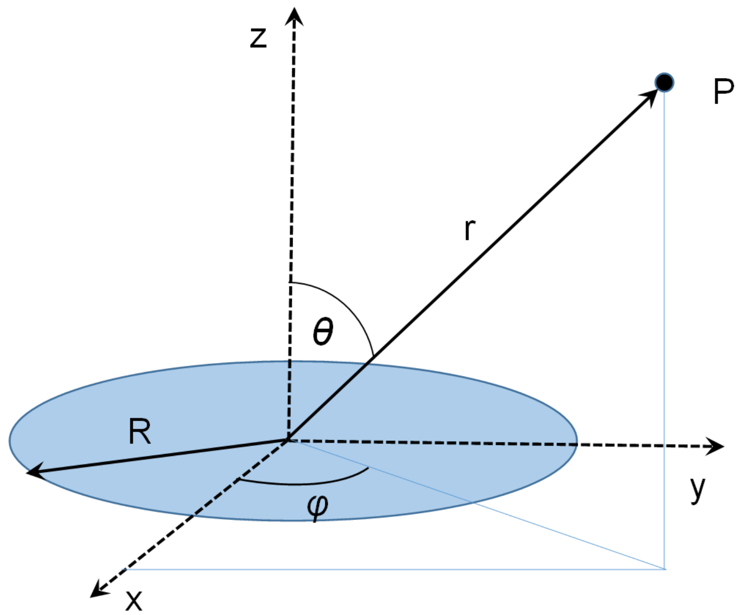
Electric Potential of a Uniformly Charged Disk of Charge — Off Axis

A disk of radius R normal to the z axis centered at the origin (i.e., lying in the x-y plane) holds a uniform charge density σ; Find and plot Vfar and Vnear — the off-axis solutions for z > 0.



HINTS:
(i) Treat as a 2D problem. Any plane through the z-axis will do — take the x-z plane (i.e., we consider y = 0). This is equivalent to saying that there is no ϕ dependence. Since any choice of ϕ will yield the same result, we can choose $\phi = 0$ (and/or π). These choices correspond to the x-z plane.

(ii) Use the solution for the Potential along the symmetry (z) axis as a boundary condition for the solution of Laplace's Equation in Spherical Coordinates.

(a) Set up the series solution for z > 0 , r > R (i.e., consider only the $\frac{1}{r^{l+1}} P_l(\text{Cos}[\theta]$ terms in the expansion).

<Enter your answer in this text cell>

(b) Use the facts that at $\theta = 0$, that all of the Legendre Polynomials = 1 when $\theta = 0$, and the hint above to find the B_l's for the desired solution. Write Vfar[r,θ] as the sum of the first four non-zero terms of the resulting series.

<Reality check>
If we set $\theta = 0$ what are the P_l's? To find out, execute: LegendreP[n, Cos[0]].

```
(* Input code below *)
ClearAll["`*"] (* Leave the ClearAll statement *)
LegendreP[n, Cos[0]]
(* For all n *)
```

Since n is arbitrary, the answer "1" implies that *every* Legendre polynomial equals one when $\theta = 0$. This is a property of Legendre polynomials. Below is an example of how this works out for n = 5: (63 - 70 + 15)/8 = 1.

```
LegendreP[5, Cos[θ]] (*   just an example  *)
```

For $\theta = 0$, all of the Legendre Polynomials = 1.

<Enter your derivation of the B_l values below>
<Create new code and text cells where it helps>

(c) Convert your Vfar[r,θ] to Cartesian coordinates for plotting. Plot the result, V[x,z], using Plot3D. You will have to choose "reasonable" values for R, σ, and ϵo. I chose R = 1, σ = 1, and ϵo = 1. (Computers and humans seem to do better with input values closer to 1 than 10^{-12}.)

```
(*Input code below *)
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(d) Now repeat the above for r < R to find Vnear[r,θ]. Keep the first six nonzero terms.

<Enter your derivation of the A_l values below>
<Create new code and text cells where it helps>

(e) Convert your Vnear[r,θ] to Cartesian coordinates for plotting. Plot the result, V[x,z]. You will have to give R, σ, and ϵo reasonable values again. The old ones have been erased.

```
(* Input code below *)
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(f) Now combine the two plots (both r < R and r > R). Reminder: here we are considering z >0 only.

For *my* sanity, I copy and paste the two equations for Vfarxz[x,z] and Vnearxz[x,z] from above, not forgetting to redefine R, σ, and ϵo in terms of actual numbers (all 1).

```
(* Input code below *)
```

(g) Interpret this Plot.

<Enter your interpretation in this text cell>

(h) Create a ContourPlot plot of the potential for points in the x-z plane. Calculate and plot the electric field for the same region using VectorPlot. Finally, produce a Streamline plot of the electric field (again, for the same region in the x-z plane).

```
(* Input code below *)
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(i) Show the ContourPlot and StreamPlot together.

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(* Input code below *)
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(i) Interpret your ContourPlot + StreamPlot. Where is the disk?

<Enter your interpretation in this text cell>