Cooperative Ph.D. Program in the School of Economic Sciences and Finance

QUANTITATIVE METHODS QUALIFYING EXAMINATION

June 2, 2025
8:45 a.m. to 1:00 p.m.

THERE ARE SIX QUESTIONS –ANSWER ANY **TWO** QUESTIONS NUMBERED 1-3 AND ANY **TWO** QUESTIONS NUMBERED 4-6. IF YOU ANSWER ALL THREE IN EITHER GROUP, ONLY THE FIRST TWO WILL BE GRADED, THERE WILL BE A TOTAL OF FOUR QUESTIONS YOU NEED TO ANSWER, TWO OF QUESTIONS 1-3; AND TWO OF QUESTIONS 4-6.

- You **must** complete the examination within four hours. You will have 15 minutes to read over the questions before starting (8:45-9:00).
- This exam is closed book. Calculators and paper will be provided.
- Read the question carefully. Allocate your time carefully. Parts within questions will often vary in difficulty and weight. Be sure to do all parts of each question chosen.
- If necessary, it is permissible to make clarifying assumptions, but be sure to label them explicitly. (Grades will not take unstated assumptions for granted.) Also, label graphs and define notation.
- Number your answer sheets consecutively. Begin your answer to each question on a new page and identify the questions number.
- Leave ½-1” spacing around the edges of your paper.

Please write your exam ID number on the top of this page.

Statistics

Q1. You are interested in analyzing customer arrivals at a large bank in downtown Seattle during the lunch hour. It is known that the number of arrivals that occur adhere to a *Poisson Process*, where some of the characteristics of the Poisson probability density function (PDF) are given as follows:

$$\textit{Parameterization}: \ \lambda \in \Omega = \{\lambda : \lambda > 0\}$$

$$\textit{Density Definition}: \ f(x;\lambda) = \begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!} & \text{for } x = 0,1,2,..., \\ 0 & \text{otherwise} \end{cases}$$

$$\textit{MGF}: \ M_X(t) = \exp\left(\lambda\left(e^{t}-1\right)\right)$$

a. Use the moment generating function (MGF) to derive the mean and the variance of the Poisson PDF.

b. Suppose you have an *iid* random sample outcome of size *n*, $\mathbf{x} = \{x_1, x_2, \cdots, x_n\}$, that indicates the number of customer arrivals during lunch hours at the bank. You want to use that random sample outcome to estimate both the mean number of arrivals and the variance of arrivals.

i. Define the Best Linear Unbiased Estimator (BLUE) of the mean number of arrivals, and justify that the estimator is, in fact, BLUE.

ii. Can you define a BLUE for the *variance* of arrivals? If so, what is it, and why is it BLUE?

c. Rather than "only" BLUE, suppose you want to use the MVUE for estimating both the mean and the variance of arrivals, so the estimator would be minimum variance and unbiased among *all* estimators, linear or not. Define the MVUE for the mean and the variance if you can, and if so, justify that the estimator is, in fact, the MVUE.

d. For all of the BLUE and MVUE estimators that you were able to define in b. and c. above, discuss their asymptotic properties. Specifically, identify whether the estimators converge in mean square, are consistent, and whether they are asymptotically normally distributed. Justify your answers.

e. Your random sample outcome is such that $\sum_{i=1}^{100} x_i = 5000$. Provide BLUE and/or MVUE estimates for the mean and variance of customer arrivals during the lunch hour at the bank.

Q2. The daily quantity of a commodity that can be produced using a certain type of production technology is given by the outcome of the random variable Q, defined as $Q = 10x_1^{.25}\,x_2^{.5}\,V$, where Q is measured in tons/day, x_1 represents units of labor per day, x_2 represents units of capital per day, and $V \sim \theta^{-1}\exp\left(-\frac{v}{\theta}\right) I_{(0,\infty)}(v)$. Note that $M_V(t) = (1-\theta t)^{-1}$ *for* $t < \theta^{-1}$.

a. Define the expected value of daily production. Define the variance of daily production.

b. What are the numerical values of the mean and the variance of daily production if labor and capital are applied at the levels $x_1 = 16$ and $x_2 = 9$, and $\theta = 1$?

c. At the value of labor, capital, and θ given in b. above, what is the probability that daily production will exceed 60 tons/day?

