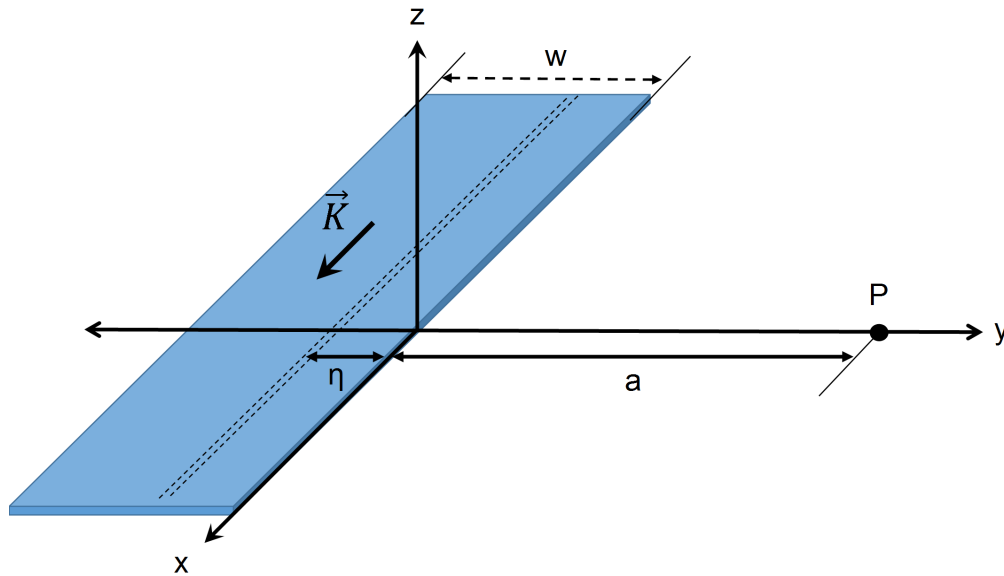


Magnetic Field due to a Thin Current-Carrying Strip in the Plane of the Strip

PROBLEM:

A thin, infinitely long, conducting strip of width w lies in the x - y plane, with its long dimension parallel to the x -axis. It carries a surface current K in the $+x$ -direction.

Find the direction and magnitude of the magnetic field at a Point P in the x - y plane a distance a from the closest edge of the strip, as shown below.



(a) Apply Ampere's law to a "wire" defined by a narrow portion of the strip of width $d\eta$ parallel to the x -axis. Assume that the wire intersects the y -axis at Point $(0, -\eta)$, where η is taken to be positive. Use the right hand rule to determine the direction of the magnetic field at Point P due to this wire. Does the direction of the field depend on the value of η , assuming that $0 < \eta < w$?

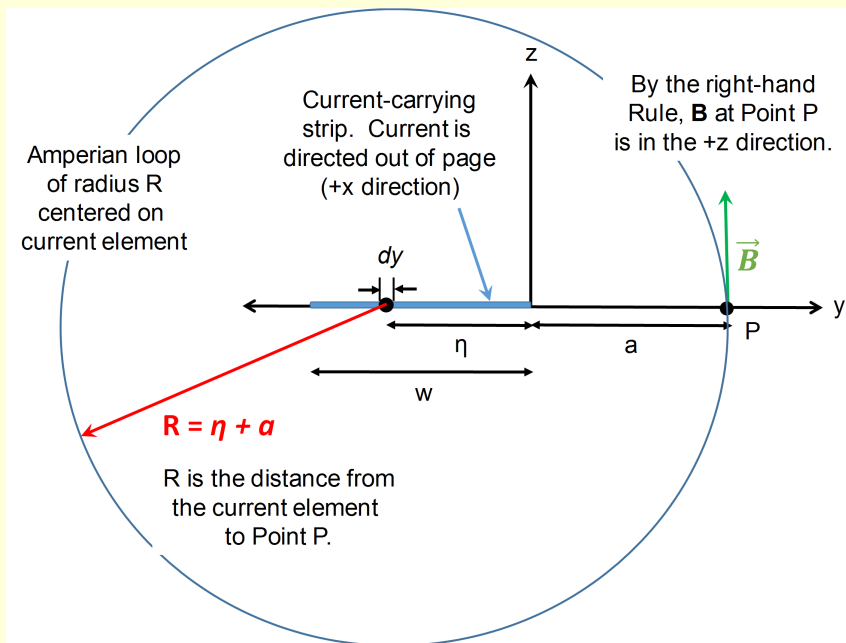
Remember that \mathbf{i} uses the symbol I for the square root of -1 . Use another symbol for the current. \mathbf{i} will work.

