Cooperative Ph.D. Program in the School of Economic Sciences and Finance

QUALIFYING EXAMINATION IN MACROECONOMICS

June 9, 2025
8:45 a.m. to 1:00 p.m.

THERE ARE FOUR QUESTIONS –ANSWER ALL **FOUR** QUESTIONS.

- You **must** complete the examination within four hours. You will have 15 minutes to read over the questions before starting (8:45-9:00).
- This exam is closed book. Calculators and paper will be provided.
- Read the question carefully. Allocate your time carefully. Parts within questions will often vary in difficulty and weight. Be sure to do all parts of each question chosen.
- If necessary, it is permissible to make clarifying assumptions, but be sure to label them explicitly. (Grades will not take unstated assumptions for granted.) Also, label graphs and define notation.
- Number your answer sheets consecutively. Begin your answer to each question on a new page and identify the questions number.
- Leave ½-1" spacing around the edges of your paper.

Please write your exam ID number on the top of this page.

1. **A Quadratic Business Cycle Model with Immigration Shocks**

Consider a one-sector business cycle model. The preference of the representative household is given by:

$$E_0 \quad \sum_{t=0}^{\infty} \beta^t u(c_t) \Bigg) .$$

The period utility takes a quadratic form

$$u(c_t) = -\frac{1}{2}(c_t - \phi)^2 ,$$

where c_t is consumption, and $\phi > 0$ is a constant preference parameter.

The household is endowed with N_t units of time in the period t, and supply it inelastically. Unlike in a standard real business cycle model, N_t is a random variable, which reflects a labor supply shock (say, due to changes in immigration policies). We assume that N_t is independently and identically distributed over time, with $\log(N_t)$ following a normal distribution $\mathcal{N}(0, \sigma^2)$.

The production technology is characterized by a standard Cobb-Douglas production function with a constant TFP $z_t = 1$:

$$y_t = k_t^{\alpha} n_t^{1-\alpha}.$$

The aggregate resource constraint reads:

$$c_t + k_{t+1} = y_t + (1-\delta) k_t,$$

where $\delta \in (0, 1]$ is the depreciation rate of capital.

At the beginning of $t = 0$, $k_0 > 0$ is given, and the household observes N_0 before making decisions.

(a) Write the Bellman equation of the social planner's problem of maximizing the welfare of the household.

(b) Derive the Euler equation for the problem in Part (a).

(c) Suppose that the average behavior of the real-world data can be captured by the social planner solution. Assume that you have observed $(c_t, i_t, y_t, k_t, N_t)$ for a long period of time. Design a procedure to calibrate σ.

(d) Consider a sequential-trading environment. The household owns and supplies capital and labor to the firm in exchange for capital rental income and labor income in each period. The representative firm chooses capital and labor input to maximize one period profit. The household purchases consumption and investment goods from the firm.

Carefully define a recursive competitive equilibrium for this economy. Be sure to write down the household's dynamic programming problem, and the expressions for wage and capital rental rate as functions of state variables.

2. **A Quadratic Continuous Time Business Cycle Model with Productivity Shocks**

Consider an infinite-horizon continuous-time stochastic growth model. The representative household has a standard dynamic utility function

$$E_0\left[\int_0^\infty \exp\left(-\rho t\right) u\left(c\left(t\right)\right) dt\right],$$

where $\rho > 0$. The period utility takes a quadratic form

$$u\left(c\left(t\right)\right) = -\frac{1}{2}\left(c\left(t\right) - \phi\right)^2,$$

where $c\left(t\right)$ is consumption, and $\phi > 0$ is a constant preference parameter.
The resource constraint at time t takes a standard form:

$$c\left(t\right) + i\left(t\right) = y\left(t\right),$$

where $i\left(t\right)$ denotes investment. The capital accumulation equation is the standard form:

$$\dot{k}\left(t\right) = i\left(t\right) - \delta k\left(t\right).$$

The production function takes an affine form:

$$y\left(t\right) = B + \exp\left(A\left(t\right)\right) k\left(t\right),$$

where $A\left(t\right)$ is a stochastic shock to the production (say, due to changes in tariffs), and $B > 0$ is a constant. We assume that $A\left(t\right)$ follows an OU (Ornstein-Uhlenbeck) Process

$$dA\left(t\right) = \kappa\left(\gamma - A\left(t\right)\right) dt + \sigma dW\left(t\right),$$

At the beginning of $t = 0$, $k\left(0\right) > 0$ is given, and the household observes $A\left(0\right)$ before making decisions.

