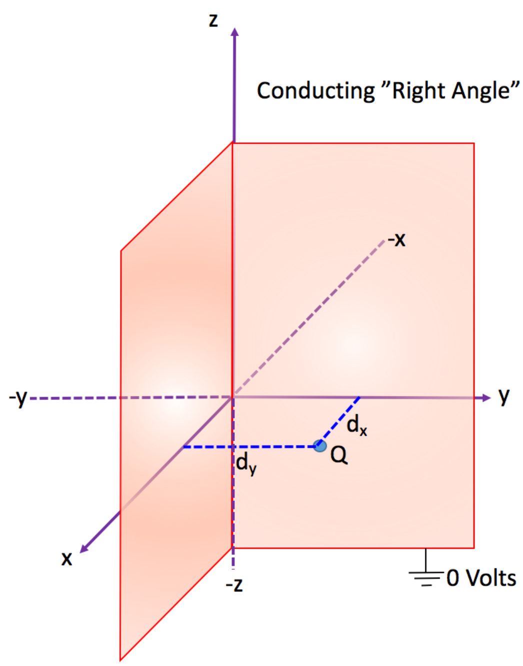
Potential and E Field due to a point charge near a grounded conducting right angle

PROBLEM:

A point charge Q is located in the vicinity of a grounded conducting "right angle" with infinite planar electrodes in the x >0, y > 0 half spaces as shown. Q lies in the x-y plane at distances d_x and d_y from the two planes respectively. We wish to determine the electric potential and field in the region where x >0 and y > 0. Realize the conductors shown in the figure extend to infinity in the ± z directions (and the +x, +y directions).



(a) What classic technique or method that is used for determining V and E for charges near certain symmetric geometries of conductors is your likely choice??

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The method of images. Our overall goal is to determine the magnitude, sign, and position of image charges that will satisfy the boundary conditions, namely that V on the two arms of the conducting right angle is zero, and then generate V (and E) in the region x >0; y > 0.

(b) In this type of problem, what role do boundary conditions play in finding V and E?

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The uniqueness of the solution to Laplace's Equation (LE) for the electric potential in a region of space arises when (say) a general solution is "hammered" into satisfying the bound-

ary conditions (e.g., potentials on surrounding surfaces). Generally this results in constants of integration taking on specific, unique values. Turning it around, if we find a solution (by hook or by crook) to LE, it is unique and we are done.

It is understood that the field of a point charge ($\propto \frac{1}{r}$) satisfies LE.
We can quickly use M to verify by using the Laplacian Function in Spherical Coordinates. If we get 0, we're done — using a superposition of point charges (one real; others image charges), we will have a solution to Laplace's Equation:

In[272]:=
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(*Execute code below*)
Laplacian[1/r, {r, θ, ϕ}, "Spherical"] (* V of a single point charge ∝ 1/r *)
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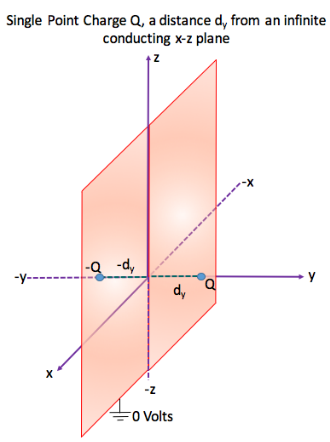
Out[272]= 0

So, *Your Response* included *Images charges??* Brilliant! The challenge is to find a set of point charges that will result in the electrodes along the x and y axes to remain at zero volts (ground). Hints:

1. We use the results of using a single image charge for the case of a single real charge Q a distance $+d_y$ in front of an infinite grounded conducting (x-z) plane (the correct image charge, $Q_{image} = -Q$, placed a distance (–) d_y *behind* the plane). This will yield a zero potential along the entire x-z plane.

Single Plane; point charge: 

Reminder: To find V[x,y,z] for y > 0 ONLY, we take simply add $V_{(Q)} + V_{(-Q)}$;

$$\text{i.e.,}\quad V[x,y,z] = k\,Q\left(\frac{1}{\sqrt{x^2+(y-d)^2+z^2}} + \frac{1}{\sqrt{x^2+(y+d)^2+z^2}}\right)$$

2. A combination of 3 image charges with magnitudes ± Q (and properly positioned) will yield infinite (x-z) and (y-z) planes at zero potential . Here is what the planes look like AND where (approximately) the charges might be placed.