When we applied Ampere's law to an infinitely long, straight wire, we found that:

$$\int \vec{B} \cdot d\vec{l} = B 2 \pi R = \mu_0 I$$

where, I is the current in the thin wire and R is the distance between the wire and the point where B is measured. In our case, we have a "wire" of width $d\eta$ located a distance η from the edge of the strip closest to Point P. The current in this wire is $K d\eta$ (the current per unit

width times the width of this wire-like part of the strip). The distance between the wire and Point P is just $a + \eta$. An edge-on view of this configuration is shown below.



By the right hand rule, a current flowing in the +x-direction will produce a magnetic field in the +z-direction at Point P, as long as Point P is located on the positive y-axis, as shown.

Because the direction of the field at Point P produced by each wire-like element of the strip is the same (+z-direction), the magnitude of the field at Point P due to the entire strip will equal the scalar sum of each contribution.

Replacing R with $(a + \eta)$ and I with $K d\eta$ in Ampere's law for a thin wire yields

$$dB_{atP} = \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 K d\eta}{2\pi(a + \eta)}$$

where dB_{atP} is the contribution of the thin wire to the field at Point P.

(b) Sum the contributions of the individual wires that make up the strip to determine the magnitude of the magnetic field at Point P. Since M integrates in complex space, you may get a complicated conditional expression. This can be avoided by including assumptions in the Integrate command. For instance, if your function is:

Integrate[Exp[- η/a], { $\eta, 0, w$ }], you can tell M that η , a , and w are positive real numbers by including an assumptions statement:

Integrate[Exp[- η/a], { $\eta, 0, w$ }, Assumptions->{ η, a, w } \in Reals && { η, a, w } > 0].

There are other ways to do this, but putting the assumptions *inside* the Integrate command means that M will forget these assumptions after solving the integral. (The assumptions are “local” to the Integrate command.) This can be handy if you use these variable names later in a different context.

With η as defined above, the wire-like element of the strip closest to P is associated with $\eta = 0$. The wire farthest from P is associated with $\eta = w$. To find the total magnetic field, we integrate from $\eta = 0$ to $\eta = w$.

$$\text{BatP} = \frac{\mu_0 K}{2\pi} \int_0^w \frac{1}{(a + \eta)} d\eta$$

In the absence of additional information, M treats the variables as complex numbers. One example is shown below.

$\frac{\mu_0 K}{2\pi} \int_0^w \frac{1}{(a + \eta)} d\eta$ (* execute - you should get a ConditionalExpression;
by adding assumptions we can often get a clean result *)

This can be avoided by incorporating the appropriate assumptions, as discussed above. In our case, a , η , and w are all positive real numbers (not imaginary).

ClearAll["Global`*"]

BatP[a_] =

$$\frac{\mu_0 K}{2\pi} \text{Integrate}\left[\frac{1}{(a + \eta)}, \{\eta, 0, w\}, \text{Assumptions} \rightarrow \{\eta, w, a\} \in \text{Reals} \ \&\& \ \{\eta, w, a\} > 0\right]$$

(c) Check this result by considering the “narrow strip limit”, $w \rightarrow 0$, while keeping the total current fixed ($K \times w = I$). The result should look like the B field for an infinitely long wire that carries current I . Recall that:

$$\lim_{w \rightarrow 0} \left(\log\left(1 + \frac{w}{a}\right)\right) = \frac{w}{a}$$

If you like, M will generate the series expansion you need with the Series command. For instance,

Series[Log[1 + x], {x, 0, 3}]

will give the first three terms of the power series expansion of the function Log[1 + x] about $x = 0$. It uses the traditional $O[x]^4$ notation to indicate that the order of magnitude of next (fourth) term grows as x^4 .

(* Input Cell *)

To drop this (the $O[x]^4$), you can write: `Normal[Series[Log[1 + x],{x,0,3}]]`

(* Input Cell *)

Now by hand, let w get really small -- pick out the “wire” solution (hint: $K w = I$)

(d) JFL == Just for Laughs: Determine BatP if you let $w \rightarrow \infty$. (use M's Limit function)

```
ClearAll["`*"]
```

$$\text{BatP}[a_] = \frac{K \mu_0 \text{Log}\left[\frac{a+w}{a}\right]}{2 \pi};$$

```
Limit[BatP[a], w -> \infty]
```

As w becomes $\gg a$, $\text{Log}\left[\frac{a+w}{a}\right]$ goes as $\text{Log}[w]$, which goes to infinity as $w \rightarrow \infty$.

So, although the “wires” making up the additional parts of the sheet (as you increase w) are further and further away from the point P (so the contribution of these parts to B is decreasing), additional wires keeps adding more and more to B.

Below is a plot of BatP for a finite w_{\max} ; it looks like it might not go to infinity but...wrong. It does. you can change w_{\max} if you wish and see what it looks like (it just keeps going up!).

```
a = 1; wmax = 100;
```

```
Plot[Log[ $\frac{a+w}{a}$ ], {w, .01, wmax}]
```