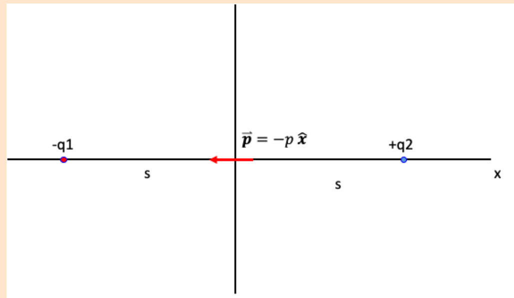
The Force on a Dipole p positioned between to point charges

Problem: A dipole p is oriented along the x axis ($\vec{p}$ = - p $\hat{x}$; p >0) between two point charges -q1 and + q2 (where q1>0 and q2 >0 are the magnitude of the charges: |q1| and |q2|) each a distance s from the dipole as shown here:



Find the force on $\vec{p}$

(a) In an input cell write down an equation for the amplitude of the E field vs. x along the x axis (treat as a 1D problem) for points on the axis near x = 0.

(For the geometry shown, we agree that the field due to the two point charges along the x axis is given by: $\vec{E}$[x] = E[x] $\hat{x}$; i.e., depends only on x and points in either the + or - x direction. E[x] is the amplitude of the vector $\vec{E}$[x].

We put the dipole at the origin as in the drawing above. We use Coulomb's Law for the E field due to the two point charges -q1 and q2 (the magnitudes q1 and q2 are +; therefore, the left hand charge is always negative and the right hand charge is always positive).

In[3]:=
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ClearAll["`*"]
EE[x_] := (- k q1)/(s + x)^2 - (k q2)/(s - x)^2 (* using Coulomb's Law;
k = 1/(4πεo); note carefully the SIGNS of each term *)
EE[x]
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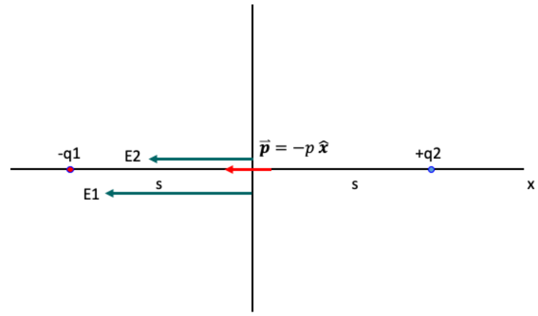
Out[5]= $-\frac{k\,q2}{(s-x)^2}-\frac{k\,q1}{(s+x)^2}$

(b) What direction does the E field point at the position of the dipole?



(Enter answer in this text cell)

For the charges shown and their given signs (the charge on the left is negative; the charge on the right is positive), both E1 and E2 (the fields from the two charges) at the dipole (0,0) point in the -x direction.

Therefore the total field at the origin, Etotal = EE[x= 0] = E1 + E2, is in the - x direction.

(c) **Given the geometry of this problem write down the Force on the dipole due to these point charges.**

(Enter answer in this text cell)

We will use the equation we derived for the force on a dipole in a E Field (derived by considering the potential U of a dipole in an E-Field).

$F = -\nabla(p \cdot E) = -\nabla(-p\,\hat{x} \cdot (-EE(x)\,\hat{x}) = -p\,\partial_x\,EE[x]$ [SIGNS CRITICAL]