(a) Write down the Hamilton-Jacobi-Bellman (HJB) Equation in the stochastic dynamic programming formulation of the social planner.

(b) From the HJB Equation, derive the Euler Equation for this problem as a function of $c\left(t\right)$ and $k\left(t\right)$.

(c) For this part, let $A\left(t\right)$ equal to a constant A so that we are in a deterministic economy. From the Method of Undetermined Coefficient, solve the optimal choice of $c\left(t\right)$ and $\dot{k}\left(t\right)$ as a function of $k\left(t\right)$.

3. A final good, denoted by y, is produced by perfectly competitive firms by combining a mass M of tasks in a set Υ. The production function is a constant elasticity of substitution (CES) aggregator with elasticity $\lambda \geq 0$

$$y = \left(\frac{1}{M} \int_{\Upsilon} (M \cdot y(x))^{\frac{\lambda-1}{\lambda}} \cdot dx \right)^{\frac{\lambda}{\lambda-1}} \tag{1}$$

where x indexes tasks and $y(x)$ is the amount of task x.

The representative firm chooses how much to buy of each task in order to maximize profits, taking the prices of the tasks, $p(x)$, as given. The final good is the numeraire.

The production of tasks is carried out by perfectly competitive, price-taking firms. Each task can be produced using capital or different types of labor (indexed by $g \in \mathcal{G} = \{1, 2, ..., G\}$). The production function of task x, i.e. $y(x)$, is specified as

$$y(x) = A_k \cdot \psi_k(x) \cdot k(x) + \sum_{g \in \mathcal{G}} A_g \cdot \psi_g(x) \cdot \ell_g(x), \tag{2}$$

where $\ell_g(x)$ is the amount of labor of type g allocated to produce task x; $k(x)$ is the amount of task-specific capital produced for and assigned to this task; A_k, A_g, $\psi_k(x)$ and $\psi_g(x)$ denote technology and factor productivities.

Capital used to produce task x, which is denoted by $k(x)$, is produced using the final good, y,

$$k(x) = q(x) \cdot y, \tag{3}$$

where $q(x)$ is task-specific technology.

(a) Write down the problem of the final good producer and obtain the demand functions for tasks. Explain and discuss.

(b) Write down the problem of task producers and characterize (1) the set of tasks allocated to labor of type g and the set allocated to capital; (2) the tasks supply functions.

(c) Use the equilibrium conditions in the task markets and the production function for tasks to obtain capital and labor demand in terms of technologies, productivities and wages.

(d) Assume now that there is a measure one of workers of each type g who supply labor inelastically to firms. Obain the equilibrium wage for each type of labor g.

4. Assume a two-period exchange economy with two types of consumers of equal measure. Each consumer maximizes

$$u(c_0^i) + \beta E[u(c_1^i)], \quad \text{for } i = 1, 2,$$

c_t^i is the consumption of a type-i consumer in period t. In period 0, type-i consumers are endowed with ω_0^i units of the (nonstorable) consumption good. Endowments in period 1 are random: with probability π_j, j = 1, 2, a type-i consumer receives ω_{1j}^i units of the consumption good in period 1. In period 0, consumers trade Arrow securities whose payoffs depend on the state of the world in period 1.

(a) Define a competitive equilibrium for this economy. Are markets complete? Explain why or why not.

(b) Suppose that $\bar{\omega}_0 = \bar{\omega}_{11} = \bar{\omega}_{12}$, where $\bar{\omega}_0 \equiv \sum_{i=1}^{2} \omega_0^i$ and $\bar{\omega}_{1j} \equiv \sum_{i=1}^{2} \omega_{1j}^i$. Show that redistributions of the period-0 endowments do not affect the equilibrium prices of the Arrow securities. (Note that a redistribution leaves the aggregate endowment unchanged.) In addition, characterize the equilibrium consumption allocation as much as possible.

(c) Now suppose that $\bar{\omega}_0 \neq \bar{\omega}_{11} \neq \bar{\omega}_{12}$. Show that redistributions of the period-0 endowments do not affect the equilibrium prices of the Arrow securities under the assumption that u has a constant elasticity of intertemporal substitution. In addition, characterize the equilibrium consumption allocation as much as possible.

(d) Now suppose that in period 0 consumers cannot trade Arrow securities but instead can trade only a riskfree bond (i.e., a sure claim to one unit of the consumption good in period 1). Carefully define a competitive equilibrium for this economy.