d. The above production technology is used in 1000 plants in a large Eastern European country. The economy is centrally planned in that country, and all of the plants are required to use labor and capital at the levels $x_1 = 16$ and $x_2 = 9$. The plants operate independently, and outputs per day can be assumed to be statistically independent and identically distributed. Assume that $\sigma_V^2 = 1$. An economist says that the *aggregate* daily production function in the country, given by

$$Q^* = \sum_{i=1}^{1000} Q_i = 10x_1^{.25}\,x_2^{.75} \sum_{i=1}^{1000} V_i,$$

is such that aggregate daily production, Q^*, can be considered to be approximately *normally* distributed. Do you agree? Why or why not?

e. What is the expected value of aggregate daily production in the country, given the aggregate production function described in d. above? What is the variance of daily aggregate production?

Q3. The "Ridge Regression (RR) Estimator" of the scalar parameter β contained in the linear model $\mathbf{Y} = \mathbf{x}\beta + \boldsymbol{\varepsilon}$ is defined as $\hat{\beta} = (\mathbf{x}'\mathbf{x} + k)^{-1}\mathbf{x}'\mathbf{Y}$, where $\mathbf{x}$ is an $n \times 1$ vector of explanatory variable values, $\mathbf{Y}$ is an $n \times 1$ random dependent variable vector, $\boldsymbol{\varepsilon}$ is a vector of residuals, and k is a scalar chosen so that $k \in [0, \infty)$. Assume all of the classical general linear model assumptions apply, and also assume that $\varepsilon \sim N(0, \sigma^2 I)$.

a) What is the expected value of the RR estimator? Is there a choice of k for which the RR estimator is the BLUE for β? If so, what is that choice and why?

b) What is the covariance matrix of the RR estimator? For the RR estimator set up as completely as possible a finite sample Wald statistic to test the null hypothesis H_0: $\beta = 0$. Define the alternative hypothesis, the test statistic, and critical values, along with its distribution under both the null and alternative.

c) Is the finite sample probability distribution of the RR estimator for β the same as the probability distribution for the OLS estimator for β? Explain and be specific.

d) Is the RR estimator a consistent estimator of β? Yes or No. Justify your answer.

e) If you are using mean squared error, $E(\hat{\beta} - \beta)^2$, to measure estimator performance, is it possible for the RR estimator to perform better than the MVUE estimator for estimating β? If so, how would one determine for what values of k this is true?

f) Now $\min_{\beta}(y - x\beta)'(y - x\beta)$ subject to $M \geq \sum_{j=1}^{p} \beta_j^2$ for some positive, finite number M. Is the resulting estimator of β related to the RR and OLS estimators of β? Be specific.

Econometrics

Q4. Suppose that you hypothesize a general linear regression model (GLM) with an autoregressive process of order 1 (AR1)

$y_t = x_t\boldsymbol{\beta} + \varepsilon_t$, where $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$

with $u_t \sim N(0, {\sigma_u}^2 I)$. Be as specific and complete as possible in answering the questions below.

a) Define the likelihood function under normality of the GLM **without** the AR1 process in the residuals. Derive the estimator of $\boldsymbol{\beta}$ and estimator of cov($\widehat{\boldsymbol{\beta}}$).

b) Does maximum likelihood (ML) under normality of the GLM **without** the AR1 process in part a) provide an unbiased and consistent estimator of $\boldsymbol{\beta}$? Explain.

c) Does maximum likelihood under normality of the GLM **without** the AR1 process in part a) provide an unbiased and consistent estimator of cov ($\widehat{\boldsymbol{\beta}}$)? Explain.

d) Now, how would you estimate ρ? Explain.

e) Assuming that you find ρ to be significantly different from zero, specify and derive a generalized maximum likelihood (GML) estimator for β? Be explicit. Is the maximum likelihood estimator $\widehat{\boldsymbol{\beta}}_{ML}$ in part a) identical to $\widehat{\boldsymbol{\beta}}_{GML}$? Explain. Compare and contrast the properties of the two estimators in small and large sample settings.

f) How would you setup and test statistical significance of ρ, using an asymptotic level α likelihood ratio test? State the null and alternative hypotheses, define the test statistic, and critical values, along with its distribution under both the null and alternative. Be sure to state any assumptions you need to answer this question.

