Cooperative Ph.D. Program in the School of Economic Sciences and Finance

QUALIFYING EXAMINATION IN MICROECONOMICS

June 5, 2025
8:45 a.m. to 1:00 p.m.

THERE ARE FOUR QUESTIONS –YOU MUST ANSWER ALL **FOUR** QUESTIONS.

- You **must** complete the examination within four hours. You will have 15 minutes to read over the questions before starting (8:45-9:00).
- This exam is closed book. Calculators and paper will be provided.
- Read the question carefully. Allocate your time carefully. Parts within questions will often vary in difficulty and weight. Be sure to do all parts of each question chosen.
- If necessary, it is permissible to make clarifying assumptions, but be sure to label them explicitly. (Grades will not take unstated assumptions for granted.) Also, label graphs and define notation.
- Number your answer sheets consecutively. Begin your answer to each question on a new page and identify the questions number.
- Leave ½-1" spacing around the edges of your paper.

Please write your exam ID number on the top of this page.

Microeconomic Theory
Comprehensive Exam - June 2025

Instructions:

- This exam has four (4) exercises.
- You must answer all exercises.
- Please read all exercises carefully.
- Answer each exercise in a formal and concise manner, but include all your steps. This will allow you to obtain partial credit.

Good luck!!

1. **Consumer theory with product differentiation.** Consider an individual with the following utility function for units of goods a and b, and for the numeraire (good c, whose price is normalized to \$1),

$$u(a,b,c) = \alpha\,(a+b) - \frac{1}{2}\left[\beta(a^2+b^2) + 4\gamma ab\right] + c$$

where $\beta > |\gamma|$ implying that the consumer regards goods a and b as differentiated. Consumer's budget constraint is $I = c + p_a a + p_b b$, where $p_a, p_b > 0$ denote prices and $I > 0$ represent his income.

   (a) Solve this consumer's utility maximization problem to find his Walrasian demand for goods a and b. Show that Walrasian demands are linear in the price of good a and b.

   (b) Interpret your results in terms of parameters β and γ.

   (c) Evaluate the Walrasian demands in the case that $\gamma = 0$. Interpret.

2. **Emission fees with asymmetric firms.** Consider two polluting firms, firm 1 and firm 2, producing a homogeneous good, and facing inverse demand function

$$p_i(q_i, q_j) = 1 - q_i - q_j$$

where $i = \{1, 2\}$ and $j \neq i$. Firms are asymmetric in their production costs, that is $1 > c_1 > c_2 > 0$. Every unit of output generates one unit of emissions, $e_i = q_i$, but every firm i can invest in abatement z_i that helps reduce its net emissions from $e_i = q_i$ to $e_i = q_i - z_i$. Consider that the abatement cost function is $\frac{\gamma_i}{2}(z_i)^2$ where $i = \{1, 2\}$, with firm 1 having a lower per-unit abatement cost than firm 2, i.e., $\gamma_1 < \gamma_2$.

Finally, social welfare is defined as

$$SW = PS + T - Env$$

where PS is the producer surplus, $T = t(e_1 + e_2)$ is the tax revenue from emission fees, and $Env = d(e_1 + e_2)^2$ is the environmental damage from pollution. where q_i denotes output.

(a) Assume that the regulator sets an emission fee t per unit of net emissions e_i. Every firm observes t and responds simultaneously choosing its output and abatement, q_i and z_i. Find the output functions $q_1(t)$ and $q_2(t)$, and the abatement functions $z_1(t)$ and $z_2(t)$. Are they increasing or decreasing in the emission fee t. Interpret.

(b) Identify the socially optimal output level for firm 1, q_1^{SO}, and for firm 2, q_2^{SO}, that maximize social welfare. Interpret.

(c) Find the socially optimal fee t^* that induces firms to choose the social optimum found in part (b), that is, t^* solves $q_1(t) + q_2(t) = q_1^{SO} + q_2^{SO}$. For simplicity, assume $\gamma_1 = \gamma_2 = 1$ and $d = 1$.

