Cooperative Ph.D. Program in the School of Economic Sciences and Finance

QUALIFYING EXAMINATION IN MACROECONOMICS

August 11, 2025
8:45 a.m. to 1:00 p.m.

THERE ARE FOUR QUESTIONS –ANSWER ALL **FOUR** QUESTIONS.

- You **must** complete the examination within four hours. You will have 15 minutes to read over the questions before starting (8:45-9:00).
- This exam is closed book. Calculators and paper will be provided.
- Read the question carefully. Allocate your time carefully. Parts within questions will often vary in difficulty and weight. Be sure to do all parts of each question chosen.
- If necessary, it is permissible to make clarifying assumptions, but be sure to label them explicitly. (Grades will not take unstated assumptions for granted.) Also, label graphs and define notation.
- Number your answer sheets consecutively. Begin your answer to each question on a new page and identify the questions number.
- Leave ½-1" spacing around the edges of your paper.

Please write your exam ID number on the top of this page.

1. **Business Cycles with a CARA Utility and Capital Share Shocks**

   Consider a one-sector business cycle model. The household is endowed with 1 unit of labor in every period. The preference of the representative household is given by:

$$E_0 \left. \sum_{t=0}^{\infty} \beta^t u(c_t) \right) .$$

   The period utility function takes an exponential functional form

$$u(c_t) = -\frac{1}{\theta} \exp(-\theta c_t),$$

   where c_t is consumption, and $\theta > 0$ is a constant preference parameter.

   The production technology is characterized by a Cobb-Douglas production function:

$$y_t = k_t^{\alpha_t} n_t^{1-\alpha_t},$$

   where $0 < \alpha_t < 1$ is the capital share parameter at t. Unlike in a standard real business cycle model, α_t is a time-varying random variable. We assume that the stochastic process α_t is independent and identically distributed over time, with a mathematical expectation $E(\alpha_t) = \overline{\alpha}$.

   The aggregate resource constraint reads:

$$c_t + k_{t+1} = y_t + (1 - \delta) k_t,$$

   where $\delta \in (0, 1]$ is the depreciation rate of capital.

   At the beginning of $t = 0$, $k_0 > 0$ is given, and the household observes α_0 before making decisions.

   (a) Write the Bellman equation of the social planner's problem of maximizing the welfare of the household.

   (b) Derive the Euler equation for the problem in Part (a).

   (c) Design a procedure to calibrate $\overline{\alpha}$.

   (d) Consider a sequential-trading environment. The household owns and supplies capital and labor to the firm in exchange for capital rental income and labor income in each period. The representative firm chooses capital and labor input to maximize one period profit. The household purchases consumption and investment goods from the firm.

   Carefully define a recursive competitive equilibrium for this economy. Be sure to write down the household's dynamic programming problem, and the expressions for wage and capital rental rate as functions of state variables.

2. **A Continuous Time Consumption-Saving Model with a CARA Utility and Labor Income Risks**

Consider an infinite-horizon continuous-time stochastic consumption-saving model. The representative household has a standard dynamic utility function

$$E_0 \left[\int_0^\infty \exp(-\rho t) \, u(c(t)) \, dt \right],$$

where $\rho > 0$. The period utility function takes an exponential functional form

$$u(c(t)) = -\frac{1}{\theta} \exp(-\theta c(t)),$$

where $\theta > 0$.
The budget constraint at time t takes a standard form:

$$c(t) + \dot{k}(t) = rk(t) + w(t),$$

where $r > 0$ is a constant interest rate on $k(t)$, and $w(t)$ denotes wage income at time t. The wage $w(t)$ follows a stochastic process defined by

$$w(t) = \exp(A(t)),$$

where $A(t)$ is a stochastic shock to wage income at time t. We assume that $A(t)$ follows an OU (Ornstein-Uhlenbeck) Process

$$dA(t) = \kappa (\gamma - A(t)) \, dt + \sigma dW(t).$$

At the beginning of $t = 0$, $k(0) > 0$ is given, and the household observes $A(0)$ before making decisions.

(a) Write down the Hamilton-Jacobi-Bellman (HJB) Equation in the stochastic dynamic programming formulation of the household.

(b) From the HJB Equation, derive the Euler Equation for this problem as a function of c and k.

(c) For this part, let $A(t)$ equal to a constant A so that we are in a deterministic environment. From the Method of Undetermined Coefficient, solve the optimal choice of $c(t)$ and $\dot{k}(t)$ as a function of $k(t)$.

3. **A Pure-Exchange Economy**

Consider a pure-exchange economy with infinitely-lived consumers. Consumer i, $i = 1, 2$, is endowed with ω_t^i units of a nonstorable consumption good in period t, $t = 0, 1, \ldots$, and has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i,$$

where $0 < \beta < 1$. The sequences of endowments are

$$(\omega_0^1, \omega_1^1, \omega_2^1, \omega_3^1, ...) = (5, 4, 5, 4, \ldots)$$
$$(\omega_0^2, \omega_1^2, \omega_2^2, \omega_3^2, ...) = (4, 5, 4, 5, \ldots).$$

(a) Define an Arrow–Debreu (time-0 trading) equilibrium.

(b) Characterize the solutions to the consumers' problems.

(c) Calculate the Arrow–Debreu equilibrium.

(d) Prove that the equilibrium allocation is Pareto-efficient.

4. **Asset Pricing**

Consider the following version of the Lucas tree model. In each period t, $t = 0, 1, \ldots$, there is a continuum of possible events. Let s_t denote the event that occurs in period t, let $s^t = (s_0, s_1, \ldots, s_t)$ denote the state in period t, and let $\pi(s^t)$ denote the p.d.f. over states. The initial state is given. There is measure one of identical consumers, each with expected utility function

$$\sum_{t=0}^{\infty} \int_{s^t} \beta^t \log(c(s^t))\pi(s^t)ds^t,$$

where $0 < \beta < 1$. There is a tree that produces $a(s^t)$ units of nonstorable fruit in state s^t. Each consumer is initially endowed with one share of the tree. Suppose that the amount of fruit evolves stochastically according to

$$a(s^{t+1}) = a(s^t)e^{\varepsilon(s_{t+1})},$$

where $\varepsilon(s_{t+1})$ is i.i.d. according to a normal distribution with mean μ, standard deviation σ, and p.d.f. $f(\varepsilon)$.

(a) Define a sequential markets equilibrium. Include Arrow securities.

(b) Characterize the solutions to the consumers' problems. Specify the Euler equations for the tree and the Arrow securities.

(c) Define a recursive competitive equilibrium. (Be sure to include both individual and aggregate state variables.) Do not include Arrow securities, but briefly explain why they are not needed for complete markets here.

(d) Calculate the price of the tree. How does the price depend on μ and σ?