R. OK; I'll do it for you. Look how I defined B[z_,d_] := ...... and wrote Manipulate with the Manipulate[d;.......]. This is a trick that makes Manipulate accept the function written for B.

In[11]:=
```
Remove["`*"]
```

$$B[z\_, d\_] := \frac{1}{2}\, II\, R^2 \left(\frac{1}{\left(R^2 + \left(\frac{d}{2} - z\right)^2\right)^{3/2}} + \frac{1}{\left(R^2 + \left(\frac{d}{2} + z\right)^2\right)^{3/2}} \right) \mu zero$$

```
II = 1; R = 1; μzero = 1;
B[z, d]
Manipulate[d;
 Plot[B[z, d], {z, -1.5 R, 1.5 R}, PlotRange → {0, 1}],
 {d, 0, 2 R, Appearance → "Open"}]
```

(f) Sticking with two coils separated by d = R, reverse the current in ONE of the coils. Derive a new B[z] and again use Manipulate to vary d and examine the changes in the Plots of B[z] (again, I assumed II, R, and μzero to be all equal to 1).