FYI : $\partial_x = (\partial / \partial x) = (d / dx)$

NOTE: we take the derivative first THEN let x-> 0

(d) Use M to find an equation for F

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In[6]:= (*  input cell  *)
F = - p ∂x EE[x];
(* fyi:  for the derivative could also use EE'[x] or D[EE[x],x] *)
F = F /. x → 0  (* this evaluates the force on the dipole located at x = 0 *)
```

Out[7]= $-p\left(\frac{2\,k\,q1}{s^3} - \frac{2\,k\,q2}{s^3}\right)$

Therefore: $\vec{F} = -p\left(\frac{2\,k\,q1}{s^3} - \frac{2\,k\,q2}{s^3}\right)\hat{x}$

(e) Comment on the direction of the force $\vec{F}$ on $\vec{p}$.

remember q1 and q2 are the magnitudes of the charges:

If q1 > q2, $\vec{F}$ is in the -x direction; If q1 < q2, $\vec{F}$ is in the +x direction.

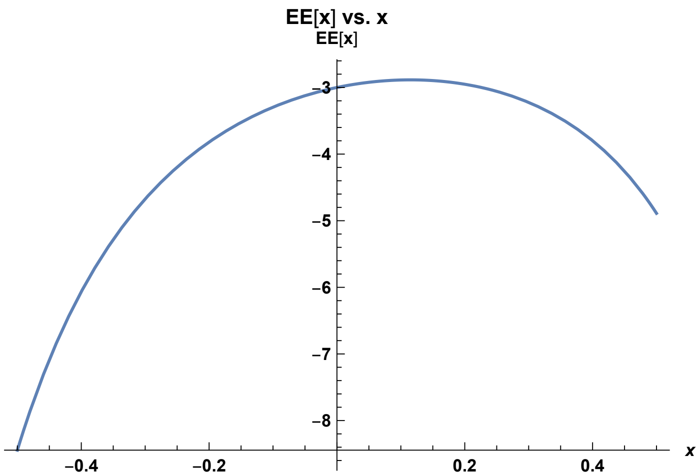
(f) Examine the slopes of EE[x] (by plotting EE[x] vs. x) near x = 0 For q1 > q2; I take q1 = 2; and q2 = 1 for an example where q1 > q2.

Note that the Sign of the slope = the Sign of $Grad_x$(EE[x] = Sign of ∂_x EE[x]

In[8]:=
```
k = 1; s = 1; q1 = 2; q2 = 1;
∂x EE[x] /. x → 0
Plot[ EE[x], {x, -s/2, s/2}, AxesLabel → {x, "EE[x]"}, PlotLabel → "EE[x] vs. x"]
```

Out[9]= 2

Out[10]=



From the Plot we see that the Slope at x = 0 is +; Likewise, ∂_x EE[x] /. x → 0 = +2 which is positive,

(g) Deduce the direction of $\vec{F}$ on $\vec{p}$ from the slope of EE[x] at x = 0.

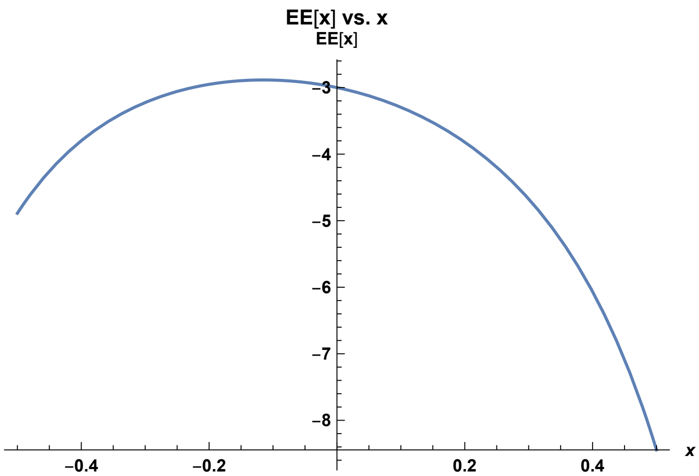
with - sign in the expression for the force ($- p\,\partial_x$ EE[x]), F is in $-$ x direction for q1 > q2

(h) Repeat (f) and (g) for q1 = 1; q2 = 2; i.e., for q1< q2

In[11]:=
```
q1 = 1; q2 = 2;
∂x EE[x] /. x → 0
Plot[ EE[x], {x, -s/2, s/2}, AxesLabel → {x, "EE[x]"}, PlotLabel → "EE[x] vs. x"]
```

Out[12]= -2

Out[13]=



From the Plot we see that the Slope at x = 0 is -; Likewise, ∂_x EE[x] /. x → 0 = -2 which is negative.

For q1< q2 slope reverses, Force reverses direction: F is in the +x direction.

(i) Using Manipulate, show the changes in slope (~Grad[EE[x] = ∂_x EE[x]) and resulting force on the dipole (~ - Grad[EE[x] = - ∂_x EE[x]) due to various q1 and q2

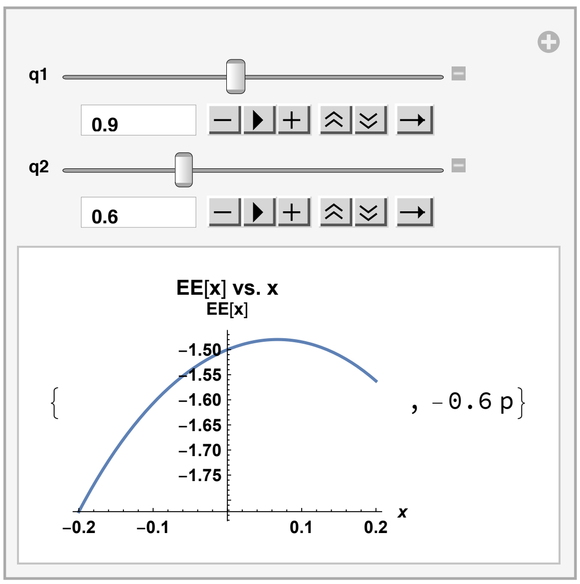
I.e., Display {A Plot of EE[x], and the Resulting Force [should be a number]} while manipulating q1 and q2.

In[14]:=
```
Clear[q1, q2]
 k = 1; s = 1;
Text[Style["Plot of EE[x] and the Resulting Force
     (note SIGN).  You can Manipulate q1 and q2.", 18, Red, Bold]]
Text[Style["Remember, the dipole is located at x = 0:", 18, Blue, Bold]]
Manipulate[{Plot[(- k q1)/(s + x)^2 - (k q2)/(s - x)^2, {x, -0.2, 0.2},
    AxesLabel → {x, "EE[x]"}, PlotLabel → "EE[x] vs. x"], -p ((2 k q1)/s^3 - (2 k q2)/s^3)},
 {{q1, 0}, 0, 2, .1, Appearance → "Open"}, {{q2, 0}, 0, 2, .1, Appearance → "Open"}]
```

Out[16]= **Plot of EE[x] and the Resulting Force**
(note SIGN). You can Manipulate q1 and q2.

Out[17]= **Remember, the dipole is located at x = 0:**

Out[18]=



Summary:

The SIGN of the Slope of EE[x] at x = 0 determines direction of force. (remember the - sign: F = - Grad[p.E])

i.e., if q1 > q2, ∂_x EE[x] is positive, F is in the -x direction; If q1 < q2, ∂_x EE[x] is negative, F is in the +x direction. Note the shape of the curve when for a given q1 you sweep q2 past q2 = q1; i.e., note the SLOPE of the curve at the position of the dipole (x = 0), as you do so.

The magnitude of the force ~ $\frac{1}{s^3}$