

MgtOp 470—Business Modeling with Spreadsheets
TOPIC 5: WAITING LINES AND QUEUING THEORY
Solutions to Practice Problems

1. a) $P_0 = 1 - 24/30 = 1 - 0.8 = 0.2$
 b) $P_0 = 0.2$ (from part a))
 $P_1 = (24/30)^1(0.2) = (0.8)(0.2) = 0.16$
 $P(< 2) = P_0 + P_1 = 0.2 + 0.16 = 0.36$
 c) $24 / (30 - 24) = 24 / 6 = 4$ people
 d) $24^2 / [30(6)] = 576 / 180 = 3.2$ people
 e) $1 / (30 - 24) = 0.16666667$ hours $\times 60$ min./hr. = 10 minutes
 f) $3.2 / 24 = .13333333$ hours $\times 60$ min./hr. = 8 minutes
 g) $24 / 30 = 0.8$

2. $\mu = 60$ customers per hour
 a) $P_0 = 1 - 24/60 = 1 - 0.4 = 0.6$
 b) $P_0 = 0.6$ (from part a))
 $P_1 = (24/60)^1(0.6) = (0.4)(0.6) = 0.24$
 $P(< 2) = P_0 + P_1 = 0.6 + 0.24 = 0.84$
 c) $24 / (60 - 24) = 24 / 36 = 0.66667$ people
 d) $24^2 / [60(36)] = 576 / 2160 = 0.26667$ people
 e) $1 / (60 - 24) = 0.02777778$ hours $\times 60$ min./hr. = 1.66667 minutes
 f) $0.26667 / 24 = .01111$ hours $\times 60$ min./hr. = 0.66667 minutes
 g) $24 / 60 = 0.4$

3. Use Model 3, $\mu = 60 / 4 = 15$ customer per hour
 a) $12^2 / [2(15)(15 - 12)] = 144 / [2(15)(3)] = 144 / 90 = 1.6$
 b) $1.6 + 12/15 = 1.6 + 0.8 = 2.4$
 c) $12 / [2(15)(15 - 12)] = 12 / [2(15)(3)] = 12 / 90 = 0.133333$ hours $\times 60$ min./hr. = 8 min.
 d) 0.133333 hours + $1/15$ hours = 0.2 hours $\times 60$ min./hr. = 12 min.

4. $P_0 = 1 - \lambda / \mu$
 $0.5 = 1 - 40 / \mu$
 $40 / \mu = 0.5$
 $\mu = 40 / 0.5 = 80$

 $L_q = 40^2 / [80(80 - 40)] = 1600 / 3200 = 0.5$ customers

5. a)
$$P_0 = \frac{1}{\frac{(10/4)^0}{0!} + \frac{(10/4)^1}{1!} + \frac{(10/4)^2}{2!} + \frac{(10/4)^3}{3!} \left[\frac{3(4)}{3(4)-10} \right]} = \frac{1}{1 + 2.5 + 3.125 + 15.625} = 4.49\%$$

 b)
$$L_q = \frac{(10/4)^3(10)(4)}{(3-1)! [3(4)-10]^2} (.0449) = \frac{625}{8} (.0449) = 3.51$$
 customers
 c) $3.51 + 10/4 = 6.01$ customers
 d) $3.51 / 10 = 0.351$ hours $\times 60$ min./hr. = 21.06 minutes
 e) $0.351 + 1/4 = 0.601$ hours $\times 60$ min./hr. = 36.06 minutes