Q5. Suppose you are interested in estimating the relationship:

$$Y_i = \beta_0 + \beta_1 \cdot X_i + \varepsilon_i$$

Unfortunately for your sample with N observations of Y, you only observe a subset N* of the variables X, with M* corresponding to the number of observations with X missing (N = M* + N*). Further, the likelihood X is missing is **strictly increasing** in an unobserved variable W $\frac{dPr(X\ missing)}{dW} > 0$, where W has the following characteristics $Cov(X, W) > 0$, $Cov(W, \varepsilon) \neq 0$.

i) Given $Cov(X, W) > 0$ and $\frac{dPr(X\ missing)}{dW} > 0$, can you say anything about the mean of the missing X values compared to the mean of the X values observed in the data? What can you say about how the unobserved variable W may differ between observed and unobserved X values?

ii) Can you obtain an unbiased estimate of β_1 using OLS for the N* subsample where both X and Y are observed? Prove your answer.

iii) In order to utilize the full sample of size N, suppose you first construct an indicator variable $Q \in \{0,1\}$, where Q_i takes a value of one if X_i is missing, and zero if observed. Next, you construct a new variable $\tilde{X}_i = (1 - Q_i) \cdot X_i$, taking the value of X_i when it is observed and taking the value of zero when X_i is missing. Using these two new variables you intend to estimate the following:

$$Y_i = \beta_0 + \beta_1 \cdot \tilde{X}_i + \beta_2 \cdot Q_i + \varepsilon_i$$

Can you obtain an unbiased estimate of β_1 using OLS for the full sample N in this case? Prove your answer.

iv) Discuss the interpretation of β_2? Does the statistical significance of β_2 (evidence for whether or not $\beta_2 = 0$) tell us anything about the validity of estimating β_1 in this setting? Explain your reasoning, you do not need to provide a proof.

Q6: Suppose you observe results from an ordinary least squares regression model examining individual hourly wages, which was estimated using cross-sectional data.

$$\widehat{W_i} = \underset{(2.1)}{8.1} + \underset{(0.3)}{1.8}Age_i - \underset{(0.01)}{0.02}Age_i^2 + \underset{(1.9)}{7.2} \cdot College_i - \underset{(1.3)}{3.7} \cdot Female_i + \underset{(0.2)}{1.1} \cdot College_i \cdot Female_i$$

$$N=2{,}000 \qquad \overline{R^2}=0.47$$

Where W_i is the hourly wage ($ per hour), Age_i is the age of the ith individual, $Female_i$ is a dummy variable that takes a value of one if the ith individual identifies as female, and $College_i$ is a dummy variable that takes a value of one if the ith individual has a college degree.

i) Evaluate the equation. How do the adjusted $\overline{R^2}$ and the signs and significance of the coefficients compare with your expectations?

ii) Derive the marginal effect of age on wage and carefully interpret using the information provided. At what age do you expect individuals to maximize their hourly wage?

iii) Using the information provided, define and set up a Wald test for whether age matters for determining wage. Assume normally distributed i.i.d. error terms. Specify the significance level of the test.

iv) Carefully interpret the impact of the dummy variable $College_i$ on the hourly wage. In addition, what does the sign of the interaction term indicate?

v) Fully explain how you would test for heteroskedasticity in the residuals of the above model.

Cumulative Standard Normal Distribution (Table entry is $\Phi(z) = \text{Prob}[Z \le z]$)

z	0.0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.4878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Right-Hand Tail Critical Values for the Student t-distribution

	<-------------------- alpha -------------------->									
df	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	1.376	1.963	3.078	6.314	12.706	31.821	63.656	127.321	318.289	636.578
2	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.089	22.328	31.600
3	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
31	0.853	1.054	1.309	1.696	2.040	2.453	2.744	3.022	3.375	3.633
32	0.853	1.054	1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
33	0.853	1.053	1.308	1.692	2.035	2.445	2.733	3.008	3.356	3.611
34	0.852	1.052	1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
35	0.852	1.052	1.306	1.690	2.030	2.438	2.724	2.996	3.340	3.591
36	0.852	1.052	1.306	1.688	2.028	2.434	2.719	2.990	3.333	3.582
37	0.851	1.051	1.305	1.687	2.026	2.431	2.715	2.985	3.326	3.574
38	0.851	1.051	1.304	1.686	2.024	2.429	2.712	2.980	3.319	3.566
39	0.851	1.050	1.304	1.685	2.023	2.426	2.708	2.976	3.313	3.558
40	0.851	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
50	0.849	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.848	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
80	0.846	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
100	0.845	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
150	0.844	1.040	1.287	1.655	1.976	2.351	2.609	2.849	3.145	3.357
Infinity	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.290