(d) Evaluate output at the equilibrium fee found in part (c), $q_1(t^*)$ and $q_2(t^*)$. Discuss under which conditions on c_1 one or both firms are active.

3. **Entry decisions under incomplete information.** Consider a monopolist operating in an industry with inverse demand function $p(Q) = 1-Q$, where Q denotes aggregate output. A new firm, the entrant, is choosing whether or not to join the industry, with fixed cost F, where $F \geq 0$. In the first stage, the entrant observes its marginal production cost, $c \in [0,1]$, but doesn't observe the monopolist's marginal production cost, c_m, although the entrant knows its distribution, $F(c_m)$, which has positive density in all its support, i.e., $f(c_m) > 0$ for all $c_m \in [\underline{c}_m, \overline{c}_m]$. In the second stage, if the entrant doesn't enter, the monopolist keeps its monopoly power; but otherwise, firms observe each other's costs, and compete à la Cournot.

   (a) *Second stage.* In the event of entry, if the entrant's cost is c and the incumbent's is c_m, find the equilibrium output and profits for entrant and incumbent. [*Hint:* This is a Cournot duopoly game under complete information but asymmetric costs.]

   (b) *First stage.* Anticipating the profits found in part (a), for each realization of c_m, the entrant can anticipate its expected profit, which can be written as a function of the monopolist's expected cost, $E[c_m]$. Find the highest realized cost for the entrant that induces this firm to join the industry, denoting it as $\overline{c}$.

   (c) *Special case.* Assume that the monopolist's cost is binary: either high, c_m^H, with probability p; or low, c_m^L, with probability $1-p$. The potential entrant's realized cost, c, satisfies $1 > c_m^H > c > c_m^L > 0$. Evaluate the expected marginal cost $E[c_m]$ in your results of part (b) using this binary distribution function. Evaluate the maximal entrant's cost inducing entry that you found in part (b), $\overline{c}$, in this special case. How is $\overline{c}$ affected by an increase in p, c_m^H, c_m^L, or F? Interpret.

4. **Moral hazard in the farm.** Consider a setting between a land owner and a farmer, who does not own the land but works in it, and suppose that both of them are risk neutral. The farmer chooses an effort level of either $e = 0$ (at zero cost for the farmer) or $e = 1$ (at a cost $c > 0$ for the farmer). When the farmer exerts a positive effort, $e = 1$, the harvest is low, x_L, with probability p_1, and high, x_H, with probability $1 - p_1$. Similarly, when the farmer exerts a zero effort, $e = 0$, the harvest is low, x_L, with probability p_0, and high, x_H, with probability $1 - p_0$; where $p_0 > p_1$. Assume that the farmer's reservation utility from rejecting the contract is zero, and use w_L to denote the salary that the farmer receives when the harvest is low and, similarly, w_H when the harvest is high, which satisfy $w_L, w_H \geq 0$ (limited liability).

   (a) *Symmetric information.* As a benchmark, let us first solve the principal's problem when he can perfectly observe the effort level that the farmer exerts. Find the contract (w_H, w_L), specifying the salary to the farmer when the harvest is high and low. Show that the land owner's expected profits are higher when he induces effort $e = 1$ than when he induces $e = 0$.

   (b) *Asymmetric information.* Find the contract (w_H, w_L), specifying the salary to the farmer when the harvest is high and low, respectively, where $w_L \geq 0$ by assumption (limited liability). Recall that the land owner prefers to induce effort $e = 1$. For simplicity, you can assume $p_0 = 1/2$ and $p_1 = 1/4$ for the remainder of the exercise.

   (c) *Crop sharing.* Real-life contracts to tenant farmers are, however, different from that found in part (b). In particular, most contracts specify that the tenant farmer keeps a fixed proportion of the harvest (i.e., a harvest share). Find the contract that the principal offers if he must provide a harvest share to the farmer.

   (d) Is the harvest share contract of part (c) better for the farmer than the optimal contract found in part (b)? Compare the farmer's expected utility in each setting.