Probability:
1) commonly means a chance.
2) a mathematical means of studying uncertainty, and used as a tool for inferential statistics.

Random experiment: any activity from which an outcome, measurement, observation, or result is obtained, and outcomes cannot be predicted with certainty.

Basic outcome (**elementary outcome**): each possible outcome of a random experiment.

Sample space: the set of all possible basic outcomes, usually denoted by *S*.

An **event** is a subset of the sample space, usually denoted by *A, B, C, ...*

An **event occurs**, if any one of the basic outcomes in the event occurs.

A simple event (**elementary event**) is an event with only one basic outcome, usually denoted by E_i.

Compound event

Two events are **mutually exclusive**, if one and only one of them can occur.
A set of events is **collectively exhaustive** if one of the events must occur.

The **probability** of an event *A* is a measure of our belief of or the likelihood that an experiment will result in event *A*, and can be assessed by the following methods.
1. ***A priori* probability** (or classical method): the probability of an outcome is based on the prior knowledge of the experiment (process) involved.
2. **Empirical probability** (relative frequency or empirical method): the probability of an outcome is the relative frequency of the outcome, that is, the frequency of occurrences of the outcome divided by the number of trials repeated.
3. **Subjective probability**: the probability of an event reflects a person's degree of belief that the event will occur.

Basic rules of the probability
1. For each simple event E_i, $0 \le P(E_i) \le 1$.
2. For each event *A* which consists of *k* different elements of simple events $E_1, E_2, \ldots, E_k$, that is, $A = \{e_1, e_2, \ldots, e_k\}$ for $E_i = \{e_i\}$,
$$P(A) = \sum_{i=1}^{k} P(E_i) = P(E_1) + P(E_2) + \cdots + P(E_k)$$
3. $0 \le P(A) \le 1, \quad P(S) = 1, \quad P(\phi) = 0$

Venn diagram

Probabilities of **compound events**

The **complement** of an event *A*, denoted by A' or $\bar{A}$ consists of all basic outcomes in *S* that do not belong to *A*. $P(A) = 1 - P(A')$.

The **intersection** of two events *A* and *B*, denoted by *A and B*, $A \cap B$, or *AB*, consists of all basic outcomes that belong to both *A* and *B* simultaneously, and is called the joint event. The probability of the intersection of two events, denoted by *P(A and B)* is called the **joint probability**.

The **union** of two events *A* and *B*, denoted by *A or B* or by $A \cup B$ consists of all basic outcomes that belong to either *A* or *B* or both.

Additive rule:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

MSL Homework: 4.1, 4.3, 4.5
HC Homework: 4.2

The **conditional event** of *A* given that *B* has occurred is denoted by *A*|*B*, and its probability is

$$P(\mathrm{A}|\mathrm{B}) = \frac{P(A \cap B)}{P(B)}$$

Example 1: In a certain city, 35% of the households subscribe to the Times, 50% to the Journal, and 20% to both.

a). Find the probability that a randomly selected household subscribes to at least one of the newspapers.
b) Find the probability that a randomly selected household subscribes to neither of the newspapers
c) A randomly selected household subscribes to the Journal. Find the probability that this household also subscribes to the Times.

Example 2: Refer to Example 4.2 on p 154 of the textbook. Recall that a table of cross-classification like the one in this example is called a contingency table.

a) Find the probability that a randomly selected household in the sample planned to purchase and actually purchased a big screen TV.
b) Find the probability that a randomly selected household in the sample planned to purchase and did not actually purchase a big screen TV.
c) Find the probability that a randomly selected household in the sample actually purchased a big screen TV.
d) Find the probability that a randomly selected household in the sample did not plan to purchase a big screen TV.
e) A randomly selected household actually purchased a big screen TV. Find the probability that this household planned to purchase a big screen TV.
f) A randomly selected household purchased a big screen TV. Find the probability that this household did not plan to purchase a big screen TV.

MSL Homework: 4.10, 4.11, 4.16, 4.23
HC Homework: 4.17, 4.24

The **marginal probability** is the probability that an observation will show any single specific characteristic and is obtained by summing the appropriate joint probabilities over all values of other variables.

Multiplicative rule:

$P(A \cap B) = P(A)P(B|A)$

$P(A \cap B) = P(B)P(A|B)$

Example 4. A bag of M&M's contains 6 green and 4 red M&M's. Two M&M's are randomly drawn *without replacement* from the bag.

a) Find the probability that both M&M's are green.

b) Find the probability that the second M&M is green.

c) Find the probability that the first one is green given that you know that the second one is green.

Example 5. Repeat Example 4 when random selection is done *with replacement*.

Two events A and B are **independent** if and only if one of the following holds:

1. $P(A|B)=P(A)$ when $P(B) \neq 0$,
2. $P(B|A)=P(B)$ when $P(A) \neq 0$,
3. $P(A \cap B) = P(A)P(B)$

MSL Homework: 4.28
HC Homework: 4.29

Example 6: At Cheers you have a 90 % chance of running into Cliff. When Cliff is present at the bar you have a 95% chance of running into Norm; when Cliff is not present you have only 10% chance of running into Norm.

a) Find the probability that you will run into Norm at Cheers.

b) One day you walk into the bar and spot Norm. Then find the probability that you run into Cliff.

c) Are the event of running into Cliff and the event of running into Norm independent?

Example 7: Of members of the population in the city of interest, 1% are infected with AIDS virus. The probability that a person with the AIDS virus will have a positive reading on a AIDS virus test is 0.98; the probability that a person without the AIDS virus will have a positive reading is 0.05. Find the probability that a person with a positive reading on the test is infected with the AIDS virus.

HC Homework: 4.34, 4.37
MSL Homework: 4.35, 4.36

https://www.yahoo.com/finance/news/poll-33-nfl-fans-purposely-stopped-watching-season-split-trump-kaepernick-110215783.html

Poll: 33% of NFL fans 'purposely stopped watching' this season

Throughout this 2017 NFL season, television ratings have declined and fans, TV pundits, and reporters have speculated as to the biggest cause. On social media, many people railed that players protesting during the national anthem led outraged Americans to boycott.

A new survey from SurveyMonkey and Ozy Media, shared first with Yahoo Finance, finds that 33% of NFL fans boycotted the league this year—but not entirely because they were outraged by the player protests. Nearly equal proportions boycotted in support of Colin Kaepernick or the protests as boycotted in support of President Trump, who vocally opposed the protests.

The survey, conducted from Dec. 8—11 of 2017, polled a national sample of 1,726 adults ages 18 and up. It found that 1,233 of those people identifed as football fans.

The survey then asked the football fans: "Did you purposely stop watching or attending NFL games this season for any reason?" 33% of respondents said yes.

That group, which the survey labeled as "boycotting," was asked why, and was given multiple options. Note: Respondents were allowed to select multiple answers; they were not asked which was the biggest factor, just which factors contributed.

They answered as follows: 32% said they stopped watching or attending NFL games "in support of Donald Trump"; 22% said "in solidarity with players kneeling"; 13% said "no interest in the teams playing"; 12% said "in support of Colin Kaepernick"; and 11% said "news about traumatic brain injuries among players." Another 8% said "games are boring." 46% chose "some other reason."

The results also show an interesting difference between male and female respondents: more men said they turned away from the NFL in support of Trump (35% to 25%), while more women said they did it in support of the players kneeling (30% to 17%) or in support of Kaepernick (17% to 10%).

If you randomly select one among those who participated in the survey, find the probability that you will select a respondent who:

1. boycotted NFL games and had no interest in the team playing;
2. boycotted NFL games, in support of the players kneeling, and is a woman.