F Table for alpha=.05 Prob(F>entry)=0.05

	df num. -->																		
df den.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**12**	**15**	**20**	**24**	**30**	**40**	**60**	**120**	**INF**
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.277	9.117	9.014	8.941	8.887	8.845	8.812	8.786	8.745	8.703	8.660	8.639	8.617	8.594	8.572	8.549	8.526
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.912	5.858	5.803	5.774	5.746	5.717	5.688	5.658	5.628
5	6.608	5.786	5.410	5.192	5.050	4.950	4.876	4.818	4.773	4.735	4.678	4.619	4.558	4.527	4.496	4.464	4.431	4.399	4.365
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.000	3.938	3.874	3.842	3.808	3.774	3.740	3.705	3.669
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.575	3.511	3.445	3.411	3.376	3.340	3.304	3.267	3.230
8	5.318	4.459	4.066	3.838	3.688	3.581	3.501	3.438	3.388	3.347	3.284	3.218	3.150	3.115	3.079	3.043	3.005	2.967	2.928
9	5.117	4.257	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.073	3.006	2.937	2.901	2.864	2.826	2.787	2.748	2.707
10	4.965	4.103	3.708	3.478	3.326	3.217	3.136	3.072	3.020	2.978	2.913	2.845	2.774	2.737	2.700	2.661	2.621	2.580	2.538
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.788	2.719	2.646	2.609	2.571	2.531	2.490	2.448	2.405
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.687	2.617	2.544	2.506	2.466	2.426	2.384	2.341	2.296
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.604	2.533	2.459	2.420	2.380	2.339	2.297	2.252	2.206
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.534	2.463	2.388	2.349	2.308	2.266	2.223	2.178	2.131
15	4.543	3.682	3.287	3.056	2.901	2.791	2.707	2.641	2.588	2.544	2.475	2.403	2.328	2.288	2.247	2.204	2.160	2.114	2.066
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.425	2.352	2.276	2.235	2.194	2.151	2.106	2.059	2.010
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450	2.381	2.308	2.230	2.190	2.148	2.104	2.058	2.011	1.960
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.342	2.269	2.191	2.150	2.107	2.063	2.017	1.968	1.917
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.308	2.234	2.156	2.114	2.071	2.026	1.980	1.930	1.878
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.278	2.203	2.124	2.083	2.039	1.994	1.946	1.896	1.843
21	4.325	3.467	3.073	2.840	2.685	2.573	2.488	2.421	2.366	2.321	2.250	2.176	2.096	2.054	2.010	1.965	1.917	1.866	1.812
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297	2.226	2.151	2.071	2.028	1.984	1.938	1.889	1.838	1.783
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275	2.204	2.128	2.048	2.005	1.961	1.914	1.865	1.813	1.757
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255	2.183	2.108	2.027	1.984	1.939	1.892	1.842	1.790	1.733
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.237	2.165	2.089	2.008	1.964	1.919	1.872	1.822	1.768	1.711
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.266	2.220	2.148	2.072	1.990	1.946	1.901	1.853	1.803	1.749	1.691
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204	2.132	2.056	1.974	1.930	1.884	1.836	1.785	1.731	1.672
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190	2.118	2.041	1.959	1.915	1.869	1.820	1.769	1.714	1.654
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177	2.105	2.028	1.945	1.901	1.854	1.806	1.754	1.698	1.638
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165	2.092	2.015	1.932	1.887	1.841	1.792	1.740	1.684	1.622
40	4.085	3.232	2.839	2.606	2.450	2.336	2.249	2.180	2.124	2.077	2.004	1.925	1.839	1.793	1.744	1.693	1.637	1.577	1.509
60	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097	2.040	1.993	1.917	1.836	1.748	1.700	1.649	1.594	1.534	1.467	1.389
120	3.920	3.072	2.680	2.447	2.290	2.175	2.087	2.016	1.959	1.911	1.834	1.751	1.659	1.608	1.554	1.495	1.429	1.352	1.254
inf	3.842	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.880	1.831	1.752	1.666	1.571	1.517	1.459	1.394	1.318	1.221	1.000